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Mining Subsidence Engineering

Translated by R. F. S. Fleming

With 380 Figures

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Preface

This book originally appeared in German in 1974, under the title “Bergschadenkunde” (mining subsidence engineering), and then in Russian in 1978, published by Nedra of Moscow. When the German edition was almost out of print, Springer-Verlag decided to bring out a new edition, this time in English. For this English version the text has been thoroughly revised, enlarged, and supplemented by over 100 new figures.

The book deals with the current state of international knowledge on strata and ground movement over mine workings, with its damaging effects on mine shafts and the land surface, and with measures for regulating mining damage in law and reducing it in practice. Discussion begins with the mine excavation underground – the cause – and ends with the damage to surface structures – the effect. Methods of roof control, including the subject of rock bursts, are not discussed, since that is a field concerned more with the safety of underground workings than with minimizing damage at the surface. Of the 500 literature references in the German edition, only the more important for an international readership have been retained, but no value judgement on the many publications not mentioned should be read into this.

The book is principally intended as a working aid for the mine surveyor, the mining engineer, the architect, and the civil engineer. For the student and the post-graduate researcher, it offers a summary and guide to this whole field of knowledge. In addition, it should provide a rapid survey of this special field for those who are temporarily concerned with mining damage – in law, for example, or in town planning, or as landowners.

To translate the German text for this English edition, the publishers were able to secure the services of Mr. Richard F. S. Fleming in London, who, as a well-known translator for publishers of technical journals and books in the mining field, was particularly well qualified to provide an accurate English rendering both of the specialized mining terms and of the difficult technical discussion. Thus, in two years of conscientious translating, an English technical manual has been produced which faithfully reflects the now revised and enlarged German original. For this, and for his close collaboration, I am greatly indebted to Mr. Fleming. The new figures have again been drawn by Mr. Horst

Conrad, who also designed the cover. Many colleagues and many technical experts abroad, above all in Poland and Britain, have given me their advice and assistance on special aspects. I thank them all, and not least the publishers, who have all along advanced the project with great energy, for their valuable assistance.

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Helmut Kratzsch

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Introduction

Wherever minerals are won by underground mining extending over any significant area, the overlying rock mass subsides into the underground cavities opened up by mining, and the upper surface of the ground subsides correspondingly, forming hollows and trenches, open cracks in the earth, abrupt steps, and extensive subsidence troughs. Ground can sink vertically or be displaced horizontally – or both – by as much as several metres. Since the middle of last century, and particularly over the coalfields of Europe, ground movements on this scale have led to severe damage to buildings, communications, and agriculture, for which the aggrieved land and property owners have demanded compensation from the mine operators and, when necessary, have pressed their claims in the courts. To be in a position to present an effective defence against unjustified claims, mine surveyors have since the beginning of this century made numerous measurements of underground excavations and observed ground movements with extreme care. From the experience thereby gathered, and the theoretical conclusions drawn on how ground movements develop, there has gradually been evolved a new branch of science and technology – *mining subsidence engineering* – which has been taught in German mining academies since 1931.

At first all that was required was a knowledge of the extent of surface damage and the duration of surface influence of a mine working, together with the ability to attribute an appropriate share of costs to neighbouring mines, which very often were jointly responsible for the damage; but with increasing mining activity underground and increasing residential development on the surface, it became necessary, both to the mine operator and to the surface developer, to do everything possible to minimize mining damage. Today therefore, it is the *task of subsidence engineering* to develop procedures for –

- a) predicting strata and ground movements over mine workings;
- b) ascertaining the effects of such movements on building structures, mine shafts, etc.;
- c) minimizing subsidence damage by means of improvements in mining, protection of structures, regional planning etc.

From this description of its role, it follows that mining subsidence engineering, although certainly to be counted among the earth-engineering sciences, takes in not only the study of ground movements and rock structure but also areas bordering on the sphere of mine surveying, like mining law, property law, mining engineering, constructional engineering, rock and soil mechanics, communications engineering, agriculture, hydrology, town planning, etc. (see Table 1).

Table 1. Division of the earth-engineering sciences as between construction and mining

Geomechanics (earth engineering)			
Construction (foundations, embankments, tunnels)		Mining (underground and opencast)	
Soil mechanics (loose ground)	Rock mechanics (firm rock)	Strata control (in vicinity of workings)	Subsidence engineering (remote from workings)

With the increasing depth of mining, its impact began to be felt even by the numerous mine structures, and especially by *mine shafts*; and so steps had to be taken to protect shafts and other important installations in mines from the damaging effects of a mine’s own excavations. Since the 1930s therefore, the planning of mining procedures near the shaft and of safety measures at the shaft itself, both designed to protect the structure from mining damage, were added to the responsibilities of subsidence engineering. One result was that a planned and economic recovery from among the large reserves of unworked mineral locked up in shaft pillars became possible for the first time.

Finally, subsidence engineering also has a safety role. Its measures in relation to both mining and construction help to protect communications networks, public utilities installations, important public buildings and historic monuments within mining areas from damage which could impair their functioning or even render them dangerous.

Subsidence engineering can be differentiated from the closely related field of *strata control* in the following way. In strata control, what is chiefly studied is stress changes and rupture processes in the immediate vicinity of underground excavations, with the object of minimizing convergence in mine roadways, avoiding rock bursts, sacrificing as little mineral as possible in support pillars, and utilizing rock pressure in the dislodging of mineral for extraction. It is thus the interaction between solid rock and roof supports or pillars in circumstances of disturbed load equilibrium which is at the centre of consideration in strata control (i.e., a load model). In subsidence engineering, on the other hand, what is principally being investigated is the interaction between loose ground and structural foundations or shaft linings under the influence of strata movements at a distance from underground workings (a movement model). The inclusion of strata control in the sphere of subsidence engineering as “subsidence engineering underground”, which was still being advocated by O. Niemczyk in 1949 in his textbook on subsidence engineering, “*Bergschadenkunde*”, cannot be sustained today in view of the very different legal position and practical objectives – leaving aside the common objectives of minimizing costs and preventing accidents – of the two disciplines. In the discussion which follows therefore the only thematic distinction drawn is between strata movements, triggered off by mining, in the interior of a predominantly solid rock body stretching from workings to caprock and shaft side (Part I), and on the other hand, movements in the loose ground of the upper surface layer in which surface structures have their foundations (Part II).

At the centre of the subsidence engineering stage stand the thickly populated coal-mining areas of Europe. Their numerous coal seams, often metres thick, are today mined across extensive areas 300–1000 metres underground, in fronts of 200–300 m length, without supporting pillars. The result is that the rock mass overlying them drops like a sagging plate behind the advancing coal-faces, immediately after extraction, and breaks up only on reaching the floor of the workings. The rock strata overlying massive orebodies and salt deposits subside in a similar way when mined, except that parts of these deposits, left standing like islands, often retard the progress of subsidence. Even in opencast mining, significant settlement and consequent damage can occur in ground adjoining a mine as a result of a fall in the water-table. Thus the ground movements, visible damage, and protective measures so familiar in relation to coal-mining are basic attributes also of other branches of mining – even of oil and natural gas production, which can lower the surface of the ground by a metre or more. It is only in the working of vein ore occurring in strong country rock that mining damage is negligible.

In today's active competition between domestic mining and foreign sources of energy and minerals, subsidence engineering is of mounting importance. More than ever is it becoming necessary to hold down the costs occasioned by mining damage – which average, for example, 6–8% of the per-ton cost of coal production – and to ensure the workability or reserves even under built-up areas. Mining subsidence engineering thus has a positive contribution to make to securing a competitive domestic supply of raw materials.

Part I Strata Movement

1. Strata Movement at the Mining Horizon

1.1 Mining Methods, from the Standpoint of Subsidence

The field of subsidence engineering stretches from the firm rock at the mining horizon to the surface layer of loose ground on which buildings stand and farming is conducted. The cavity artificially created underground by the extraction of mineral removes the natural support from the overlying strata. As a result, successive layers of rock over the mine workings bend under the influence of gravity, until finally the movement reaches the upper earth surface. At the same time, the underground cavity is closed up to a greater or lesser degree. The extent of movement in the upper layers thus depends on the closing up, over a period of time, of this cavity. The size of the latter is consequently the basic dimension in the calculation of strata and ground movement. By contrast, convergence between roof and floor in mine roadways plays only a minor part in movements occurring in the upper layers of overlying rock, because of the insignificant width of roadways.

Before the evolution of movement at the mining horizon can be discussed, it is first necessary to consider the various methods of winning mineral and to clarify certain mining concepts¹.

For the purposes of subsidence engineering, *mining methods* can best be classified according to the treatment of the roof and the degree of filling in mine excavations. By “roof” is meant the surface forming the upper limit of the mine excavation.

Roof treatment (control) can be differentiated into three categories (see Table 2):

a) Supporting the roof with pillars (stoping with *permanent pillars*). Parts of the deposit are left standing between the working fronts (the “stopes”) to guard against movement of the overlying rock during mining, either because the rock is difficult to contain, or because it is water-bearing. This procedure of partial extraction is chiefly employed in mining massive deposits of salt minerals or metallic ores in firm wall rock. Long, parallel “pillars” are formed in the deposit by driving entry roads through it, which are then widened into “chambers” (chamber working, Fig. 1). In the case of high-value minerals, in order to limit losses of material to between 30% and 50%, only small, square pillars are left standing, by widening the entry roads into “rooms” or “stalls” and mining in strips on a grid pattern (*room-and-pillar working*, Fig. 2).

¹ For geological concepts, see Figs. 35 and 48

Table 2. Classification of mining methods

Manner of working	Treatment of roof		
	With permanent pillars (partial extraction)	With filling ^a	With caving (total extraction)
In long fronts		Longwall mining (Fig. 3) Rill stoping Overhand stoping Underhand stoping	Longwall caving (Fig. 5)
In chambers	Chamber workings (Fig. 1) Room or stall workings (Fig. 2) Pillared open stopes	Room and temporary pillar Sublevel stoping (Fig. 4)	Chamber working with caving Room-and-pillar with caving (Fig. 6) Open-stope caving (Fig. 7) Sublevel caving Block caving (Fig. 8)
In blocks		Bench stoping Overhand shrinkage stoping Cross-cut stoping	
In single faces			Cross-cut stoping with caving

^a In each case, with either simultaneous or subsequent filling ("stowing" in coal-mining)

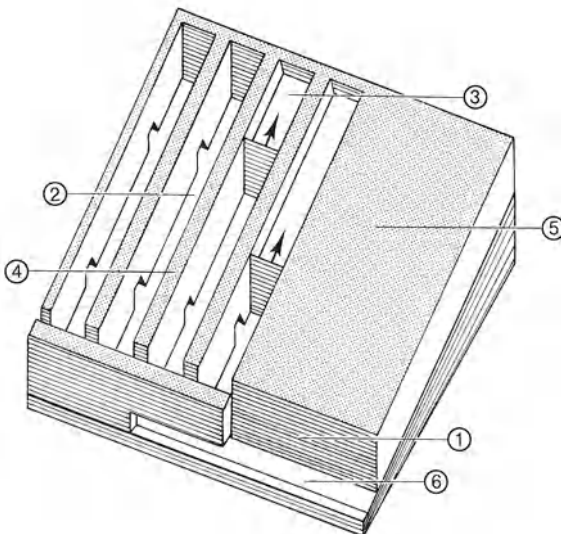


Fig. 1. Mining a massive deposit 1, in chambers 2, by breast-and-bench stoping 3, between permanent pillars 4. 5 Fore-field (zone in front of face); 6 Haulage roadway

The pillars between chambers, rooms or stalls remain as a permanent and natural support for the roof strata, but they are liable to become gradually crushed later on by the weight of overlying rock. In salt-mining, for example, the final shape of the trough of subsidence at the surface often does not become apparent

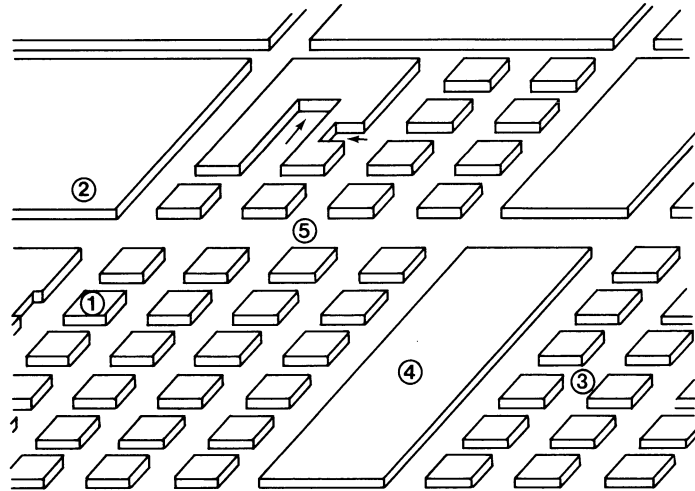


Fig. 2. Mining a flat-lying deposit 2, by room 3 and regular, permanent pillar 1 working. 4 Barrier pillar; 5 main roadway

for a hundred years, i.e. not until the pillars of salt have finally been completely compressed. The extent to which the underground cavities have by then been filled depends on the volume of the pillars and chambers respectively. With room-and-pillar workings therefore, the subsidence engineer must bear in mind the long-term deformation behaviour, or rheology, of the rock mass, i.e. the time factor in relation to failure and flow criteria.

In a certain sense, the extraction of petroleum and natural gas from porous reservoir rocks can be regarded as “pillar working”, because their supporting granular framework is left intact. Furthermore brining, which leaves undissolved supporting ribs of salt behind, can be included in this category.

b) Lowering the roof on to fresh support (*stowing*). In this procedure, the mine excavation is filled with crushed rock or other waste material to support the roof artificially in good time as it goes down, so that it will settle almost without a break. The stowing material will “give”, but its support reduces the degree of sagging and of opening-up in the bonds between overlying layers. This technique is known as stowing and can be applied to room-and-pillar, longwall, and short-wall workings. For example, in mining extensive but relatively thin coal-seams by the longwall method, waste fill is introduced into the resulting 1–3 m high excavation as a supporting bed for the roof. This procedure is also followed in some other mining methods (see Table 2). The degree of filling achieved is around 50%, but with hydraulic stowing it can be as much as 80% or 90%.

Longwall mining (Fig. 3) is the method of total extraction principally employed in the European coal-mining industry. The “*long wall*” is the working face, which may be anything up to 300 m long and, depending on mining progress, may advance laterally towards the mine boundary by several metres a day. The narrow open strip, or “face working”, between the “goaf” – the mined-out seam – and the coal-face is protected against roof falls by an array of vertical

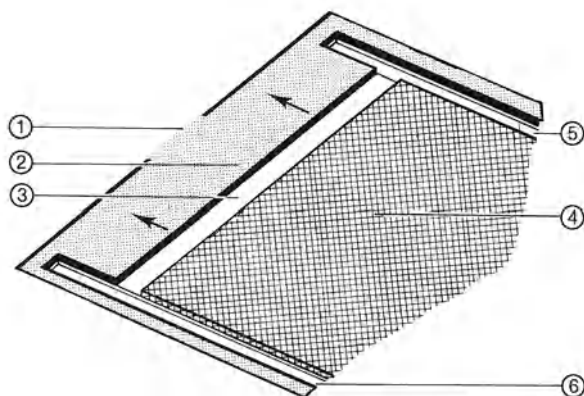


Fig. 3. Longwall mining of a flat-lying seam in direction of strike, with stowing or caving. 1 Forefield in seam (zone in front of working face); 2 extraction front; 3 face working, or face area; 4 mined-out area; 5 top road; 6 bottom road with conveyor belt

props capped with horizontal bars, or by composite supports having broader roof canopies. Mineral is both won from the face and transferred to the conveyor mechanically.

The newly mined and abandoned strip behind the face working must be filled immediately, and when mining makes good progress, perhaps advancing by as much as 5 m a day, this can present technical problems. The usual solution today is to employ pneumatic stowing, blowing in waste through special pipes. The "gate roads" leading on either side of the goaf to each end of the working face — the "top" and "bottom" road respectively — serve for ventilation, haulage, and manriding. In steeply dipping seams the working face is set at an angle to the gate roads to lessen the gradient.

Longwall mining is thus characterized by the long working front, the large area worked at a time, and the rapid extraction of the seam. The disturbed rock mass comes to rest again within 1 to 3 years of mining, leaving a flat trough at the surface.

Steep-dipping evaporite deposits are mined by sublevel stoping (Fig. 4). Intermediate ("sub") levels divide a deposit into horizontal slices, which are then blasted down from the bottom up, on an angled front. The steep, mined-out cavities thus formed are filled with residues from the related potash or other salt plant.

c) Caving the roof. In the *caving method*, the roof of the mined-out and abandoned working area is systematically caused to collapse, in order to avoid leaving large cavities with projecting slabs which could suddenly fall and endanger further working. The plan is that the immediate roof layer breaks off in large lumps and fills the mine excavation as a heap of rubble — a so-called self-stowing medium which provides a yielding underlay for the main mass of overlying rock. The latter then sags almost unbroken on to this and settles without subsidence delay directly behind the working front (see Fig. 5).

Because the immediate roof layers increase in bulk as they break up, the main body of overlying rock settling into the mined area subsides by only between 70% and 95% of the height M of the excavated cavity. The first segment of the roof ("I" in the Figure) collapses when the span of the open face-working reaches between 10 and 50 m.

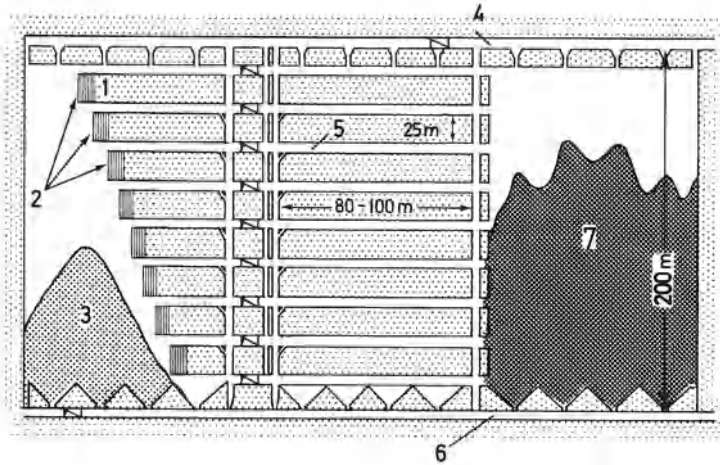


Fig. 4. Mining a steeply dipping, massive orebody or salt deposit by sublevel stoping, with filling (shown in vertical section). 1 Horizontal slices of deposit; 2 drill holes; 3 broken ore/salt brought down by shot-firing; 4 upper level (for bringing in waste fill); 5 sublevels; 6 main haulage level; 7 mined-out and filled cavity

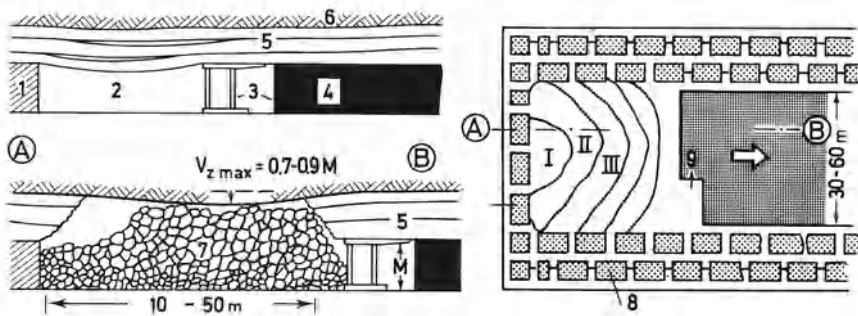


Fig. 5. Mining with caving, illustrated as a shortwall face (in plan on right, and in vertical section A-B, twice, on left). 1 and 8 barrier pillar; 2 open face-working; 3 coal-face and roof support; 4 seam; 5 immediate roof layers; 6 main roof layer, or roof rock; 7 roof layers, caved; 9 coal-getting with shuttle-car miner; I, II, III roof zones of progressive caving. (After L. J. Thomas, 1978)

Longwall mining comes within the category of caving methods if the overlying rock is such that it settles easily, and so also does room-and-pillar working in cases where, to save the cost of stowing, or to avoid sacrificing a proportion of the deposit, the pillars of mineral left standing between workings are ultimately mined. Figure 6 depicts a conventional procedure in U.S. coalmining, in which the pillars of coal, starting from the edges of the working panel, are systematically “robbed” without loss of mineral.

Two further methods which belong in this category are open-stope caving and block caving, both practised in ore mining. The first (Fig. 7), adopted in steep-dipping deposits having a certain minimum strength of wall rock, provides for an underground excavation, or “open stope”, over 15 m high, from which the

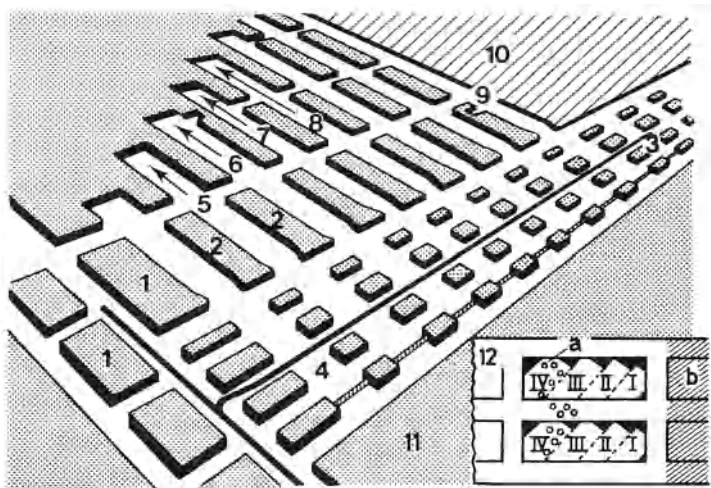


Fig. 6. Room-and-pillar mining, with caving. 1 Permanent pillar for main entry road; 2 temporary pillars; 3 entry road; 4 section conveyor belt; 5 undercutting; 6 face drilling; 7 blasting; 8 coal loading; 9 pillar recovery; 10 caved area; 11 coal-seam; 12 pillar recovery in slices I to IV; 12a props; 12b caved. (After L. J. Thomas, 1978)

mineral is worked laterally by caving at several sublevels, to be collected at a lower level for hauling.

In block caving (Fig. 8), a massive orebody of considerable height is freed from the country rock surrounding its vertical sides, is undercut across its entire base, and is then brought down in sections from lower levels by shot-firing and simple gravity.

This is similar to the procedure followed, with comparatively thin horizontal slices, in sublevel caving. In the latter method, as caving progresses upwards to the roof – the upper floor pillar – the waste rock, or “goaf”, and the main body of overlying strata follow the collapsing ore down. Troughs or trenches form at the upper earth surface.

Certain of the mining methods listed in Table 2, and so far unmentioned, proceed by the advancing of horizontal faces, or “breasts”, in single strips one to three times the width of the average drift, either alongside or over each other. These methods generally presuppose a steep-dipping and competent orebody, such as a vein deposit in strong wall rock. There is usually no noticeable strata movement, because the excavations made are so narrow. These methods are consequently of little concern in subsidence engineering.

Figure 9 brings out the spatial relationship between underground workings and surface installations in a diagrammatic representation of a coal-mine. Inclined coal-seams are opened up by means of vertical shafts and horizontal entries, providing mining levels at vertical intervals of 100 to 200 m. “Face” or “gate” roads and inclined raises give access from the entry roads to the working faces.

Fig. 7. Open-stope caving in a steeply dipping orebody. 1 Sub-levels; 2 open stope; 3 pillar; 4 caved goaf; 5 upper floor pillar; 6 haulage level; 7 ore in place; 8 wall rock. (After Dorstewitz, Fritsche, and Prause, 1959)

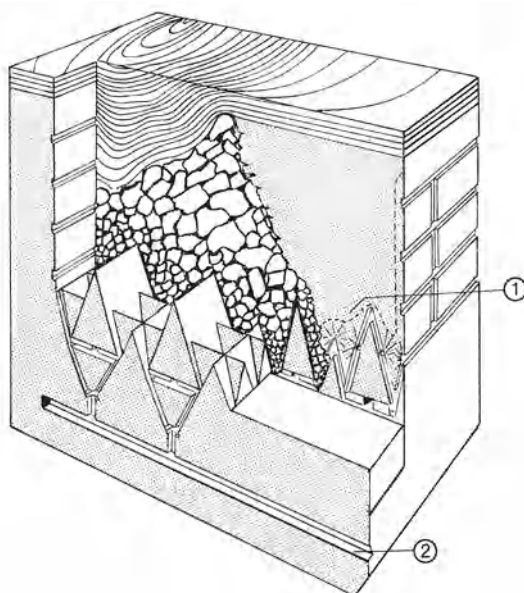
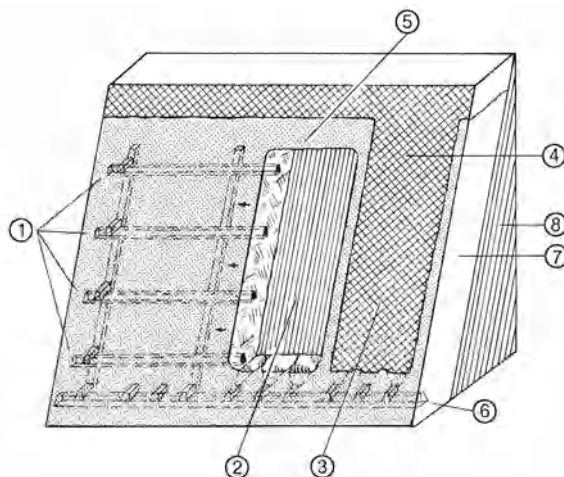


Fig. 8. Block caving. 1 Causing a section of the deposit to cave by shot-firing, after cutting round, and under, a large block. 2 Haulage level

1.2 Convergence in the Workings

It is a common element of all mining procedures that, over a greater or lesser area A , a body of rock is undermined at a depth h . This causes the undermined rock body, the weight force of which had previously borne evenly on the deposit being mined, to lose its firm support.

The vertical weight force G of this undermined rock body is the product of its mass m in kg and the acceleration of gravity $g = 9.81 \text{ m/s}^2$; that is to say

$$G = m g \quad (\text{in N}). \quad (1)$$

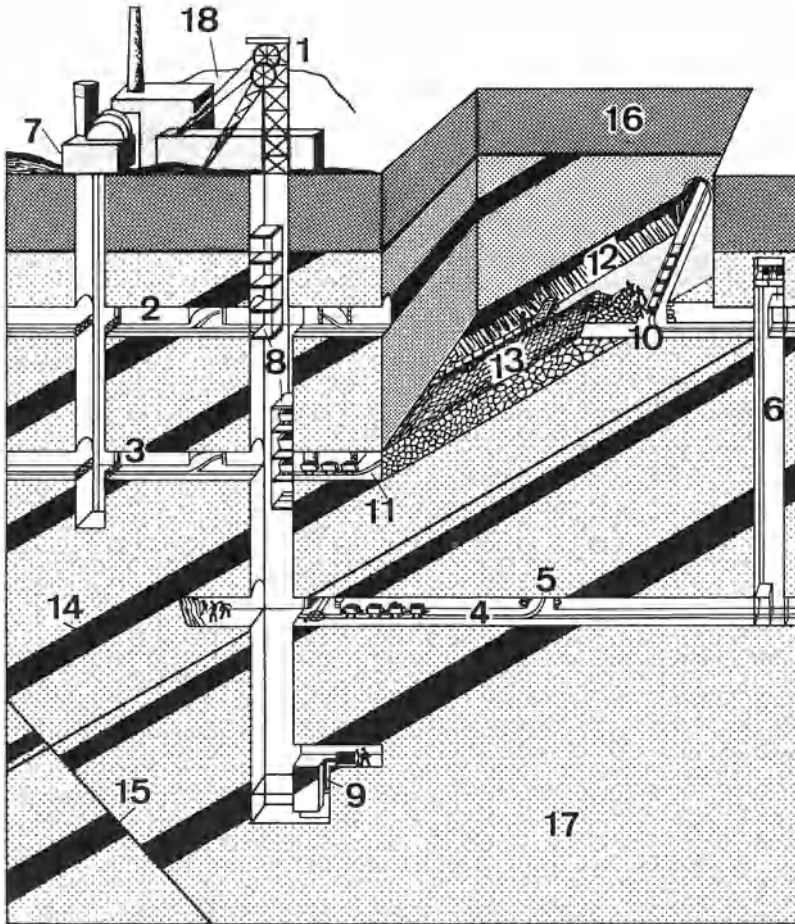


Fig. 9. Vertical section through a typical coal mine. 1 Winding tower over main shaft; 2 level station and crosscut, 1st level; 3 2nd level; 4 3rd level; 5 main-level road; 6 blind shaft from 2nd to 3rd level; 7 ventilation shaft with fan; 8 hoisting cage; 9 power station underground; 10 upper mining level; 11 base level; 12 coal face, and face area; 13 mined-out and stowed area; 14 inclined seam; 15 fault; 16 cap rock, overburden; 17 beds of the Carboniferous; 18 waste tips and coal tips

Given the mass of the rock per 1 m³ of its volume, i.e. given its density

$$\rho = m/V \quad (\text{in kg/m}^3) \quad (2)$$

the pressure exerted by a rectangular column of rock of height h (in m) on an area of $A = 1 \text{ m}^2$ can be determined:

$$p = h\rho g \quad (\text{in Pa}). \quad (3)$$

From the values $\gamma = \rho g = 25000 \text{ kg/m}^3 \times 9.81 \text{ m/s}^2 \approx 2500 \text{ kg/m}^2\text{s}^2$, there can be obtained, in terms of SI units of force

$$1 \text{ Newton} = 1 \text{ N} = 1 \text{ kg m/s}^2$$

and of pressure

$$1 \text{ Pascal} = 1 \text{ Pa} = 1 \text{ N/m}^2,$$

a rough formula for the normal pressure exerted by overlying rock at a depth h (in m), namely:

$$p = \gamma h \approx \frac{h}{40} \quad (\text{in MPa}) \quad (4)$$

with the specific weight force (gravity) $\gamma = G/V = 0.025 \text{ MN/m}^3$. For example, assuming the overlying rock pressure, derived from its own weight alone and disregarding the effect of mining, is approximately 20 MPa at a depth of 800 m, that implies $20 \text{ N/mm}^2 = 2000 \text{ N/cm}^2$ or – in earlier units – 200 kp/cm^2 (200 bar)².

The undermined rock body therefore has to be supported not only by the waste filling or caved waste in the goaf but also by the ribs and pillars. This is because the roof, which continues to be bound in parallel stratification to the rest of the rock mass, does not subside as an unattached block but bends like a thick board over the whole working area and is supported only in part by the goaf. The other part of the load is transferred by the rigid bonding of the strata to the rib-sides in the “forefield” – the zone in front of the face – or to the pillars, and a very high rock pressure results at the points concerned from a combination of cover-load pressure and abutment pressure. The new and uneven pressure distribution, with a peak over the edge of the forefield (A_1 in Fig. 10) and a zone in the goaf which is partially relieved of cover-load pressure (A_2 in Fig. 10), leads to perpendicular compression of the seam edge, increasing compaction of the loose fill or caved waste towards the centre of the workings, and heaving of the floor in the zone of lowered pressure. Table 3 sets out the interrelationship of these rock processes at the mining horizon, from the artificially created disequilibrium of forces in Stage 1, to the forces of deformation operating in the transitional phase, and the rock deformation resulting from the newly established equilibrium in Stage 2.

1.2.1 Mining Without Permanent Pillars (Total Extraction)

Taking coal-mining as a starting point, three *zones of movement* can be distinguished at the mining horizon of a longwall operation, divided from each other both in time and in place.

a) Convergence in the forefield. The immediate roof layer begins to subside in the forefield, some 30 – 100 m ahead of the working face. At two points, P_1 and P_2 , perpendicularly opposite each other in the roof and floor of a seam 1 – 3 m thick, are located at the edge of where subsidence begins, the distance between them will undergo a change as the face approaches and then reaches them – namely, a perpendicular contraction, or convergence c_0 , made up of roof settlement c_r and,

² Conversion: $1 \text{ kp/cm}^2 \approx 10 \text{ N/cm}^2 = 0.1 \text{ MPa} = 1 \text{ bar}$; $1 \text{ N/cm}^2 = 10 \text{ kPa} \approx 0.1 \text{ kp/cm}^2$;
 $1 \text{ MPa} = 1 \text{ N/mm}^2 = 100 \text{ N/cm}^2 = 10 \text{ bar}$

to a lesser extent, floor heave c_f . This initial convergence c_0 amounts to between 10% and 20% of the original seam thickness M .

Convergence in the forefield is caused by abutment pressure, which acts on the comparatively soft seam, compressing it and causing it to protrude 10 cm or so into the face area. The immediate roof settles by the same amount as the seam “gives” in the forefield. At the same time there is a heaving of the floor at the seam edge. Since the seam’s lateral support has been removed along that edge, the abutment pressure proves too much for its reduced breaking strength, and it slides down sloping fracture planes (cleats) into the face area, once frictional resistance has been overcome. As it expands transversely and breaks up along planes parallel with the stratification, the floor of the excavation is able to bulge upwards (floor heave); but the vice-like grip of roof and floor on the seam edge prevents any further transverse expansion.

b) Convergence in the face area. From the ribside to deep in the goaf the roof has no firm support, and the strata have to bridge the gap. In the process they bend under their own dead weight. As the 4–6 m wide face-working advances through the pair of reference points, the rate of settlement of the immediate roof accelerates to perhaps 10 cm a day. Convergence in the face area c_1 , reflecting this settlement, increases to several tens of cm, as measured from the time the two reference points are reached by the advancing face to the time when the last row of supports in front of the goaf is withdrawn. The floor heaves in the face area by 10–20 cm.

With an average yield load in hydraulic props of 240 kN and a prop density of around 1 prop per m^2 , the support system for the face-working is not capable of bearing the whole roof over that area and maintaining it at its original height. The role of the support system is simply to prevent premature detachment of the lower layers of roof rock and the premature opening-up of joints, and thus, by maintaining formational and frictional restraints on fracture planes and bedding planes, to inhibit roof settlement.

c) Post-convergence (after the face has gone through). The roof finds support again on the fill or the caved waste in the goaf, and its curve of settlement is slightly convex at that point. This so-called post-convergence c_2 is almost complete some 100 m to the rear of the last row of props, and at that point, P'_1 , the immediate roof pressure is again the same as the cover load. Floor heave amounting on average to 6% of M continues at first in an elastic manner, as a stress-relieving, flexural deformation, in the moderately compacted border area of the goaf, but is completely suppressed again by the middle of the panel under the influence of the strong roof pressure. Thus, at the reference points P_1 and P_2 , a total convergence of

$$C = c_0 + c_1 + c_2 \quad (5)$$

occurs, as measured from when those points stand unaffected in the forefield to when their relative movement is complete in the goaf. With stowing, this convergence amounts to some 45% of the extracted seam thickness, and with caving, to around 90%. In the middle of a “critical”, or of a “supercritical”, area of extraction (see Sect. 7.2 in Chap. 7), total convergence C in the mine excavation corre-

sponds to the maximum possible subsidence (“complete subsidence”) or “full subsidence” at the central point on the surface, because the column of rock at that point, after temporary compression or extension, has resumed its original length (i.e. depth) as measured from the surface to the workings.

1.2.1.1 Fill’s Resistance to Deformation

The fill, consisting of crusher or washery wastes up to 70 mm in particle size, which is introduced loose into the mine excavation – for example, by *pneumatic stowing* – provides an effectual support for the roof only after initial compaction, i.e. only when the gap remaining between the upper surface of the fill and the lower surface of the roof, amounting to perhaps 15% of the depth of fill, has been closed by roof pressure, and when the individual particles of the fill, after further convergence of around 14%, lie in close and tight-fitting contact with each other. With increasing roof pressure and increasing compaction, more and more particles share in the support of the roof. In this process the fill’s resistance to deformation grows in proportion to the friction surface area of the particles and to the perpendicular pressure on their contact surfaces. The pressure/convergence curve of the fill is consequently, after the initial compaction resulting from around 29% convergence, logarithmic in shape. In the Ruhr District, for example (see Fig. 11), it has been found to conform to the expression

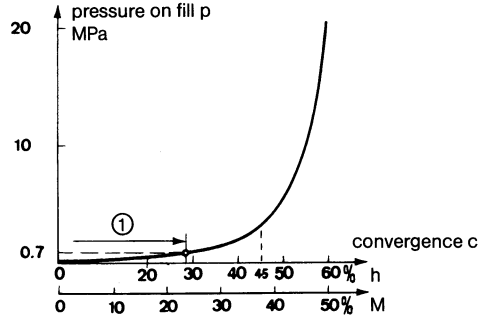
$$c = 15.6 + 13.1 \log (p - 70) \quad (6)$$

where c represents convergence in the fill as a percentage of its original depth (thickness) and p the pressure on the fill in N/cm^2 . When convergence on this basis has reached 45%, the fill is capable of only a further 15% compression, even if high pressures of over 2000 N/cm^2 (20 MPa) are applied. The fill curve rises steeply in this pressure bracket. For example, the stiffness index E_s of the fill increases from 6 N/cm^2 (0.06 MPa) at 10% convergence to 240 N/cm^2 (2.4 MPa) at 40% convergence. The bedding index C_b however, which indicates the required increase in pressure in N/cm^3 for each additional 1 cm of convergence, rises in almost linear fashion from 180 N/cm^3 at a pressure of 1000 N/cm^2 (10 MPa) to 350 N/cm^3 at a pressure of 2000 N/cm^2 (20 MPa). From Fig. 11 it can be seen that the cover-load pressure of 20 MPa at a depth of 800 m corresponds to a convergence value of approximately 60%. This in turn, allowing for convergence in the forefield and the face area of around 20% (implying a depth of fill of 80% M), corresponds to a total convergence at the centre of the panel, in terms of the seam thickness M , of $80 \times 60/100 = 48\%$, or say 50%.

Concrete fill, consisting of approx. 60% mine waste of under 60 mm grain size, 30% fly ash, 3% Portland cement 350, and water, is capable of only 4% – 12% compaction from its original depth, or thickness. It has a compressive strength of 1400 N/cm^2 (14 MN/m^2) and sets in 16 h at the most (Šiška, 1975). *Hydraulic fill* consisting of sand alone has a similarly low compressibility.

In stowage procedures, even though the immediate roof fractures, it does settle on the fill as a cohesive, rigid unit, closely followed by the main roof. In *caving*, on the other hand, it is the formational and frictional cohesion of the main roof which is maintained, so that it remains rigid and can bridge the face

Fig. 11. Pressure/convergence curve for fill consisting of small-sized rock fragments, as a percentage both of the original depth of fill h and of seam thickness M . (After Jacobi and Holtz, 1960). 1 Initial compaction as result of gap in fill



area instead of weighing down on it. This is achieved by providing the main roof with a supporting bed – in this case the immediate roof itself, which is usually of shaly structure and collapses after the last row of props has been removed, or “robbed”. The role of the fill is thus taken over by the immediate roof beds, which break off in large lumps, sometimes to three times the height of the seam itself. It is on these rock lumps, grading 1.5 to 2.5, that the main roof settles after gradually sagging. Over caved waste the main roof will bend almost twice as much as over stowed fill. Consequently, a roof of brittle sandstone will break at frequent and regular intervals under the bending stress of an advancing face, and the load on the working area from such main-roof breaks is abrupt and periodic. The props standing at the waste edge, preferably on hard footings, have to take the weight of some quite thick blocks formed as the roof beds break off (see Fig. 12).

In flat-lying seams, caving methods predominate in coal-mining today; in strongly inclined or steeply *dipping seams*, by contrast, stowing is the rule, with fill being introduced by being either dumped down the waste slope or – in hydraulic stowage – ducted in pipes. Stowage takes place by sections along the face, to avoid too much desegregation by grain size in the course of dumping. Because of the dumping or pouring action, the degree of stowing is greater in steep seams than in flat-lying ones, and there are no gaps under the roof or pneumatic-stowage “shadows” behind props. The stowage density of fill compacted by its own weight in steep-lying situations increases only in the upper 20 m, below the top road. In the middle and lower zones its density remains constant, because the weight of fill is absorbed to a large extent by friction along the hanging and foot walls – the so-called silo effect. The degree of convergence is affected not only by the deformation resistance of the fill but also by rock pressure normal to the stratification. Where the strata are steeply inclined, this pressure, in accordance with the theory of elasticity, diminishes with the angle of dip α by approximately

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\alpha, \quad (7)$$

where σ_1 represents the perpendicular cover load in flat-lying measures [as in Eq. (3)], σ_2 the evenly distributed horizontal rock pressure

$$\sigma_2 = \frac{\sigma_1}{m-1} = \lambda_0 \sigma_1 \approx 0.25 \sigma_1 \quad (8)$$

$m = 1/\nu$ the transverse expansion ratio of coal-measure rocks ($m \approx 5$), and λ_0 the ratio σ_2/σ_1 of the pressures at rest.

On the basis of a total convergence of 50% in flat-lying seams, and assuming that, for moderately to steeply inclined seams at a depth of 800 m, convergence in the forefield and face will decrease from 20% of seam thickness at a dip of 0 gr³ to 15% at 30 gr, 10% at 40 gr, and 5% at >50 gr; assuming also that the gap in the fill will decrease from 15% of the depth of fill at 0 gr dip to 10% at 30 gr, 5% at 40 gr, and 0% at >50 gr; and also that compression from strata pressure will decrease from 14% of the depth of fill at 0 gr dip to 12% at 30 gr, 8% at 40 gr, and 6% at >50 gr, then it is possible from Table 4 to calculate final convergence C at the centre of the area of fill in terms of seam thickness M, depending on pressure normal to the stratification, in accordance with Eqs. (6) and (8). Thus, where seams are steeply inclined, the overlying strata will sag less than over flat-lying seams (see also Fig. 31).

1.2.1.2 Settlement Profiles – Hard and Soft Roofs

The movement of roof beds, which is shown in Fig. 10 in its interrelationship from forefield to goaf, produces in its vertical component an S-shaped curve, which is convex in the forefield, almost a straight line over the face-working, and concave over the area of fill. The *settlement profile* of the main roof in caved workings is of a similar shape (Fig. 12), but it subsides over the goaf rather more than with stowing – indeed to approximately 0.9 of M. Observations of this nature on the exact progress of settlement at the mining horizon or immediately above it in the main roof are, despite their significance for the calculation of strata and ground movement, still rare: once the central roadways have been abandoned there is usually no further access to the goaf, and the cost of driving fresh investigation routes, or of installing “lost” remote-measuring devices, is usually grudged. A further complication with caving is that settlement of the

Table 4. Calculated convergence of fill in moderately to steeply inclined seams (Depth: 800 m)

Dip, in “grades”	Rock pressure normal to strata MPa	Convergence C normal to strata %	Vertical subsidence $C \times \cos \alpha$ %
0	20.0	50	50
30	16.9	45	41
40	14.8	40	33
50	12.5	35	25
60	10.2	32	20
70	8.1	31	15
80	6.4	30	10
90	5.4	29	4
100	5.0	28	0

³ Decimal division of circle: 400 “grades” = 360°

all, and therefore drops comparatively steeply behind the face⁴. The bending resistance of the roof beds is thus mirrored in the shape of the settlement profile. A bend-resistant roof transmits the heavy load of the bridged excavation to a broad belt of the forefield, whereas a roof that is liable to bend transmits a much smaller load to a narrow zone along its edge. With a sandstone roof, the transition point from convex to concave in the settlement profile is roughly over the coal face; with a shale roof it is over the face area. The characteristics described here of a hard roof and a soft roof are also recognizable in the intermediate profiles “c” and “d”.

While the face is being actively advanced, the slope in both varieties of roof bed is comparatively slight, and it is only after the cessation of working that settlement and sagging over the filled area intensify. As pressures in the rock mass become redistributed, and especially as the fill in the border zones comes under an increasing load, the dynamic profiles “c” and “d” turn into the static profiles “a” and “b”. The daily increase in convergence between a pair of reference points at the top and the bottom of the seam, i.e. the rate of convergence measured in relation to distance from the advancing face, rises rapidly in advance of the face, reaches a maximum of 10 cm/d during or shortly after the transit of the face, and drops off in the area of fill (Fig. 13). The maximum value in the rate-of-convergence curve coincides with the transition point in the roof-settlement profile.

1.2.2 Mining with Permanent Pillars (Partial Extraction)

If parts of a deposit are left standing, unmined, as supporting pillars arranged in such a pattern that the roof spans the chambers without fracturing, the rock mass over the workings will subside only to the extent that the pillars yield under constant load. Provided the chambers are not too wide, the roof will remain intact like a bridge. Conversely, the pillars must not fail prematurely: which means that they must be of sufficient length and width. Pressure on the pillars increases with increasing size of the workings, up to a limit value which depends on the depth of the workings and the horizontal area of the pillars and chambers respectively, but which in all cases exceeds the cover-load pressure.

In *chamber workings for salt*, the roof begins to settle like a slab if the size of the workings reaches 300 m by 400 m⁵. It was formerly assumed that a pressure and support arch spanning the workings took the load off the inner pillars and transferred it to the outer pillars or the seam edge along with the abutment pressure, but this has not been established. “Individual arches” under a partial load are to be found only at the beginning of working. The reason why the load on the pillars from the overlying rock exceeds the cover load throughout the workings is that the pillars have jointly to carry the rock mass over the open chambers and stalls, which has been deprived of its support; but in a 150 m wide border zone

⁴ The exaggerated bend in the roof beds portrayed in Fig. 13 is in reality, with an inclination of at most 1 in 75 over the face area, hardly noticeable

⁵ See also Chap. 10.2.1.2 and Fig. 252

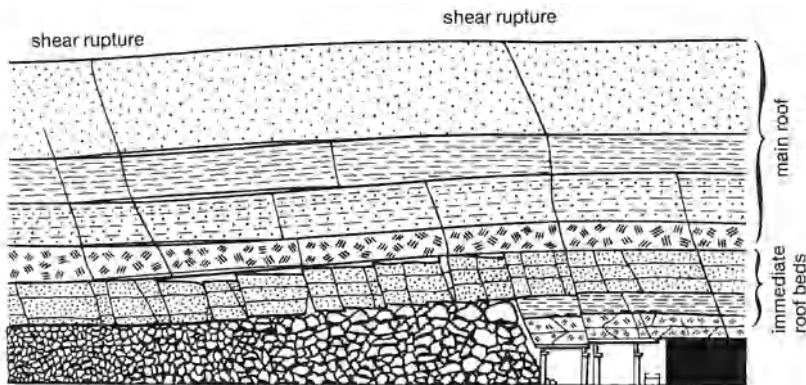


Fig. 12. The distorting effect of caving on lower roof beds in longwall caving

lower main roof can only be measured from a road driven just above the caving area and, even if this is done, it will be destroyed during undercutting. Measurements over the caving face-working are therefore conducted at a perpendicular distance of 50 m or more above it, i.e. away from the actual mining horizon. A further difficulty is that roadways are surrounded by a zone of loosening, and survey points therefore have to be established outside this zone to isolate them from secondary strata movements. Rock is loosened to a depth of at least 7 m from the sides of roadways. Measuring marks must consequently be spring-mounted in a sufficiently long drill hole and monitored with a theodolite. If the hole is drilled vertically, then vertical and horizontal movement is measured by means of a wire lead line secured at the top of the drill hole.

There are no data on the pressure/convergence relationship of coarsely lumpy caved waste consisting of roof beds that have broken off to a height of 2 to 6 m. The ability of such caved material to provide support as the main roof begins to settle is undoubtedly less than that of fill solidly stowed in the mine excavation. The permanent increase in volume of the caved waste amounts to only a few percentage points in terms of perpendicular measurement at the centre of the panel, where something approaching the cover-load pressure has been restored and the main roof has subsided by 90% to 95% of the thickness of seam mined.

Such observations as have become known — the mean values of which, suitably smoothed out, form the basis of the settlement profiles represented in Fig. 13 — make clear how much variation there can be in the settlement of roof beds with stowing, and similarly in the settlement of the main roof with caving, depending on whether the roof is of hard sandstone or of soft argillaceous shale, and on whether it is the final situation or an intermediate phase of movement that is being considered. With a sandstone roof (Profile “a”) settlement in its final state stretches 100 m into the forefield, and the fill is not fully compacted until 100 m behind the face. Within a span of 200 m therefore, a sandstone roof is only partially supported and bends over the yielding underlay of seam, face supports, and fill. A shale roof, by contrast (Profile “b”), is only capable of bridging about 25 m of the forefield and 75 m of the mined-out area, i.e. 100 m in

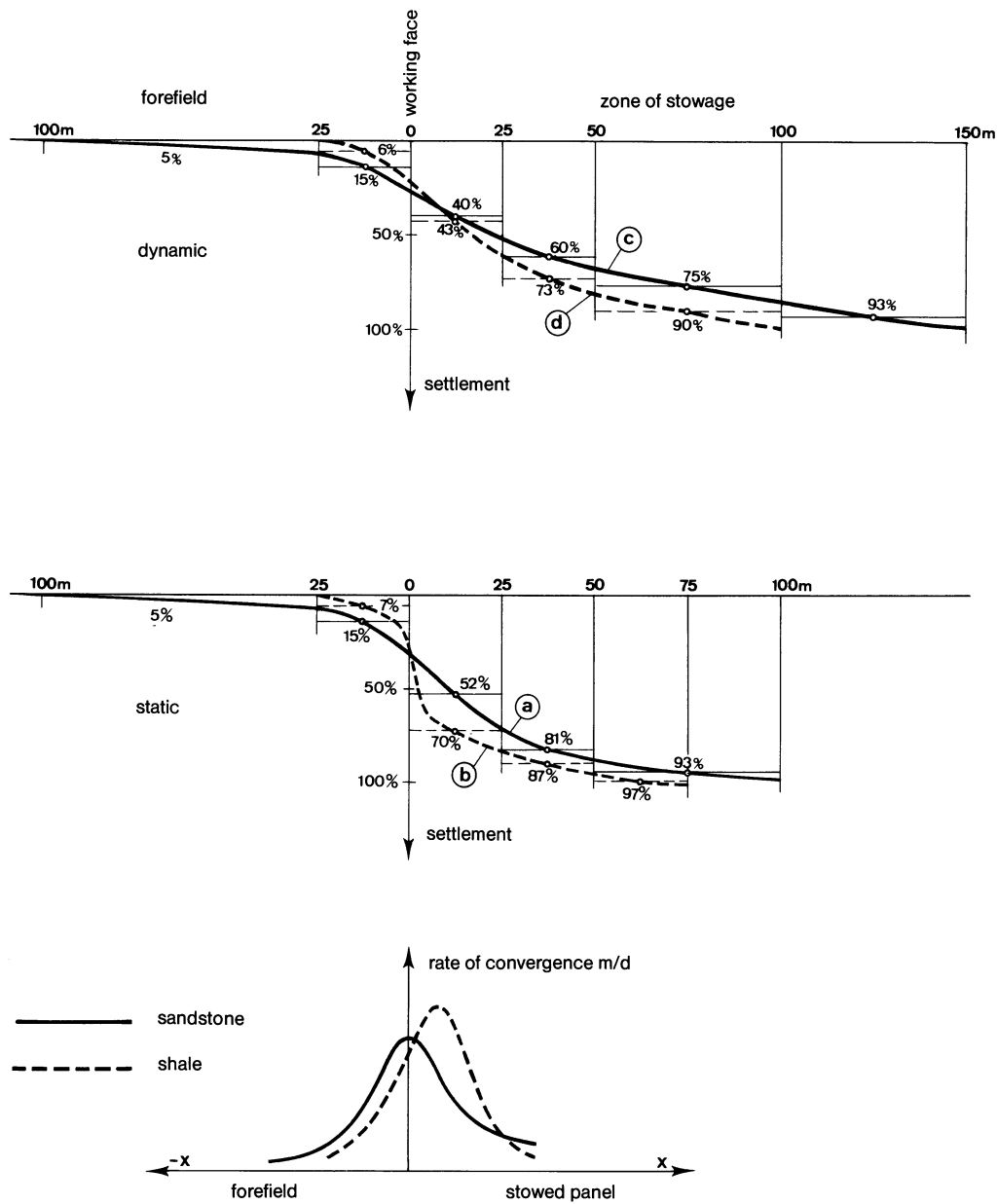


Fig. 13. The bending profile of roof beds in stowing, or of the main roof in caving, when the working face is actively advancing (*top*) and when it is stationary (*centre*). Rate of convergence in the working area, in metres per day (*bottom*)

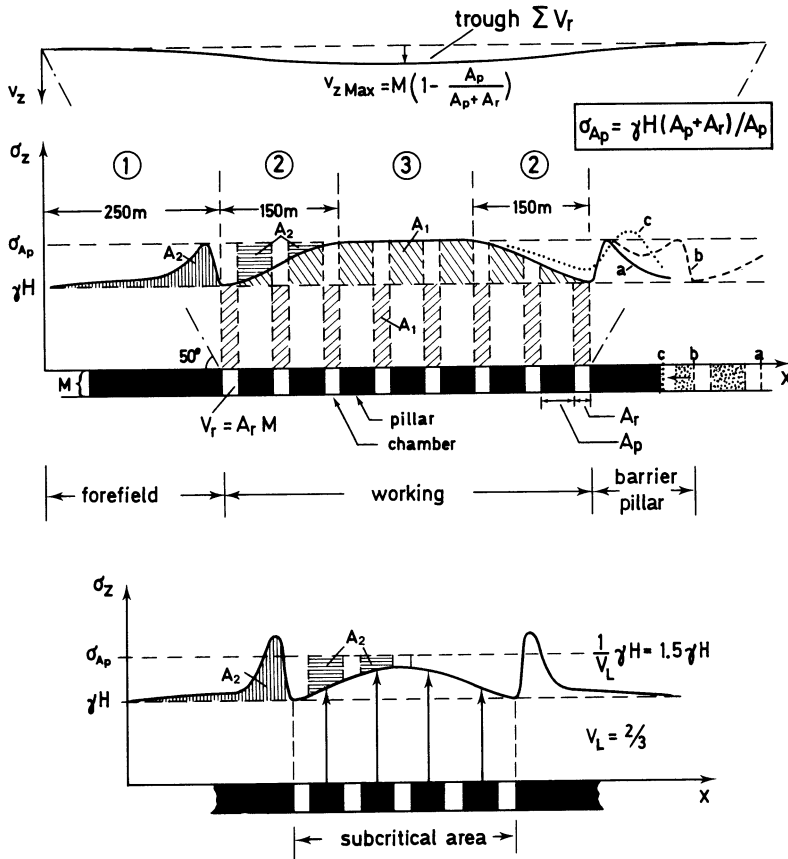
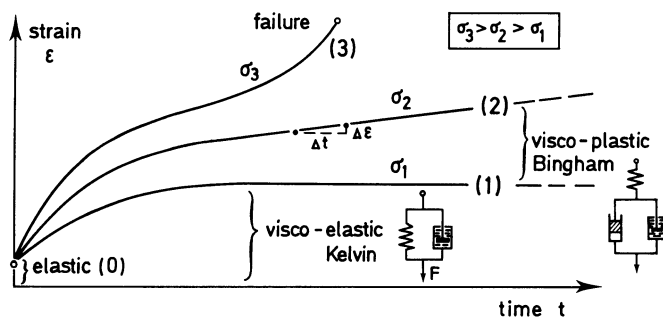


Fig. 14. Distribution of perpendicular rock-pressure in a chamber working for rock salt. *Top*: supercritical area. 1 Zone of abutment pressure; 2 zone at side of working with increasing pressure on pillars; 3 zone of full load on pillars. *Bottom*: subcritical area

the overlying load is less than in the middle of the workings and over the seam edge (Fig. 14). Up to this distance the forefield – the zone immediately adjoining the workings – acts as an abutment to the undermined roof, but the abutment pressure makes itself felt up to 250 m into the forefield (Erasmus, 1962). In contrast to longwall working without pillars, the working area in chamber working is not relieved of load (cf. Fig. 10).

Thus in chamber working the same movement processes as in longwall working are to be observed in the roof when pillars gradually give under load. Pillars of rock salt deform visco-elastically if loaded continuously with over $100 \text{ N/cm}^2 = 1 \text{ MPa}$, the limit of elastic behaviour, losing in height and gaining in width. In this primary stage of creep or flow the convergence movement in the salt pillar ceases after a certain time, due to the effect of strain hardening. From a load of approximately $2000 \text{ N/cm}^2 = 20 \text{ MPa}$ and upwards, a visco-plastic state of unending secondary flow with a constant creep rate sets in (see Fig. 15), under which, for example, 50% of the void in chambers is filled in in 50 years, and 70%

Fig. 15. Rheological deformation behaviour of saline rock under constant load. 0 Immediate, elastic deformation; 1 primary flow in salt, leading to consolidation; 2 fracture-free secondary flow in salt under a constant load of over 20 MPa; 3 tertiary flow, leading to failure, in salt under very heavy load



in 100 years. Under heavy load, tertiary flow with increasing creep rates leads to failure by rupture. Flow deformation of salt pillars is accompanied by the collapse of sheets and slabs of salt from the beds, the heaving and buckling of chamber floors, and the penetration of pillars into the soft footwall of salt by up to 20 cm.

Given a constant thickness of seam M , the maximum settlement of roof beds, amounting usually to 30% or 40% of M , is a function of the relation between the horizontal area of pillars and workings respectively. For example, if the horizontal area of chambers and pillars in the whole working originally amounted to 275 m² and 125 m² respectively, the pillars when completely flattened will fill the mine excavation up to a proportion of $125 \div 275 + 125 = 0.31$ of the original height M of both chambers and pillars. The roof will then have settled by 0.69 M .

Figure 16 compares settlement of the roof in room-and-pillar working with roof settlement in caving and stowing without pillars. At the end of the subsidence process, when the pillars have been compressed and completely fill the chambers, the degree of subsidence in chamber working and stall working depends on the thickness of mineral extracted M as well as on the horizontal area of pillars and chambers, thus:

$$v_{z \text{ end}} = M \left(1 - \frac{A_p}{A_p + A_r} \right) = M \frac{A_r}{A_p + A_r} = Ma. \quad (9)$$

If the pillar is larger in plan area than the chamber, then roof settlement will be less than 0.5 M , its value in stowing without pillars; if, on the other hand, A_p is smaller than A_r , roof settlement will exceed 0.5 M , but will be subject to the upper limit of 0.9 M found in caving without pillars.

By appropriate measurements it has been possible to establish that the perpendicular pressure σ_z on the pillar is distributed in a bell-shaped pattern and at its central point exceeds the specific surface load (Fig. 17):

$$\sigma_{A_p} = \frac{A_p + A_r}{A_p} \gamma H. \quad (10)$$

Figure 18 shows that, for a square pillar of $b = 20$ m width, the specific load at 900 m depth, and with stalls (chambers) of $B' = 14$ m width, is approximately

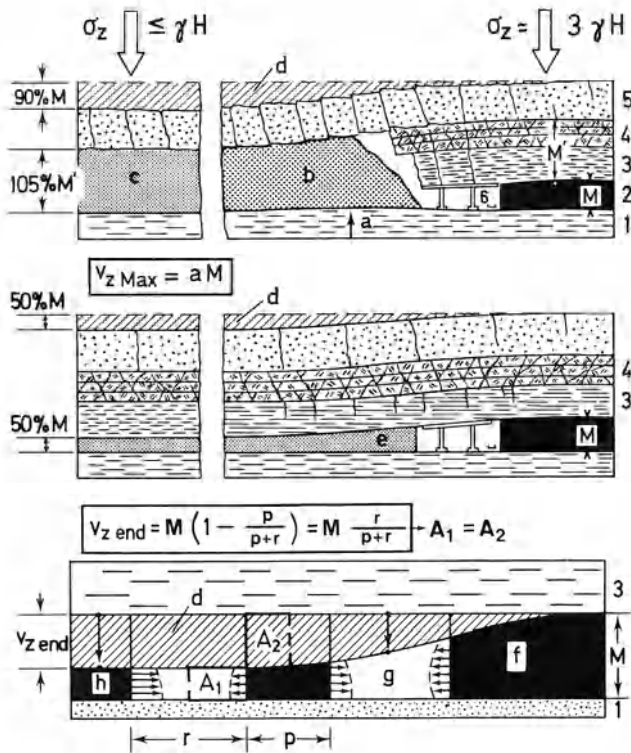


Fig. 16. Settlement of the roof in longwall mining with caving (*top*) or with stowing (*centre*), as compared with room-and-pillar working (*bottom*). 1 floor; 2 seam; 3 shaly roof beds; 4 jointed roof beds; 5 main roof (sandstone); 6 face-working with roof supports and conveyor trough; *a* heave in floor; *b* loose waste from caved roof beds; *c* caved waste compacted by roof pressure; *d* volume of roof settlement; *e* fill (stowed goaf); *f* compressed edge of seam; *g* and *h* reference points, which have settled with roof beds; *r* width of chamber; *p* width of pillar

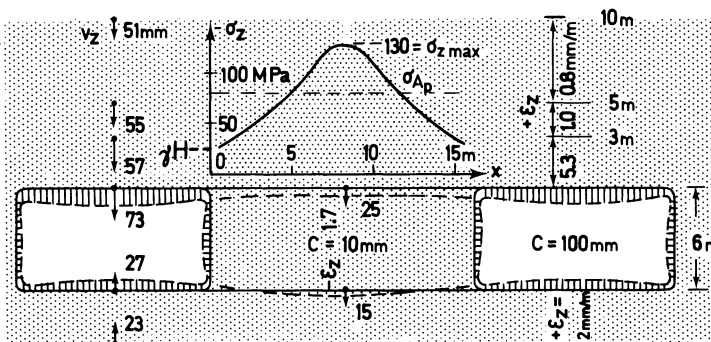


Fig. 17. Vertical deformation of the rock mass within the first 3 years in a pillar-and-chamber working for potash; and the distribution pattern of vertical compressive stress in a pillar, represented in section. (Adapted from Uhlenbecker, 1974). At a depth of 900 m, the core of the 16×16 m pillar is forced into the floor, and is compressed vertically by up to 1.7 mm/m. Both roof and floor of the 12 m wide chambers expand. (*C* vertical convergence)

Fig. 18. Nomogram for ascertaining mineral loss and specific surface-load on pillars in room-and-pillar working, given the breadth b of square pillars p and the width b' of chambers or roads r . (Adapted from Uhlenbecker, 1974)

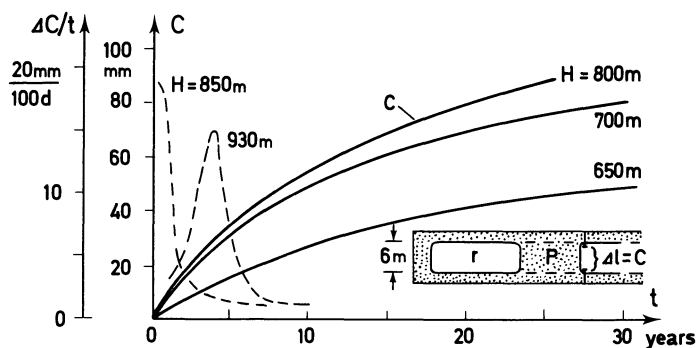
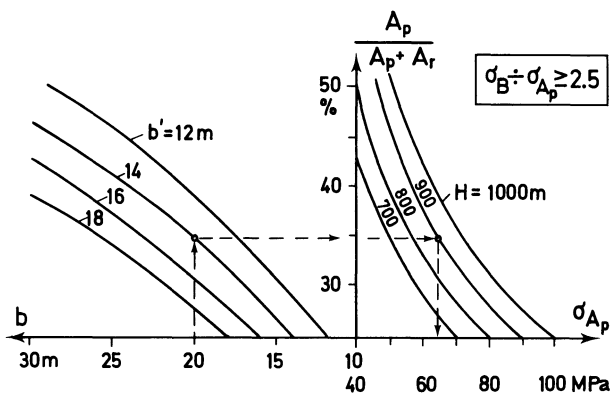


Fig. 19. Time-dependent convergence C in a pillar of potash, measured on the side, at depths of $H = 650 - 800$ m; and rate of convergence in mm per 100 days (*dotted line*). r Room or chamber; p pillar. (Adapted from Uhlenbecker, 1974)

65 MPa. In this example, the 35% sacrifice of mineral⁶ resulting from leaving pillars standing is equal to the proportion that the pillar area constitutes of the total area of the working:

$$V_L = \frac{A_p}{A_p + A_r} . \quad (11)$$

Within a space of 3 years, the roof of a chamber in workings 900 m deep settled by 73 mm, and the floor heaved 27 mm (see Fig. 17). The high load of 130 Mpa drove the core of the pillar 15 mm into the floor, and convergence C came to 100 mm in the chambers and 10 mm in the centre of the pillar. The rock mass over the chambers was extended in a vertical direction by between 0.8 and 5.3 mm/m, and the floor by 2 mm/m.

The time-dependent magnitude of the convergence C – shown in Fig. 19 as for readings taken at the edge of a pillar – is, as might be expected, influenced by the perpendicular pressure σ_z , i.e. by the depth of the workings H . The deeper the workings, the greater the time-dependent convergence, which also increases

⁶ The extraction ratio is $V_w = 1 - V_L = A_r / A_p + A_r$. Note: Symbols h or H for depth of working and symbol γ for gravity or limit angle – see Eqs. (4), (10), (161).

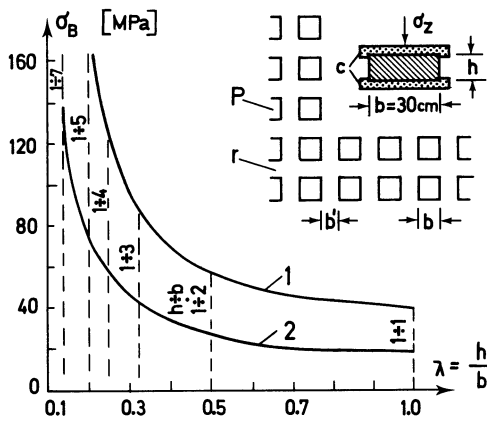


Fig. 20. Ultimate strength σ_B of square pillars p of sylvinitic (1) or carnallitic (2) in relation to degree of "slenderness" λ , as established in scaled-down model tests. (After Uhlenbecker, 1974). *Top right:* section through sheet of salt of height $h = 5-15$ cm, briefly placed under load in a press. c Top and bottom plate of rock salt; p pillar in plan; r mined-out chambers or galleries; b width of pillars; b' width of chambers

over time. The rate of convergence, expressed per 100 days, also depends on the specific surface load. In potash mining, for instance, it is usually greatest in the first years after extraction — e.g. 15 mm in 100 days.

For a pillar of given material properties, its ultimate strength depends on its degree of "slenderness" λ , i.e. the relation between pillar height h and pillar width b (Fig. 20). Whereas a pillar which is four times as wide as it is high fails only at 120 MPa, a pillar which is only twice as wide ($\lambda = 0.5$) fails as early as 60 MPa. Account must therefore be taken of the pillar's slenderness index λ in interpreting the flow boundaries in Fig. 15. For example, by adopting a pillar of sufficiently wide dimensions, and accepting a correspondingly high sacrifice of mineral, it is possible to avoid the tertiary flow in a salt pillar which leads to its failure.

On the basis of photoelastic stress investigations conducted with laboratory models, the centre of pillars is under comparatively low stress in the elastic zone. At their sides, however, the perpendicular main normal stress σ_1 rises sharply until, with increasing loading σ_{Ap} there, the flow boundary is reached (Fig. 21). Flow in the highly loaded salt rock, together with scaling at pillar edges, then relieve the peaks of stress, and at the same time vertical pressure σ_1^* increases, given an unaltered and constant specific load in the pillar interior. Horizontal compressive stress σ_2 in the zone of elastic and plastic deformation exhibits a similar bell-shaped development. The stability of the pillar is exhausted when every point of its breadth b is under a stress of

$$\sigma_1^* - \sigma_2 = 2\tau_0 \quad (12)$$

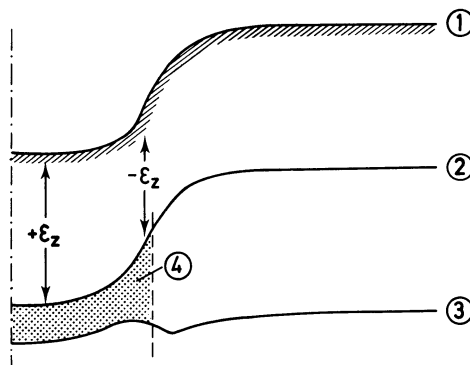
as shown shaded in Fig. 21. (τ_0 is the flow shear-stress, ascertained empirically.)

In a *brining operation* the mine cavity can be taken as resembling, in simplified form, an inverted circular pyramid or cone with a radius of

$$r = \sqrt{\frac{3Q}{\rho M \pi}} \quad (13)$$

where Q represents the quantity in Mg (1000 kg) of salt of density $\rho = 2.1 \text{ Mg/m}^3$

Fig. 26. Calculated subsidence for the Groningen natural-gas field. (After Geertsma, 1973). 1 Surface with subsidence trough; 2 roof of gas reservoir; 3 footwall; 4 diminished pressure and convergence in reservoir rock



Auger mining (Fig. 24) also counts as a mining method employing permanent pillars. Coal is mined by drilling horizontal holes of 80 cm diameter and 30 m length close to each other in the seam. Up to 40% or 60% of mineral may be sacrificed in ribs left between the boreholes and in safety pillars along the roadways. If overlying rock pressure later collapses the boreholes, the roof will settle by around 50% of seam thickness.

Strata movements also occur over the cavity formed by *underground gasification* of coal (Fig. 25). This cavity is the gas generator and is of a very irregular shape, becoming filled in to a greater or lesser extent with residues (cinder, ash, coke). Only 10% to 30% of the cavity does not become filled in this way. The compressibility of these residues is comparable to that of sand.

Parts of the seam left ungasified between gas generators act as pillars, and the consequent loss of mineral, usually 20% to 50%, depends on the distance of 25 to 40 m between boreholes. The high temperatures of 1000 to 1500 K arising during gasification heat up the wall rock for several months, producing zones of melting which can extend up to 40 cm into the roof and floor. The thermal stresses may fracture the strata and lead to rock falls in the mine cavities.

In *gas and oil fields*, the underground reservoir converges to the extent that pressure in the deposit decreases during the period of production. According to investigations and calculations carried out in the natural-gas field at Groningen, in the Netherlands, heaving occurs in the footwall and settlement in the hanging wall, just as in coal-mining (Fig. 26). In large fields, subsidence at the surface corresponds to settlement of the hanging wall but not to compression of the reservoir. Immediately over the gas or oil reservoir rock the hanging wall expands, but in the adjoining forefield it becomes compressed (cf. Fig. 10).

1.3 Factors Influencing Convergence in the Workings

Investigations up to the present into roof settlement, convergence in workings, and rates of convergence confirm the model proposed by rock mechanics. This suggests that strata movement at the mining horizon resembles the behaviour of a quasi-elastic beam bedded on a yielding underlay and is chiefly determined by the following critical factors:

a) Depth of workings. As is shown in Eq. (4), the cover-load pressure to be taken up by the immediate roof or the main roof depends on the depth of the workings. The greater the depth at which work takes place, the greater will be the pressure on the zones ahead of and behind the face – the forefield and the stowed goaf – and the greater will be the sag in the roof.

b) Bending resistance of the roof. A bend-resistant roof gives little under the load of the undermined rock mass and finds extensive support in the forefield or on pillars for redistribution of the rock pressure. A yielding roof, on the other hand, seeks to escape from the strata load by bend deformation and transfers the local load almost undiminished to the underlay, both in stowing and in caving procedures. The bending resistance N of a rock stratum can be determined from its modulus of elasticity E (Young's modulus), expressed in N/cm^2 , its thickness d , and its load-bearing width b :

$$N = EI = \frac{Ebd^3}{12} \quad (14)$$

where I = the moment of inertia. Any joints or fissures present in the rock will lower its bending resistance.

c) The underlay supporting the roof. An unyielding support for the roof will counteract both sagging and redistribution of the strata load. Given early support from a hard fill material, or a large pillar area, high abutment pressure is avoided (Figs. 10 and 14), and convergence in advance of the face remains slight. If the outer edge of the face excavation is not supported in good time with roof props, blocks of the roof stratum may slip downwards along fissure planes, and the resulting greater convergence $c_0 + c_1$ will reduce the quantity of fill that can be stowed and increase the subsidence factor.

The load-bearing capacity of imported fill or caved roof rock is related to the voids volume and compressive strength of the fill or waste. The proportion of voids in the filled excavation decreases with decreasing rock particle size, and hard waste material improves the deformation resistance of the fill's granular skeleton.

d) Seam thickness and depth of fill. In sagging, the roof compresses both the seam and the fill, and in the process mobilizes the support capacity of the bedding layer. This increases with the degree of sag in the roof, as though it were a spring being compressed. The thicker the seam or the fill, the farther the roof must sag to compact the underlay to the point where it will bear a load. For example, at a pressure of 500 N/cm^2 , a roof overlying 2 m of fill will find support after dropping 100 cm, but after only 50 cm if over 1 m of fill – i.e. in both cases when convergence in the stowed goaf has reached 50%, as represented in Fig. 11. Thus with both caving and stowing procedures, the greater the working thickness, the more the roof will sag.

e) Width of excavation and size of working. The roof has to bridge the face excavation or chamber like a cantilever beam, but even far back in the stowed goaf the load-bearing capacity of the fill is usually not enough to support the pressure

of overlying rock. An increase in the width of the face excavation and area of fill, or of the chamber, thus means an increase in the span to be bridged, and therefore in the likely flexure of the roof.

For purposes of comparison, an elastic beam will bend at the middle by the third power of the cantilever length l :

$$w_0 = \frac{1}{k} \frac{Fl^3}{N}. \quad (15)$$

f) Rate of advance and daily coal-getting time. In working without pillars, the roof can go down only to the extent that it compresses what lies under it. This compression process is time-dependent, because seam and fill do not give immediately under the load, but only gradually as their breaking strength or friction resistance is overcome. The corollary of this time-lag in convergence is that, with a rapidly advancing face, the roof goes down slowly by comparison, both ahead of the face and behind it. Less convergence $c_0 + c_1$ also means an increase in the quantity of fill introduced; and so a more rapid face advance has the effect of flattening the curve of roof flexure. Daily coal-getting time has a similar influence on the downward movement of the roof. For example, with 6 h of cutting per day, the roof has the 18 h in which the face is stationary to compact the fill, drive in the props, and compress the seam. It will therefore dip more steeply over the area of fill, and the quantity of fill stowed will be correspondingly less than with 12 or 18 h of extraction per day.

g) Nature of the floor. The heave in the floor resulting from its elastic response to a reduction in load or from water absorption contributes to compressing the seam and reducing the height of the face excavation. When working with stowing, the quantity of fill introduced is thereby also reduced.

h) Compressibility of pillars. In pillar working, the decisive factor for convergence is the compressibility of the pillars. This varies from rock type to rock type but is also influenced by the behaviour of wall rock and by slip planes in roof and floor. The amount by which a pillar is compressed is determined by its width-to-height ratio, by the specific load on it, by its flow properties in the case of an evaporite rock, and by its crushing strength. In blasting, the rock becomes fissured, and this reduces the stability of the pillars. The various ways in which a pillar may fail, and typical compression curves for some evaporite rocks, are shown in Fig. 27.

i) Length of time that a face is halted. When work at a face is suspended, the thinner the seam, the greater the depth and the pillar load, the more flexible the roof, and the more yielding the pillars, the sooner the static, i.e. final, bending profile of the roof is reached.

All in all, the interrelationship between roof settlement, behaviour of fill material, compression of pillars, geometry of workings, and rock-mechanics considerations is a highly complex one.

Just how variable *roof settlement* in coal-mining can be in practice, it has been possible to observe from a series of measurements made of the final thickness of fill at one mine, when new roadways were driven through the goaf a num-

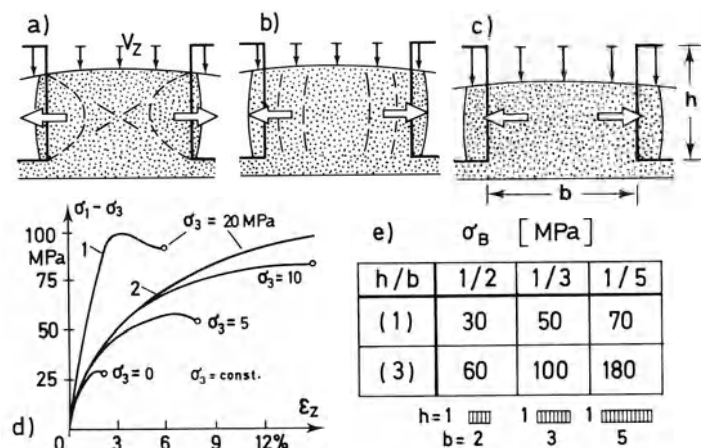


Fig. 27. Pillar failure on exceeding *a* shear strength, *b* horizontal tensile strength, *c* the flow point. *d* Deformation curves for (1) carnallite, showing sudden failure, and (2) rock salt (halite); *e* crushing strength as affected by pillar height *h* and pillar breadth *b* for (1) carnallite and (2) hard salt (sylvinite)

ber of years after extraction had ceased. Figures 28 and 29, which depict the findings, show that local influences can cause actual roof settlement to deviate greatly from the perfect S-shaped flexure profile seen in Fig. 13.

At the active face (Fig. 28), the roof-settlement profile starts as a flat arc reaching to a distance of 64 m behind the face – an indication that the roof settles only gradually at first over the stowed panel. It is only after work is suspended that a sizable delayed settlement of around 20 cm takes place there. It is quite another matter with settlement at the stationary face where working began (Fig. 29). The roof has gone down by 59 cm immediately behind that face, at only 5 m from it, and at as little as 28 m from it the roof is resting with its full weight on the fill. The repeat readings taken much later at the centre of the panel showed that the roof had sunk by only a further 4 to 14 cm. In other words, over the stationary development face the roof had had ample time to sag and settle in step with the increasing abutment pressure as working advanced through the panel. The additional subsidence of around 5% of *M* often noticed over the initial development face and the fixed outer edges of the workings, as compared with the final situation on the extraction-face side of the workings, can be ascribed to the length of time available for fracturing to develop in the roof and in the stationary abutments of the seam. The settlement behaviour of the roof stratum at the two sides of the panel is thus somewhat dissimilar and asymmetrical.

Figure 30 gives a good idea of the resistance of roadways and *roadside packs* to subsidence when workings pass along them. It shows how roadside packs separating two panels have left a subsidence ridge of over 30 cm, or approx. 30% of the maximum subsidence. In the 1940 panel, the displacement of the focal point of subsidence towards its rim is probably connected with the dip in the seam, and in the 1942 panel, with the change in the type of goaf. Laboratory tests also indicate that, in gently to steeply inclined measures, an increase in roof settle-

Fig. 28. Convergence in the mined-out panel (with stowing of the goaf) as measured from the working-face end. 1 Roof before settlement; 2 measurement 1939; 3 measurement 1943. (After H. F. Schulte, 1952)

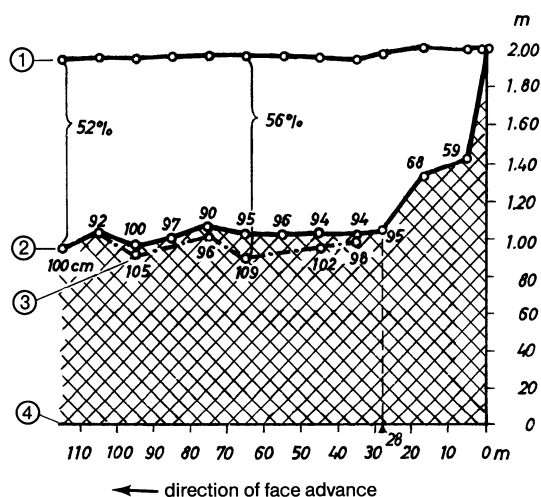
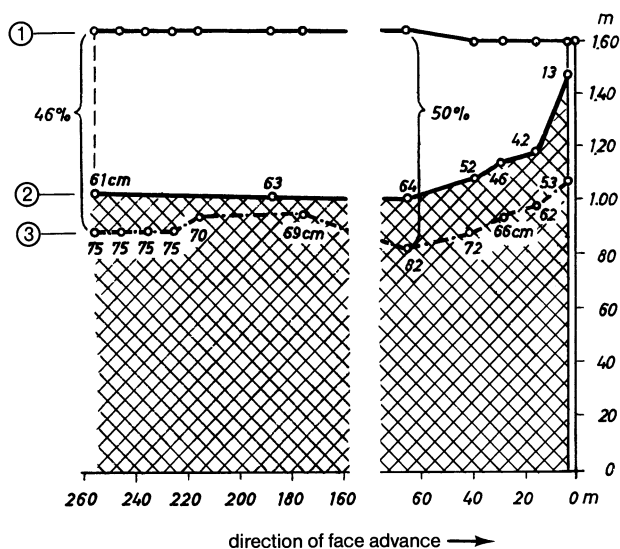


Fig. 29. Convergence in the mined-out panel, measured from the initial development-face end. 1 Roof before settlement; 2 measurement 1939; 3 measurement 1943; 4 floor of workings, represented as level

ment of around 5% of M can be expected in the lower half of the stowed panel. And because the strata pressure acting perpendicularly on the roof increases in a wedge-shaped curve when there is a dip in the formation, the point of maximum sag, which is situated at $x = 0.51$ in horizontal measures, is shifted slightly downwards to $x = 0.5251$ (Fig. 31).

Local tectonic features such as small synclines and anticlines in the working area also influence convergence. Zones of minimum roof settlement coincide with the trough lines of synclines, and the crest lines of anticlines run between the maxima and minima of subsidence curves (Fig. 32).

In underground measurements of convergence, no clear connection has been found between the amount of roof settlement at the panel centre and the rate of

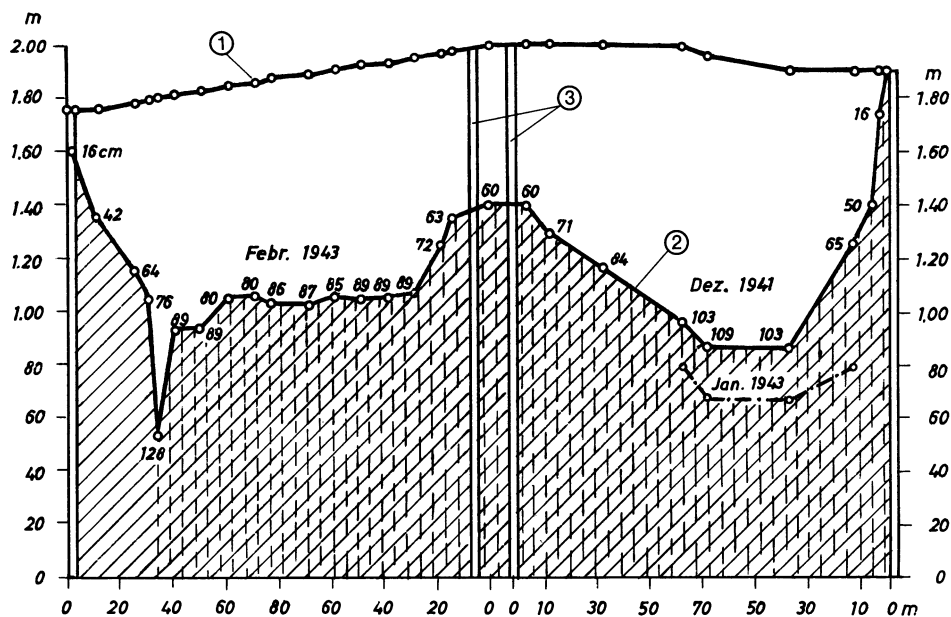


Fig. 30. A transverse section of two neighbouring panels showing the convergence in each. (After H. F. Schulte, 1952). 1 Roof before settlement; 2 roof after settlement; 3 mine roadways

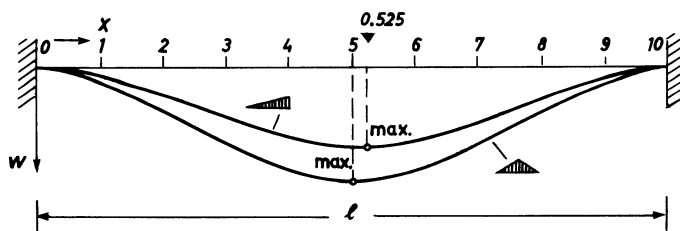


Fig. 31. The sag w of an elastic beam, fixed at both ends, under various load conditions

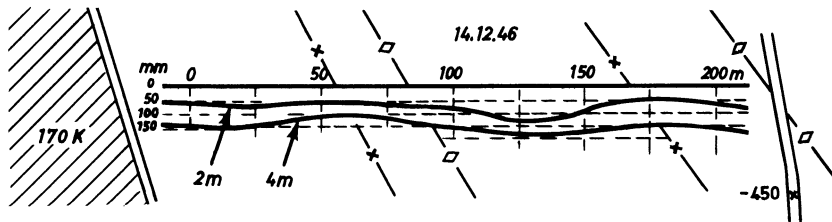


Fig. 32. The influence of local tectonic features on convergence in the face excavation, measured at 2 m and 4 m from the face. (After H. Hoffmann, 1947). Lines with crosses: troughs of synclines

face advance, nor with the nature of the wall rock. An increase of 3.6% in final subsidence at depths ranging from 400 to 800 m, and of 5.6% with panel width increasing from 150 to 300 m, has been established with some certainty for hand stowing, and a similar correlation is likely with other forms of stowing. At a depth of 750 m, maximum roof settlement as measured at the panel centre amounts to approximately 50% of M with pneumatic stowing and 95% of M with caving.

1.4 Horizontal Movement in the Roof Stratum

So far only the vertical movement of a datum point at the mining horizon has been considered. Individual points in the roof stratum do not, however, go down absolutely vertically, or even at right angles to the stratification; they follow a diagonal path downward, sometime even describing a loop. The vertical component of this movement is several times greater than the horizontal component, or “displacement”. The subsidence profiles in Fig. 13 can therefore be regarded as representing the position at the relevant time of the under surface of the roof stratum. Horizontal displacement of points in the roof has been little researched because, in contrast to subsidence, which is what determines the narrowing of the mining excavation, displacement is not required as a fundamental quantity in strata-movement calculations and is in any case subject to a variety of influences at the limit plane of the mine excavation. Displacement has to be measured on the accessible underside of the roof stratum, where interface friction and fracture-deformation are so strongly evident that the horizontal deformation effects to be expected in bending movements – namely, compressive strain ahead of the face, where the undersurface of the roof is concave, and tensile strain behind the face, where the bend is convex – are overlain by a shear movement, or transverse displacement, starting ahead of the face and directed towards the cutting zone. This “rolling-mill” effect on the roof stratum, squeezing it in the direction of the goaf, is initiated by the perpendicular pressure on the abutment and assisted by the lateral contraction of the roof stratum associated with perpendicular expansion from stress relief in the goaf.

Measurements of the displacement of observation points in the roof stratum, made in long development headings in unworked coal, revealed that the dynamics of mining produces tensile strains in the vicinity of the working face and compressive strains in the face excavation. Observation points marked ahead of the face, in the forefield, had moved 5 cm towards the goaf by the passage of the face, and 10 cm in all by the edge of the area of fill (Fig. 33). This movement towards the point of cut was confirmed by measurements made of the shift of roof-marker points in a main gate-road (Fig. 34), both along the strike (the axis of the road) and down the dip (of 15 gr.). A friable roof of sandy shale begins to shift some 30 m ahead of the working face. After passage of the face, horizontal movements down the dip increase rapidly, to 4 cm and later almost 6 cm. Along the strike, shifts of 3 cm are reached, which recede again 180 m behind the face. Tensile strains of up to 0.4 mm/m along the strike have also been established im-

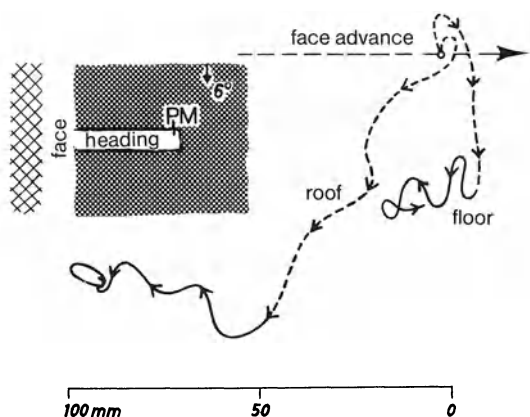


Fig. 33. The measured horizontal movement of marker points PM in the roof and floor. (After Weissner, 1934). (Dotted line: movement before passage of face)

mediately before and after the face but also far into the area of fill, even though the underside of the roof ahead of and over the face zone bears a concave curvature.

The overlapping, already mentioned, of fracture-deformation and flexure-deformation effects can be demonstrated, in the light of the jointing system, as being in this case a *sliding movement* of the pieces of rock along joint planes (Fig. 34, bottom). The seam, having undergone transverse deformation in the forefield by the heightened pressure, has dragged wall rock into the face excavation with itself. At the margin between face area and fill area, interface friction and support both disappear, and in a brittle roof there begins a “tracked sliding movement” at the fracture planes. This happens when, during roof settlement, the pressure acting normal to the joint planes diminishes as a result both of perpendicular expansion as the stress is relieved and of flexural stretching, and the weight force of the roof blocks thereby temporarily exceeds the friction forces at the fracture planes. Starting at 180 m from the face, there is a compressive strain which derives from the declining curve of displacement along the strike. It can be assumed that at that point the individual blocks of the roof stratum expand again laterally under the increasing strata pressure and become constricted.

In front of and behind the fixed rib where face was opened out, an inherent movement of blocks of roof stratum in the direction of the stowed goaf must also be assumed. The curves of displacement thus run, in a longitudinal section along the strike, from both faces to a point “A” near the development face at which the amount of displacement falls to zero. When all working has been suspended, this point moves somewhat towards the centre of the panel, until a complete stress/strain equilibrium has been reached.

In contrast to the horizontal displacement of ground at the surface, which is discussed later, the displacement of points in the roof over the area of fill increases only marginally with an increasing area of working. Such displacement depends chiefly on the amount of roof settlement and on the direction imparted by jointing in the roof. Furthermore, the point of zero displacement “A” does not move deeper into the panel in step with progress in working but follows the face only after a time-lag, and only for a short distance. Movement at the mining

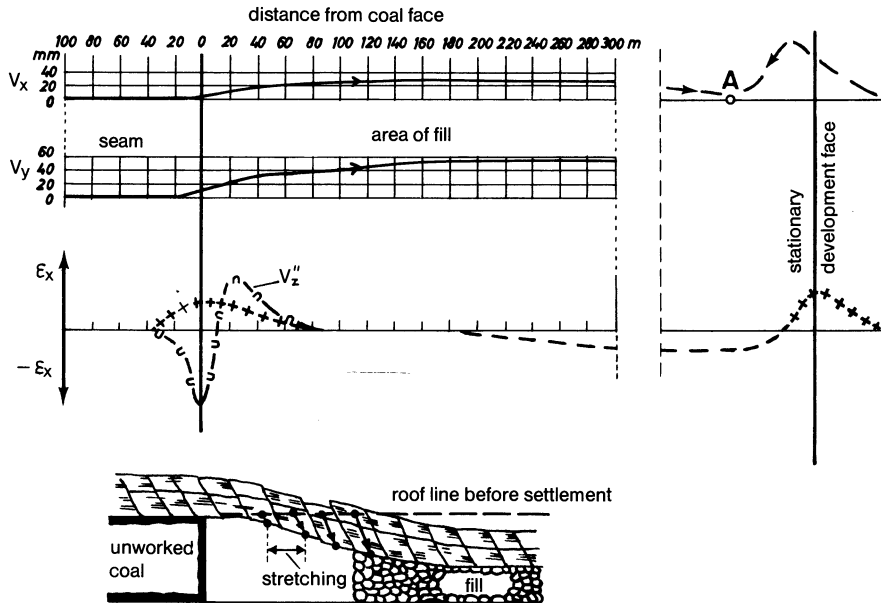


Fig. 34. Measured horizontal displacement of roof-marker points in a main gate road (v_x along the strike, v_y down the dip), and tensile and compressive strains with curvature of the roof stratum. (After Gräbsch, 1954). Blocks of roof stratum settle increasingly between sloping joints as the area of fill is approached. Despite the concave curvature, this leads to stretching (tensile strain) of the section measured

horizon is thus not subject to gravity alone: planes of fracture in fact play a larger part in determining the path of blocks in the roof stratum.

If the bonding of the immediate roof to the main roof, and the form fit of the blocks within the immediate roof, both remain intact — which can happen if there is limited settlement with pneumatic stowing or pillar working, and if there is no bed separation caused by slippage — then there will arise, on the *flexing* of the lower series of roof beds, a compressive strain over the forefield and a tensile strain over the goaf (the “after-field”, as it were) — quite the contrary to what is represented in Figs. 33 and 34. The immediate roof is tipped forward into the forefield over the yielding edge of the seam, with the result that a marker point in it moves farther into the forefield on the approach of the face. To some extent therefore, working drives marker points before it (see Fig. 44). Once the face has passed under a marker point, its displacement is reversed towards the goaf; but right up to the centre of the panel, where in theory the point should have returned to its original position (horizontally speaking) it is the movement towards the forefield which predominates. It is thus over the working face, as it passes through, that maximum displacement is reached.

The contradictory data on the measured direction of displacement which are to be found in the literature may have their explanation in this phenomenon. Results depend not only on the variation possible in the deformation of the roof stratum — elastic flexure as one member of a cohesive series of strata, or rupture

and slippage on its own – but also on the point in time selected as a reference for the investigation, whether at the beginning of the mining effect or only on exposure through passage of the face. For example, if a reference point is not marked in the roof and measured until the advancing face is under it, then all that will be observed, even with a purely flexural movement, is increasing displacement towards the “after-field”, i.e. only the return phase of horizontal movement.

With *dipping seams* there is, because the direction of roof settlement is predominantly perpendicular to the stratification, an additional displacement component going against the direction of dip; and in development headings (Fig. 33) and face gates there is also, because of the excavation effect – i.e. the stress-relief creep zone around the opening – an additional component of so-called drift convergence.

2. Deformation of the Rock Mass

2.1 Zones of Movement in the Overlying Beds

Settlement of the immediate roof over the workings, as described in the previous section, continues in higher strata if the mine excavation is so wide that it can no longer be bridged by the overlying rock and the pressure arch in the roof beds collapses (Figs. 12 and 49). In settling, the lower roof beds, if they become detached in sections from the rock bond along horizontal slip planes and loosened bedding planes, withdraw their support from higher beds. Being no longer fully supported, these immediately give under their own dead weight and the extraneous load bearing on them and lay themselves on the beds that have already gone down. The downward movement spreads in this way very rapidly until it reaches the upper earth surface (see Fig. 118). In this process six *zones of movement* ("movement" in the sense of changes in the position of points in the rock mass, independently of time) can be distinguished (Fig. 35):

- a) the floor layers, which arch elastically upwards on the relief of the perpendicular load;
- b) the seam and waste or packing layer, which is inelastically compressed both by the front abutment pressure on the solid ahead of the face and by the back abutment pressure on the mined-out goaf or, in the case of pillar working, on the pillars of salt, ore, or coal;

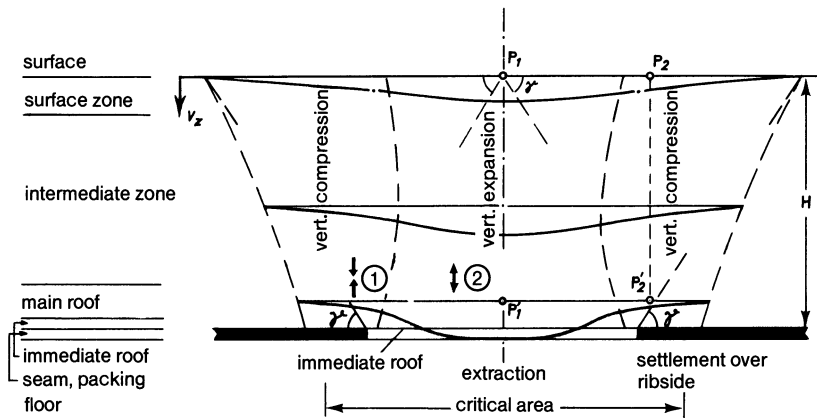


Fig. 35. Vertical deformation of an undermined rock mass in flat-lying measures (subcritical area of extraction). 1 Additional pressure on the solid resulting from abutment pressure and bending pressure; 2 load reduction and bed separation over the area of extraction

- c) the immediate roof layer, which in the area over the working detaches itself from the more rigid main roof, breaks off, and either lays itself as a laterally constrained slab on the packing or the pillars or, in mining without stowing, falls in large pieces into the excavation and fills it;
- d) the main roof, which in the case of mining without pillars settles gradually in a flat curve until wholly supported by the stowed or caved waste, and by the compressed seam ahead of the face, or which breaks off at regular intervals, leaving only a slight overhang protruding over the advancing face (Fig. 36); or which again, in the case of pillar working, sags in a wavy outline over rooms and pillars (see Fig. 41);
- e) the intermediate zone, consisting of thick beds of solid rock, which sag in a predominantly elastic fashion and in the process become detached horizontally along some bedding planes (at this level the wavy profile over rooms and pillars has largely evened out);
- f) the surface zone of loose overburden layers, which behaves plastically and follows the solid bedrock down in a trough-shaped depression.

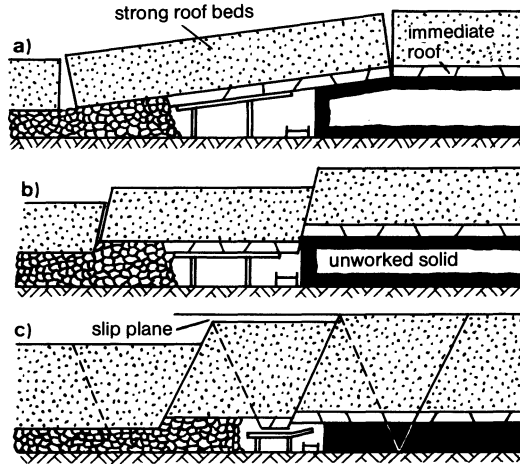
2.2 Vertical Movement and Deformation

The *lower roof beds* (the immediate roof and main roof) are exposed over the working face to high pressure, strong bending, and a rapid development of movement. Consequently their elastic stage of deformation is soon passed, even with a moderate sag (Fig. 12). In this area, elasto-plastic deformation predominates, by which is meant an initially elastic bending of cantilevered or friction-gripped sections of bed, followed by the breaking-off and down-slipping of separate blocks along structural or newly formed fracture planes. In the case of new fractures it is a matter of roof breaks in beds which, in mining without pillars, cantilever out by up to 30 m behind the advancing face. These breaks occur at regular intervals, each time precipitating an abrupt drop in the main roof. They are bending or shearing breaks, depending on whether it is bending strength at the upper surface of the bed or shear strength in the compression zone which is reached first (Fig. 36). In the latter case the roof is either sheared off at right angles to the stratification by a punching effect over a hard seam edge or, if it is able to move towards the excavation and free itself of the load while remaining parallel to the stratification (slip separation), is broken up into several wedge-shaped blocks ahead of the face by the "rolling-out" action of the advancing abutment pressure.

The relative proportions of initial elastic bending and plastic movement of fractured blocks are determined principally by such factors as bending resistance, brittleness, jointing pattern, perpendicular clamping pressure, rate of face advance, amount of bending (a function of seam thickness), and the cover load (a function of depth). In an investigation of the roof beds over a caving operation carried out 3 years later in a winze, it was established that fewer and fewer cracks were formed higher up and that visible fracturing ceased altogether 15 m above the former workings (H. F. Schulte, 1952). The quarry of paving-stone-sized pieces of rock came to an end at a height of twice the seam thickness.

Fig. 36. Principal types of main-roof break in caving or stowing without pillars. (After Everling, 1965).

a Bending fracture; **b** perpendicular shear; **c** thrust-fracture along slip planes



In the beds of solid rock in the *intermediate zone*, it is elastic deformation which predominates. The beds are not bent as strongly over the working face as the immediate roof but sag like a flat dish, without breaking. Sometimes, however, bending is accompanied by slip movements along planes of structural disturbance or along fracture planes created by earlier workings. In the lower intermediate zone, i.e., not far above the fracture-deformation zone, the bending profile corresponds to the S shape found at the working horizon (Fig. 13), as shown by measurements made in a roadway 50 m above a working (Fig. 37). In the area of transition from convex to concave curvature, i.e., in the vicinity of the “transition point” lying above the ribside, bed separation can occur if weak beds (e.g. shale) are overlain by hard, rigid beds (e.g., sandstone) and easily loosened bedding planes are thus to hand. The rigid upper beds bridge the considerable sag in the lower beds, so that millimetre-wide gaps form between them (Fig. 12), which travel with the working front and only close behind the face, over the goaf.

The solid beds of the intermediate zone can also shear along naturally existing separation planes. This is because when a thick suite of beds sags together, there arise shear stresses parallel to the stratification which, as in a beam, reach their maximum along the central axis of the suite and decrease in a parabolic curve to

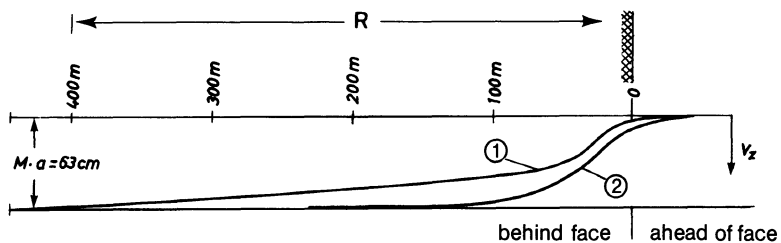


Fig. 37. Measured subsidence in the roof 50 m above an active working (*curve 1*, dynamic) and above a 16-months stationary working (*curve 2*, static), both in direction of strike. (After Spickernagel, 1964, modified). Working face at 0

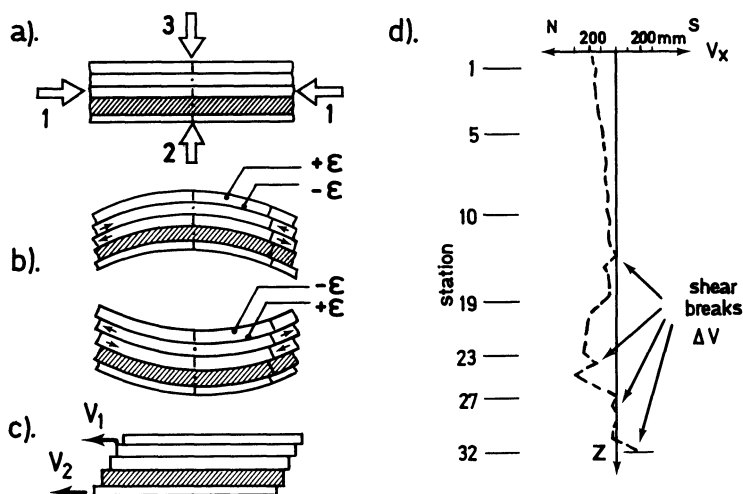


Fig. 38. Slip movements parallel to the stratification in a group of beds under tectonic or mining stress. **a** Original group of beds, of varying rock composition; **b** folding, anticlinal or synclinal – (1) from lateral thrust, (2) from upwarping, (3) from sagging. Shear movement of beds outside axis of folding. Interaction of extension (+ε) and compression (-ε) at the interface between beds. **c** “Translation”, with increasing or decreasing travel in lower layers; **d** tilting and dislocation of a mine shaft through mining-induced strata movements

zero at the outer laminae of the parting surfaces (Fig. 90). The individual beds begin to shift in relation to each other and to separate, a phenomenon which can be clearly seen at the ends of a stack of planks when made to bend (Fig. 38). The bond between the beds is broken, once the shear stresses have overcome the cohesive and frictional forces which lock their interfaces together, producing a shear fracture along those planes. The thick suite of beds is then found to be divided along the stratification into two or more thinner groups of beds. In some shafts in fact, shear movements of up to 50 cm along bedding planes have been measured.

The upper, sandy to clayey beds of the *surface zone*, or the weathering zone of outcropping Coal Measures rocks, behave in a predominantly inelastic and plastic manner in the formation of subsidence troughs. They have practically no tensile strength and follow the subsiding basement down as a flexible covering. In the process, individual grains of sand in the surface zone are thrust against each other and remain in the same position even after the ground has come to rest.

In Figure 35 it will be noticed that the area of influence of a working becomes wider and wider as it rises. The *limit of movement*, which still cannot be precisely defined in measurement terms, begins at the mining horizon some 30 to 100 m ahead of the working face and runs in a probably outward-curving line to the outer edge of the zone of surface influence, the rim of the so-called subsidence trough. The straight line joining the trough rim and the ribside of the working forms with the unworked, horizontal seam the “limit angle” or “angle of draw” γ. In flat-lying measures this angle is of the order of 40 to 65 gr, depending on the nature of the strata (in the Ruhr usually 62 gr, i.e. 55°). It serves to define the limit of surface influence – though not the limit within the rock mass. We see

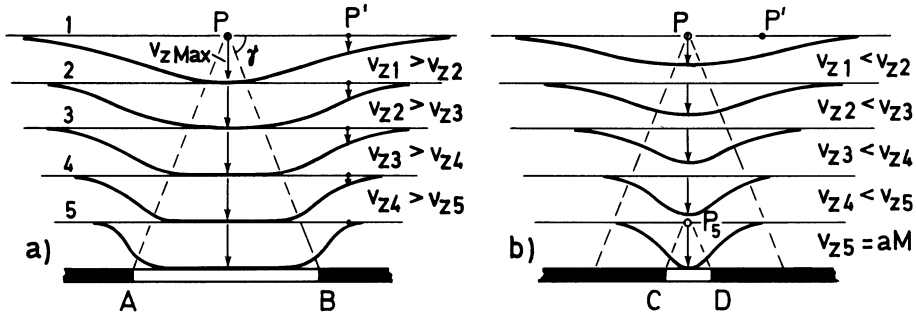


Fig. 39. The subsiding of overlying strata at horizons 1 to 5, with a critical area of extraction $A-B$ (left) and a subcritical area $C-D$ (right)

then that, with increasing height above the mining horizon, the bending of the rock strata begins farther and farther out in the adjoining zone, with the result that the subsidence trough of each bed from the main roof to the upper earth surface is wider and flatter than the one below (Fig. 39). This progressive shift of the trough rim into the zone adjoining the working is connected with the anticlinal curvature in that zone. In the same way as the seam ahead of the face becomes vertically compressed and transversely deformed, each roof bed in turn is compressed by the beds above it as they sag in an anticlinal fold; and the yielding of each bed increases the distance between supports, i.e. the span, of the bed above. In this way the strata load is spread farther and farther into the zone around the working.

Each bed both bears like a weighted beam on the bed below and at the same time acts as a yielding underlay for the bed above. The consequence of this dual load-and-support role is that the limit of underground movement is shifted farther out with each successive bed. From practical experience of the "angle of draw" it has been found that the limit line of rock deformation is steeper in hard, bend-resistant beds than, for example, in the loose ground of the surface layer – a fact which can be explained by the lower compressibility of the harder beds.

Taken in a vertical direction, the rock mass undergoes (see Fig. 35) *expansion* over the goaf and *contraction* (i.e. compression) over the unworked seam. The deformation forces resulting, on the one hand, from the anticlinal downward folding of the beds over the ribside, and on the other hand, from the additional abutment pressure which bears on the solid adjoining the working as the beds over the working are undermined, have the effect of reducing the bed thickness at this point. In contrast to horizontal compression, this is known as "compaction" (sometimes as "contraction", the converse of "expansion") and is accompanied by, in fact, a contraction in voids volume, which nevertheless remains at as much as 15% to 20% in Coal Measures rocks. In the de-stressed zone over the working, on the other hand, the overlying beds exhibit a greater thickness than before: they "expand" at right angles to the stratification. This deformation process in the undermined rock mass – vertical expansion over the goaf (except in pillar working), and vertical compaction over the unworked solid – can be explained, in terms of mining dynamics, by the flexural stress and abutment pressure prevailing over the ribside, and the perpendicular load reduction and stress relief

which take place in the intermediate zone of influence. The share of the load of the undermined strata taken over by the unworked abutment decreases with increasing height above the mining horizon, and the anticlinal curvature and bending pressure likewise become less with increasing height (see Chap. 4, Sect. 4.5). The degree of compaction will therefore diminish in the higher beds, which – as is clear from the stratigraphic sequence in most mining districts – are in general not strong enough to bear the far-reaching load redistribution and gap bridging entailed. In large part their local dead weight and extraneous load bear directly on the underlying strata, with the result that vertical expansion from load reduction must also fall off at higher levels. Below the mining horizon, the floor beds, like the roof beds, are compressed under the unworked seam by the abutment pressure, while under the working area they can expand upwards as they become de-stressed.

Although exaggerated vertically, the transverse profiles of subsidence in the main roof, the intermediate zone, and the surface layer represented in Fig. 35 are in agreement with the rock deformation processes described above. At the centre of the zone affected, the amount by which the strata drop along the vertical line $P'_1 - P_1$ decreases in an upward direction, signifying vertical expansion, whereas over the abutment, up the line $P'_2 - P_2$, it increases, signifying vertical contraction (i.e. compaction). The difference in the amount of drop at two vertically separated points in the rock mass thus indicates whether expansion or contraction has taken place, and this can be expressed as a linear change in mm/m, related to the original vertical distance. It will signify expansion if the lower point has dropped more than the upper, and compaction if the upper point has dropped more than the lower. For example: drop in upper point 30 cm, drop in lower point 20 cm, vertical distance between points 50 m; i.e. difference in drop (upper more than lower) 100 mm, therefore compaction $-\epsilon_z = 100 \div 50 = 2 \text{ mm/m}$.

In flat-lying measures, the *maximum subsidence* $v_{z \text{ max}}$ (i.e. the maximum subsidence *occurring*, as distinct from $v_{z \text{ Max}}$ – with a capital letter – the maximum subsidence *possible*) is found at each horizon at the centre of the subsidence profile (the vertical line through P in Fig. 39). Observations show that, as extraction progresses, this “maximum occurring” tends towards the “maximum possible”, or *complete*, subsidence $v_{z \text{ Max}}$. They also show that, at all horizons below a point P on the surface, this is attained when extraction covers a zone enclosed by lines projected downwards at the “angle of draw” from that point (or rather when it covers an extra 10 or 20 m around that zone). Once all rock layers on the vertical line at the centre of the circular “critical area” of extraction thus formed (A – B in Fig. 39) have sagged as far as they can, the original cover-load pressure will have been restored and both vertical expansion and horizontal compression will have been virtually cancelled out, returning to almost zero. The amplitude of this complete subsidence thus depends on the extent of convergence C at the mining horizon. It is usually expressed in terms of the original thickness M of the seam to be extracted, as measured in the solid during development and entered on the mine plan:

$$v_{z \text{ Max}} = C_{\text{Max}} = aM. \quad (16)$$

Here “a” represents the Subsidence Factor, established empirically by comparing $v_{z \text{ Max}}$ with M . As measured in the Ruhr, it is of the order of 0.90 to 0.95 with total caving, and 0.45 to 0.55 with solid stowing (see Fig. 10).

Points in the rock along the curved boundary line between expansion and contraction zones, shown in Fig. 35 above the area just within the edge of the working, will have dropped by half the amplitude of complete subsidence in the case of a critical area of extraction without pillars.

If within the cone formed by dropping the limit angle γ from the point P in all directions only a part $C - D$ of the critical area $A - B$ (Fig. 39, right) is worked – that is to say, if only a “subcritical area” is worked – then the maximum subsidence $v_{z \text{ max}}$ occurring in the upper four horizons on the central line through P will remain less than the maximum possible subsidence $v_{z \text{ Max}}$. In other words, v_{z1} to $v_{z4} = v_{z \text{ max}} < aM$. Since in this case the amount of subsidence experienced by points lying one above the other decreases in an upward direction, the rock mass over the subcritical area has been vertically extended. Only the central point P_5 , near the bottom, has subsided by the greatest possible amount $v_{z \text{ Max}} = aM$, because for this point the area $C - D$ worked constitutes a critical area: lines erected inwards from C and D will meet at P_5 . On the other hand, the amount of subsidence on the vertical line from P' to the ribside increases in an upward direction, even when the area of extraction is subcritical: which means that the rock mass here has been vertically compressed, just as with a critical area of extraction.

As already mentioned, strata which have been compressed from all sides by the cover-load pressure expand on the creation of an underground cavity (cf. the expansion theories of Heim in 1878 and Trompeter in 1899). Insofar as an elastic movement on the reduction of load can be assumed in this process, the maximum possible expansion on relaxation is calculated on the basis of Hooke's Law:

$$\epsilon_z = \pm \frac{1}{E} \left(\sigma_z - \frac{\sigma_x + \sigma_y}{m} \right). \quad (17)$$

This gives a figure of approx. 0.90 mm/m when perpendicular pressure σ_z drops from 20 to 0 MPa, given a reciprocal Poisson's ratio of $m = 5$ for Coal Measures rocks, a modulus of elasticity of $E = 2,000,000 \text{ N/cm}^2$ or 20,000 MPa, and a horizontal pressure in accordance with the formula for surrounding pressure (8), namely $\sigma_x = \sigma_y = 0.25 \sigma_z$ at a depth of 800 m. If the vertical pressure over the unworked seam rises to three times the cover-load pressure, and if horizontal expansion remains inhibited, then there is an increase in compaction of 1.8 mm/m. Greater deformations than this, which have been measured underground above the working area (Potts 1964) and in shafts, are due to faulting, bed separation, creep phenomena, and plastic displacement.

The extent of the movement on the reduction of load 20 m under an active working can be seen in Fig. 40. On the passage of the working front (at $x = 0$), observation station 1 in the floor of a roadway under the working has heaved upwards by about 100 mm, whereas station 2 in the roof of the roadway has dropped about 50 mm because of the effect under the working of the abutment pressure above it; but 40 m behind the face (at $x = 40$) the roof station also follows

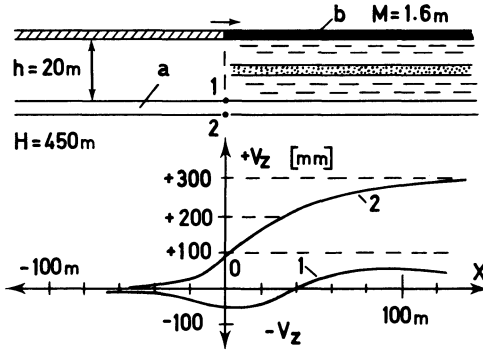


Fig. 40. Heaving ($+v_z$) or lowering ($-v_z$) of an observation station (1) in the roof and (2) in the floor of a roadway a 20 m under a caving operation advancing in the same direction as the road. Top: cross-section; b seam. (After Labanowicz, 1979)

the upward movement from the reduction of load, which in the case of the floor station ultimately amounts to 300 mm.

In *room-and-pillar working*, the roof over the working area is supported from pillar to pillar. The pillars themselves are vertically compressed and horizontally extended by the specific pressure exceeding γH (normal cover load), a fact that can be seen from the way the sides of the pillars bulge out into the chambers (Fig. 17). The roof over the pillars up to three or four times their height h shares in their lateral expansion $+\epsilon$ (Fig. 41). Above the chambers, the immediate roof is squeezed between the compression zones of the pillars ($-\epsilon$), whereas vertically it expands into the chambers on the removal of load ($+\epsilon_z$). The main roof settles in a wavy outline, with the hollows over the chambers and less settlement over the pillars, corresponding to the compaction both of the pillars and of the floor under them. When the movement reaches the higher roof beds, this wavy outline is smoothed out. Above the main roof, the subsidence and the deformation – which consists only in expansion over the working area and compaction over the unworked seam – are similar to those in the roof of a working without pillars (Fig. 35).

Subsidence measurements taken in a borehole over a potash mine revealed, as depicted in Fig. 42, decreasing subsidence towards the higher levels, from 90 mm at a height of 42 m (point P_1) to 45 mm at a height of 137 m (point P_8). The hatched areas illustrate the difference in subsidence between neighbouring points. Between points P_1 and P_2 vertical expansion amounts of 1.8 mm/m.

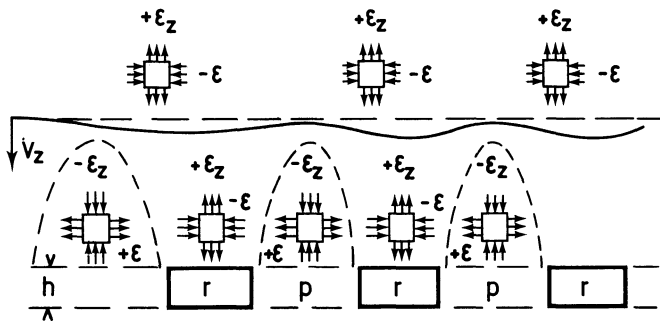
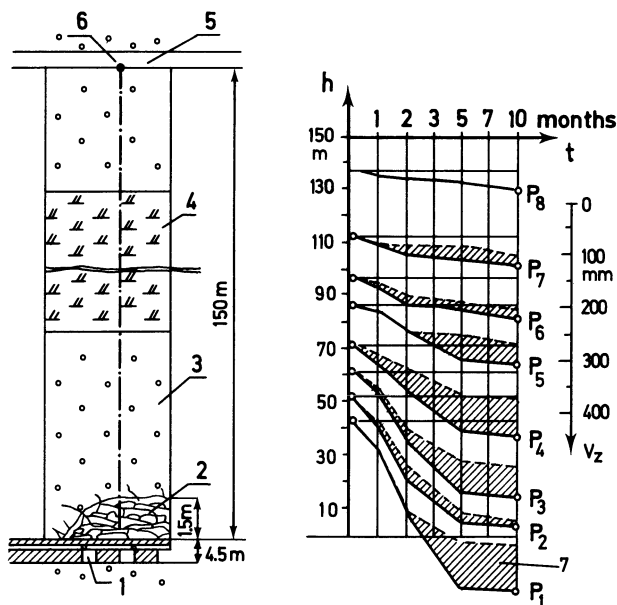


Fig. 41. Deformation of the roof over rooms (r) and pillars (p). Wavy outline of settlement at four times room height ($4h$) above the workings and the adjoining solid. (After Fenk, 1974, modified)

Fig. 42. Subsidence in the overlying beds of a potash mine, measured in a borehole. *Left:* borehole and strata section. *Right:* subsidence at stations P_1 to P_8 over 10 months. 1 Room-and-pillar working with caving of the top slice of 1 m; 2 caved area; 3 salt formation; 4 dolomitic marl; 5 exploration drift; 6 borehole; 7 bed separation (vertical expansion) between two observation stations. (After Permjakow and Nesterow, 1977)



2.3 Horizontal Movement and Deformation

In considering components of horizontal movement — lateral displacement v_x and v_y , and differential displacement, designated as extension $+\epsilon$ and compression $-\epsilon$ — a distinction has to be drawn between the “rolling-out” of brittle-roof and main-roof beds, mentioned earlier, and their elastic sagging. The abutment pressure ahead of the face, both perpendicular and parallel to the stratification, forces the blocks of roof bed, which have been broken off over the face edge, along slip planes and joints into the excavation area. This is known as “strata displacement”. There the roof bed, which has become somewhat relaxed and stretched in the course of settlement and bending, is recompacted in parallel with the stratification by the displaced blocks of main roof (Fig. 36c). The displacement of marker points in a brittle roof will thus be in the direction of the goaf, as illustrated in Fig. 34. Otherwise behaviour is as in elastic bending.

First, let a rock mass be considered which is made up of beds that are capable of slipping. On bending, such a rock mass would experience movements parallel to the stratification, as in Fig. 38. In the course of subsiding, each individual bed could shift in relation to the bed above and below without appreciable hindrance from friction along bedding planes. On the concave upper surface of each bed there would arise, as in a beam that is bent, a horizontal compressive strain $-\epsilon$, and on the convex underside a tensile strain $+\epsilon$. At the bedding planes therefore (looking at the vertical central line $P'_1 - P_1$ in Fig. 35), there would be an unstable change from compression to tension at the point of transition to the next higher bed, while above the middle of the bed — its neutral axis — this would become compression again, and so on upwards. It would be the same at the margins of

the affected rock. Abrupt transitions like this from compressive to tensile strains within a short range of depth have not so far been observed in rock. They are only imaginable at the shearing surfaces of bedding planes: within the rest of the rock mass the bond between the various beds remains intact, under the pressure of the overlying strata and the friction along bedding planes.

It follows that the rock mass deforms as a whole, like a very thick slab in which extensive zones of tension and compression form during bending, without any neutral axis parallel to the stratification becoming discernible. It is true that on the vertical lines $P_1 - P'_1$ and $P_2 - P'_2$ there are various preliminary signs of horizontal deformation, but the zone of extension, contrary to the bending deformation of a thin, elastic beam, stretches diagonally from the central portion of the underside to the outer area at the top. This can be related to the torsional shear stresses produced when a thick slab is buckled. The outer compression zones can be explained by the bending of the roof over the seam edge, as a result of which points in the rock are moved towards the adjoining zone (Fig. 44, top left). The other zones of deformation — extension in the centre of the underside, near P'_1 , compression in the middle zone, under P_1 , and extension at the top near the outer edge — are similar to those in a curved beam.

Calculations on the basis of the finite-elements method for homogeneous, elastic belts of strata reveal that horizontal displacement v_x immediately above and below a working is in an outward direction, towards the adjoining zone (Fig. 43). The fact that displacement over the goaf increases in degree from the centre outwards to right and left implies that the rock mass is being stretched, or “extended”, by several mm/m; and in the area of the falling displacement curve over the adjoining solid, where an inner point is displaced more strongly outward than another lying a metre beyond it, the roof is clearly under compression (cf. Potts, 1964; Ortlepp and Cook, 1964; Oglobin, 1960). At the upper earth surface, on the other hand, displacement is in the direction of the trough centre, with the result that a tensile strain arises at its periphery and a compressive strain at its

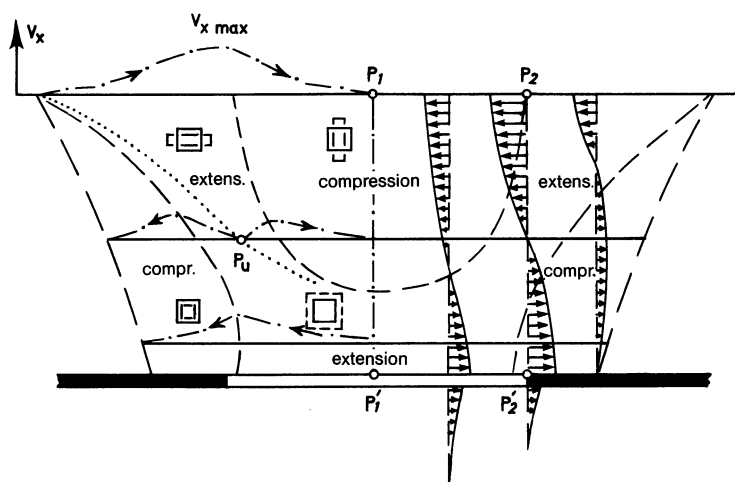


Fig. 43. Horizontal deformation of a rock mass affected by underlying workings, with no bedding-plane slip (i.e. as a continuum)

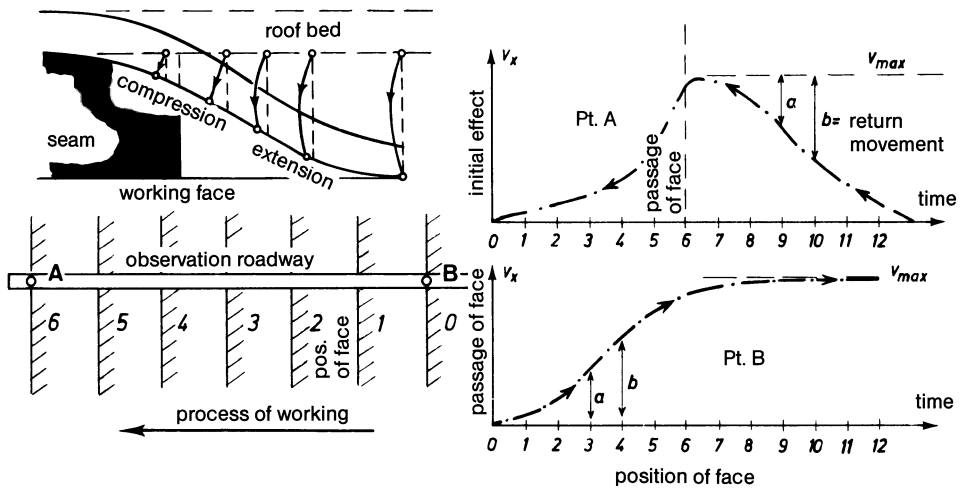


Fig. 44. The displacement v_x of the marker points, A and B, in a compact roof bed, as measured in the direction of the roadway axis from the moment of exposure by the face (0) onwards (right). Left: section and plan

centre, from the point of maximum displacement on. In the intermediate zone, displacement is outward at the periphery and inward over the area of extraction. The point of reversal for displacement P_u follows the dotted diagonal from horizon to horizon through the zone of lateral extension.

There have been many observations of the movement of stations in an elastically bending main roof towards the zone adjoining the working (e.g. Niemczyk, 1949, his Figs. 177 and 178). Nevertheless, such displacements have often been measured, as was the case with point B in Fig. 44, only on the passage of the face, from the moment of exposure, with the result that the return displacement gave the impression of movement towards the goaf alone.

Points in the surface zone, the main roof, and the floor attain their maximum displacement over or under the working face (Fig. 43). In the intermediate zone, displacement maxima lie over the periphery of the goaf and the adjoining zone. Except at the centre of the working, originally straight lines like $P_2 - P'_2$ become twisted into an S shape from the working horizon to the upper surface and incline inwards. Similarly, a cross-sectional plane which was vertical before bending ceases to be a plane and becomes buckled, contrary to Bernoulli's theorem covering thin beams. At the seam horizon, the vertical-line tracing displacement is off-set laterally, because the floor moves outwards less than the roof layer.

As regards a comparison of deformation in a vertical and a horizontal direction, it has been found that a cube of rock is compressed from all sides in the lower abutment zone, drawn outwards from all sides in the lower roof zone, deformed into a horizontally elongated block in the upper abutment zone, and deformed into a vertically elongated block in the upper roof zone (Fig. 43). Over a critical area of extraction, the original state of no tensile or compressive strains is restored on the central line $P_1 - P'_1$.

Except at the surface, few measurements have been made of the horizontal component of displacement v_x in a – generally inaccessible – rock mass affect-

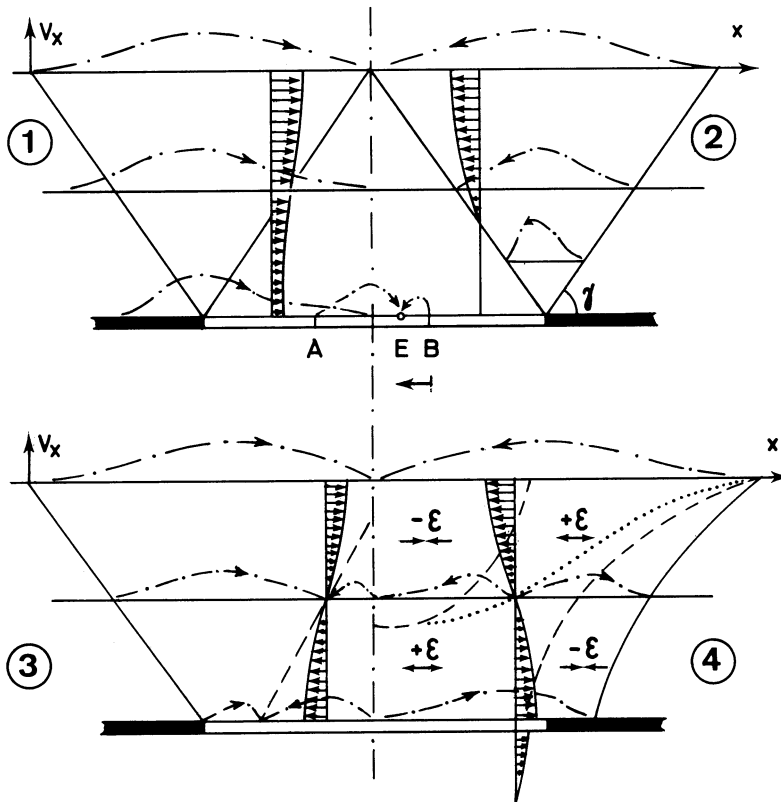


Fig. 45. Horizontal displacement of points in the rock mass above workings, after observations or models proposed by (1) Gräbsch, (2) Bals, (3) Avershin, and (4) Kratzsch (finite-elements method)

ed by underlying workings; and displacements measured in roadways and shafts affected by mining have themselves been affected by load reduction and fracture deformation in the rock (Fig. 33). Consequently, other models than that represented in Fig. 43 have been proposed for horizontal movements in a rock mass with underlying workings. Thus some authors assume a displacement v_x which is in the same direction in the rock body as at the surface, and in connection with which, solely with a critical area of extraction A–B, the zero point in the intersection of the contrary movements at E lies excentrically on the side of the stationary rib where the face was opened out (Fig. 45, example 1). Another conception is that, within the limit-angle cone erected above the working area, complete subsidence occurs, and all points for which the working area amounts to a critical area of extraction return to their original horizontal position x, y . On this model, a zone of horizontal displacement arises only in the inverted limit-angle cone opening out above the face edge (Fig. 45, example 2). In an equivalent model of the rock mass, displacements in the direction of the panel centre were observed at its periphery, and others in an outward direction were observed at its centre (Fig. 45, example 3).

In a static model, the excavation is opened over its whole length at once. In reality, however, the working area grows in size only slowly, and only a narrow

strip of the face working is opened at first. Nevertheless, in the direction v_y , parallel to the working front, points in the rock mass are affected by working along the full and final length of the face from the start. Consequently, there is a difference between the amount of initial displacement v_x and v_y , and even after the passage of the face there is no return movement in the component v_y , as there is in v_x (see Fig. 47). In any case the direction given to movements in advance by the existing jointing of the rock is something which cannot be simulated in a model. For instance, displacements in ground which has been worked through several times before and is consequently thoroughly fragmented will differ greatly, both in degree and in direction, from those in ground which is being mined for the first time.

A comparison of Figs. 35 and 43 will show that compressive strains are paired with expansive, and tensile with contractive. The same applies to the main roof in room-and-pillar working (Fig. 41). The zones of tensile and expansive strain, which in salt rock are so conducive to the opening of cracks, are separated by a neutral zone (indicated by a dotted line) in which deformation is practically zero.

2.4 Rock Kinematics in Steep-lying Measures

In a steep-lying group of rock beds, the point of maximum subsidence occurring, $v_{z \max}$, alters from bed to bed, at first perpendicularly to the bedding, then – in the middle and upper roof beds – upwards in a curved, “shovel-shaped” line lying between the bed perpendicular and the vertical, from the point P_m of maximum settlement of the immediate roof (Fig. 46). The *subsidence trough* for each horizon is therefore unsymmetrical in horizontal terms. Strata movement which in the main roof is still conditioned by sag perpendicular to the bedding acquires in higher strata a direction which is nearer and nearer to the vertical action of gravity. The subsidence curve in the upper zone of influence slopes more on the footwall side than on the hanging-wall side. Furthermore, the limit angles¹ γ_L and γ_H carried from the upper and lower ribsides of the working to the rim of the surface trough differ in size, as strata movements over the lower end of the face spread far out into the overlying beds and produce a very flat trough on the hanging-wall side. The strata over a steep-lying stowed goaf sag less than when it is flat-lying, because the component of pressure at right angles to the stratification is weaker, as is clear from Eq. (7). The load-relieving movement of the footwall as it heaves into the mine excavation can sometimes, in shallow workings, exert an influence right through to the surface, with the result that the footwall limit angle in such cases is smaller than the angle of dip α of the seam (Fig. 155).

In the horizontal representation of the displacement of points within the rock body (Fig. 47), a movement directed against the dip predominates. This “uphill”

¹ The limit angle γ_L from the horizontal at the surface point P leads to the lower end of the working face, the limit angle γ_H to the higher end (see Fig. 155). In the delimitation of the subsidence trough at the surface, the horizontal arm of the angle γ_L at the upper face leads to the deeper strata, and that of the angle γ_H at the lower face to the higher strata.

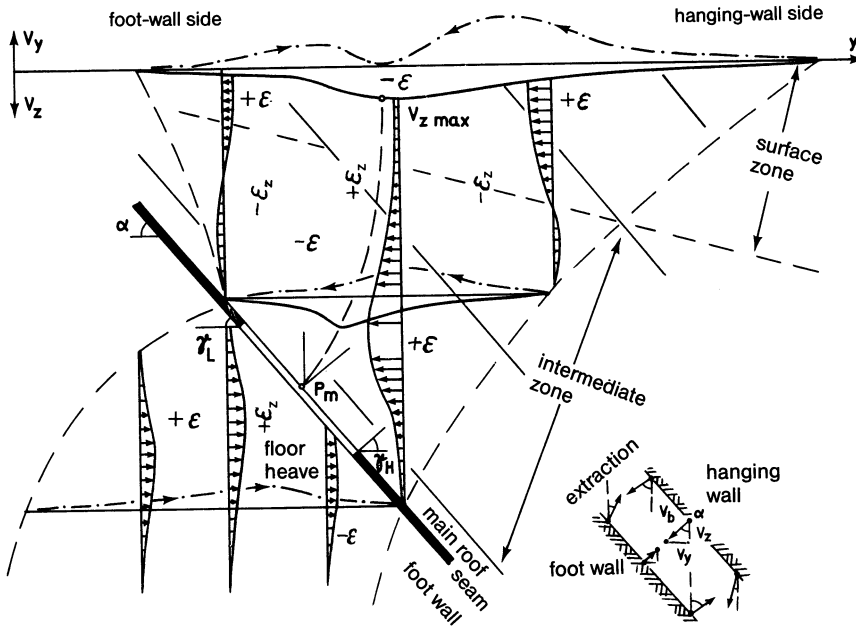


Fig. 46. Movement and deformation in steep-lying beds, calculated in accordance with the finite-elements method

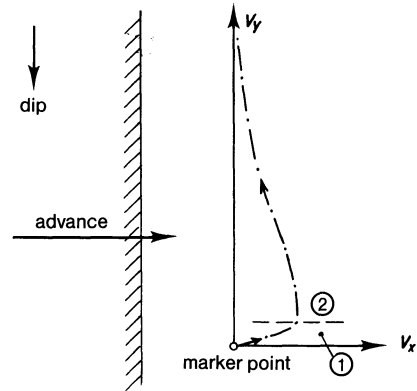
pointing component of displacement arises with a roof which goes down over the excavation more or less at right angles to the stratification (Fig. 46, inset), thus:

$$\begin{aligned} v_y &= v_b \sin \alpha \\ v_z &= v_b \cos \alpha. \end{aligned} \quad (18)$$

In the transverse section portrayed in Fig. 46, the greater the dip, the more this movement increases the horizontal displacement towards the footwall in the main roof and intermediate zone. The angle between the vertical and the resultant movement increases from the bottom up in the roof of the excavation, and from the top down in the footwall (Fig. 46, inset). In the direction of the strike (v_x), the marker point is shifted somewhat towards the zone ahead of the face as the latter approaches and then, after the face has passed, back again (Fig. 47). In a transverse direction (v_y), the displacement is not reversed, even with a critical area of extraction (which in any case is only encountered, in steep-lying measures, if the workings are at a shallow depth). This has been shown by underground measurements (e.g. Spickernagel, 1964). Where the dip is steep, horizontal displacement along the strike amounts to only about one-tenth of transverse displacement. Consequently, displacement along the strike, which is commonly observed in roadways, is strongly influenced – and can be overlain – by the loosening of the ground to a depth of 5 or 7 m which takes place at the sides of roadways. Beneath the workings, where the beds expand and heave, displacement is towards the hanging-wall side.

The zone of vertical contraction, or compaction, over the ribsides reaches into the stowed goaf, making the zone of expansion over the goaf narrower than

Fig. 47. Horizontal displacement of a marker point in the roof (1) before and (2) after passage of the face, in steep-lying measures



in flat-lying measures. The zones of compression and extension are indicated in Fig. 46, but a precise delineation is not possible. It will be noticed that tensile strain reaches as far as the working on the footwall side, whereas on the hanging-wall side compressive strain predominates. In the surface zone a certain similarity with deformation conditions in flat-lying measures is found.

In Fig. 46 the dip of beds within the zone affected by workings remains unchanged. In practice, however, the rock mass is usually folded and dislocated. Sometimes the dip becomes gradually flatter with depth, as in Working 1 in Fig. 48, which lies on the southern limb of a syncline; or the direction of dip changes within the zone affected, as is the case with Working 2, which lies at right angles to the beds on the other side of the bottom line of a syncline. Often the upper strata have slipped down fault planes which dip to the workings; these planes then form the outer limits of movement and bring about abruptly large subsidence movements and tensile strains (Working 3). A further factor which may vary is the thickness of overburden within the affected zone. Strata movements in such a variety of bedding conditions can be very irregular and have been little researched.

The following phenomena must be reckoned with in *folded strata*: horizontal strata pressure derived, in accordance with Eq. (8), from cover-load pressure can

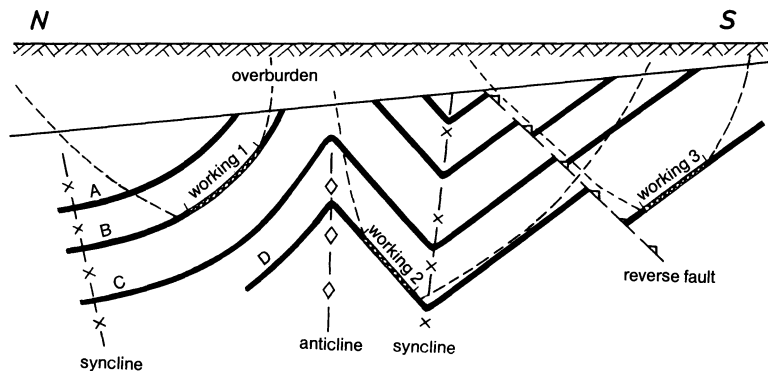


Fig. 48. Section through a folded and structurally disturbed rock mass, showing four seams, A to D. Broken line: limit of influence of workings 1 to 3

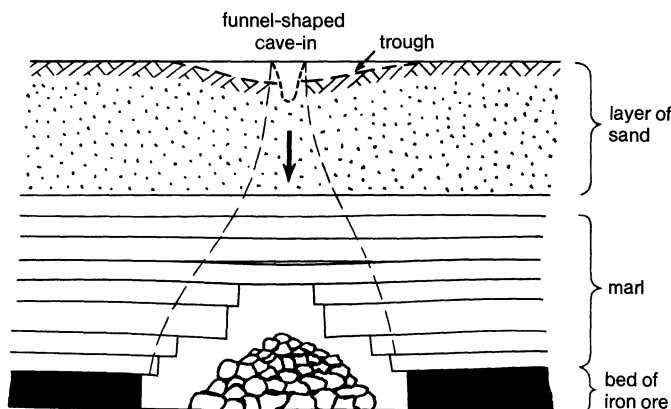


Fig. 49. The dome-like collapse of beds overlying a narrow excavation near the surface

be less strong down the dip, in the x direction, than along the strike; and horizontal compressive stresses in a syncline can be greater, and in an anticline smaller, than predicted by Eq. (8). In dislocated strata, similar stress anomalies occur in the fault blocks, grabens, horsts, and step faults (Kratzsch, 1979). Added to these strata-load stresses are latent structural forces tending to folding, and also so-called inherent stresses, arising from forces which cause the upthrow of a fault block or an anticline and which cannot then “unwind” because of the geometry of the movement caused; or other, diagenetically induced stresses. For all these reasons, horizontal strata pressure can in some areas be greater than vertical pressure.

2.5 Surface Fracture over Shallow Workings

When working is near the surface, there is a possibility of surface fracture, either before or after the surface itself has subsided. From observations in iron-ore mining, a roof of marl beds caves in a dome shape over the mine excavation (Wagner, 1956). When the dome of projecting beds of marl has reached a height and a width at which it can no longer support the weight of the overlying beds of diluvium, it caves to the surface. The stresses in the rock are thereby relieved, and the surface subsides around the point of rupture (Fig. 49). *Surface fracture* does not occur if enough caved and loosened waste accrues to provide fresh support for the projecting roof beds. Surface fracture can also be avoided by a long and narrow working face, giving rise to no dome of caving over it. Overlying beds subside in a trough-shaped depression over a working which is kept in a straight line; but if it develops an irregular pattern (Fig. 50), the roof, which is bound on three sides to the solid rock mass, spans the salient A of the workings as a single slab; and when this load-bearing slab breaks, a caved dome forms.

The subsidence factor in these conditions amounts with caving to only 0.7, instead of 0.9 as with workings at depth. At depths of 50, 100 and 200 m, the cavity of 30 cm between lagging and roof over a development heading gives rise to subsidence at the surface of 2 cm, 1 cm, and 0.5 cm respectively.

In ore-mining also, with steep-lying veins, there can be a collapse of the unmined vein overhead. If this happens in competent rock, vertical-sided cracks 40 m deep, 100 m long, and a metre wide will open up at the surface (Reichenbach, 1979). Less competent ground over old, unfilled workings will collapse at three and a half times the height d of the mine cavity when the voids volume of the roof, or "back", as verified by drilling, increases from $n_1 = 10\%$ to $n_2 = 30\%$ (Fig. 51).

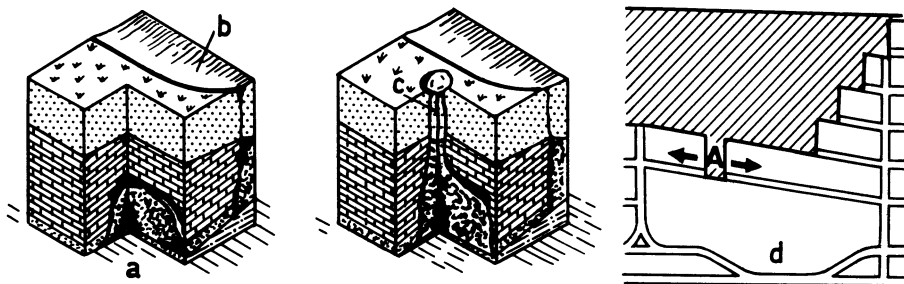


Fig. 50. Stages in surface fracture over a shallow working. (After Wagner, 1956). *a* Dome-shaped caving over working chamber; *b* trough over linear working front; *c* caving of dome to the surface, forming a chimney with funnel at top; *d* iron-ore workings in plan

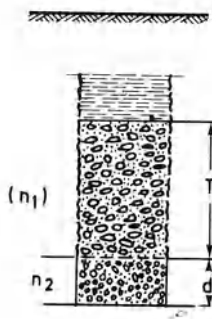
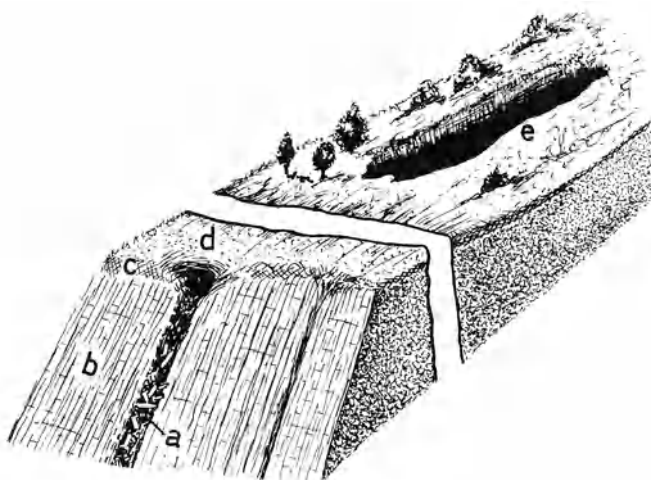


Fig. 51. Estimating the height T of the fracture zone over a former excavation d . (After Rom, Nendza, and Hollmann, 1978). n Porosity

$n_1 = 10\%$
$n_2 = 30\%$
$\Delta n = 20\%$
$d + 0.10 T = 0.30 (d + T)$
$0.7d = 0.20 T$
$T = \frac{0.7}{0.2} d$
$T = 3.5 d$
$d = 2.5m \quad T = 8.75m$

Fig. 52. Surface fractures over old and steep workings. *a* Old open working with timber supports; *b* carbonaceous rock; *c* soil layer; *d* funnel-shaped cave-in; *e* elongated surface fracture along strike of steep coal-seam. (After Hollmann, 1968)



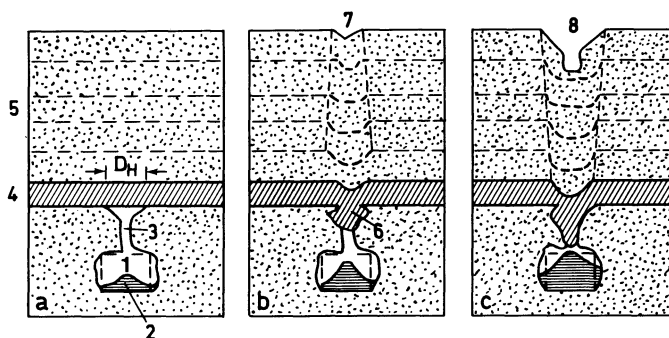


Fig. 53. Stages in the development of a surface fracture in loose ground. **a** Collapse of roof over a rectangular working; **b** collapse of cohesive bed (shown *hatched*) into chimney over workings; **c** cone-shaped cave-in at the surface. (After Fenk, 1977)

In steep coal seams, the upper sections left unmined for safety may later find their way into the excavations below if these have not been filled and the timber supports have rotted away. The surface then caves in and forms elongated hollows along the strike of the seams (Fig. 52).

The sand model depicted in Fig. 53 gives an insight into the timing of the development of a cave-in in loose ground intersected by a cohesive bed. After the failure of the immediate, non-cohering roof, material from it (2) runs into the mine excavation (1), forming a chimney (3) up to the cohesive bed (4). If the top of this chimney funnels out sufficiently to expose the cohesive bed to a critical width D_H , the bed collapses (6) and the overlying layers of sand (5) subside within a cone described by the angle of break. At the surface a depression forms (7). This widens and deepens into a cone and then into a bell-shaped cave-in (8) with steep walls, until the underground excavation has filled up. Over shallow lignite workings, surface cave-ins 2 m wide and up to 17 m deep have been observed, leading here and there to open chimneys.

The critical span D_H is reached when, as set out in Fig. 54, the dead weight $V_D \gamma_b$ of the cohesive bed (2), increased by the load p_0 of the overlying ground (3), creates a tensile stress σ equal to the tensile strength of the cohesive bed at the conical plane M_D . This statement leads to the equation

$$p_0 + V_D \gamma_b = \sigma_v M_D \quad (19)$$

in which shear stress and bending stress on the collapse of the bed are ignored. σ_v is the vertical component of the tensile stress σ , related to the angle of inclination

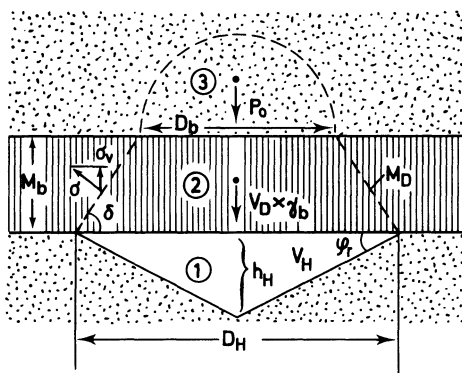


Fig. 54. The critical span D_H for the collapse of the cohesive bed (2) into the cone-shaped opening (1). (After Fenk, 1978)

δ of the outer plane of the conical body (2). The solution of Eq. (19) to obtain D_H leads to a cubic equation (Fenk, 1978).

Of decisive influence on the probability of a cave-in in the cohesive bed is – quite apart from the age of the workings – the volume V_H of primary caving, i.e. the volume of the cone-shaped enlargement at the top of the chimney (1). The deeper the workings, the thicker the cohesive intermediate bed M_b , and the more jointed the immediate roof, the less likely is it that roof fractures will occur. A lowered ground-water table increases the likelihood.

2.6 Chronological Stages in Strata Movement

The strata movement set off in the mine excavation is propagated to the surface only when the area of extraction has reached a certain size. Thus measurements of changes in shaft length taken at the edge of a 120 m wide face which had advanced 100 m showed subsidence and vertical expansion reaching only 100 m up the shaft (curve 1 in Fig. 55). During the year's interruption of working which followed, vertical expansion in strata and shaft extended farther upwards (curve 2). The first signs of major subsidence at the pit-head level were observed 5 months after the resumption of working, when the panel had reached a length of 350 m in the direction of advance.

Consequently, during the stage of opening out a face, only the lower part of the intermediate zone participates in the subsidence of the main roof, and the settlement occurring at the mining horizon is compensated for and absorbed by expansion and loosening in the next higher layers. The increasing volume of subsidence accompanying the progress in working cannot be absorbed by vertical ex-

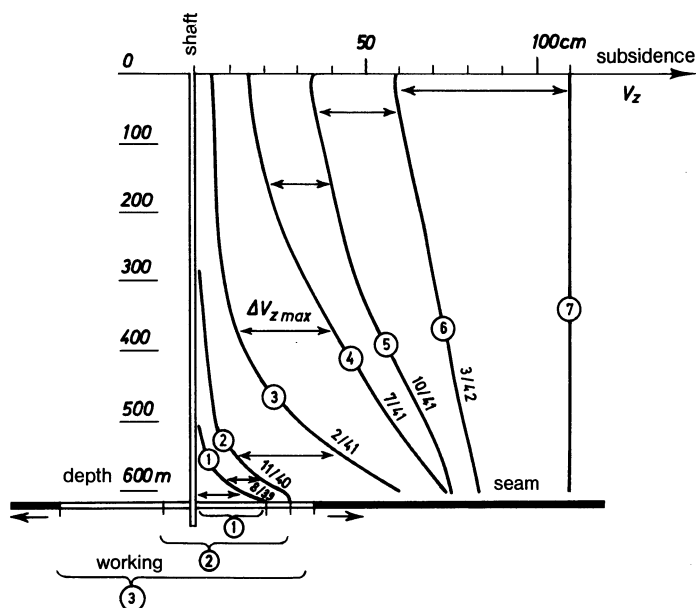


Fig. 55. Changes in shaft length with mining of shaft pillar conducted from the shaft outwards. (After Caye). 7 Subsidence after extraction of critical area

pansion once the panel has reached a length of some 300 m, and subsidence in the uppermost layers therefore follows. From this size of panel onwards the whole sequence of beds underlain by the workings sags, but because of internal deformation the amount of subsidence varies at different horizons, so long as the area of extraction remains subcritical. Every irregularity in the progress of working, for example on rest days, is reflected in delayed or accelerated movement. In parallel with a working rhythm governed by shifts, subsidence at points in the rock proceeds jerkily – a fact which can be verified seismically. A temporary “hang-up” of the subsidence process in an upper layer is imaginable only if the upper layers are particularly rigid and consequently settle as a single, uninterrupted length on the already subsided rock mass beneath, when the panel has reached a greater length or span than usual.

The rate of subsidence measured down a vertical series of points in the rock mass varies continuously according to progress in the working and the intensity of strata deformation (see Fig. 42). For example, if working is conducted as a double-unit face in an outward direction from the vertical series of points, the area of most rapid subsidence at any given moment travels from the bottom up (Löffler, 1951). The difference in subsidence between two consecutive measurements, as recorded in Fig. 55, is greatest first in the main roof, then in the intermediate zone, and then, as a critical area of extraction is approached, in the surface zone. With working in an inward direction, the reverse is the case: the zone of most rapid subsidence travels from the top down, during the period of influence of the workings. In both cases the most rapid rate of subsidence of all occurs in the immediate and main roof at the time the marker point is being undercut. At the centre of a critical area of extraction, the final amount of subsidence is the same at all horizons (curve 7).

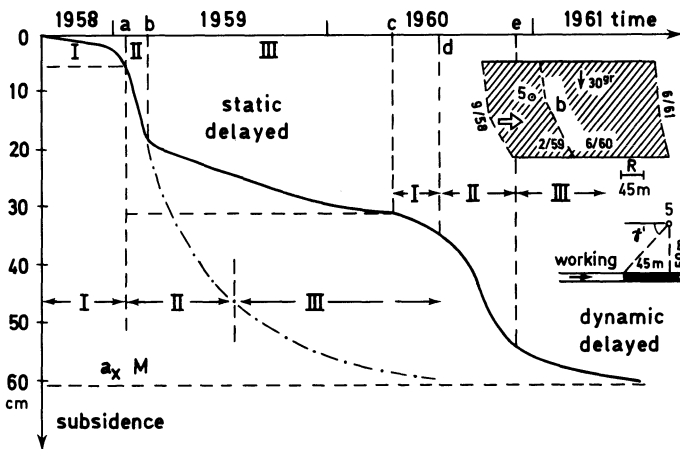


Fig. 56. Subsidence over a period at a point (No. 5) in the rock mass 50 m above the workings. (After measurements by Spickernagel, 1960). Working halted from February 1959 to June 1960. *a* Undercutting of Point No. 5; *b* face stationary; *c* 400 m³ of coal cut; *d* working resumed; *e* work finally halted. *Broken line*: presumed subsidence if working had continued without a halt. Rate of advance: 0.8 m/d

Figure 56 shows the development of movement over a period of time at a point in the rock which is undercut. Some 50 m above workings at a depth of 600 m, strata movement begins as soon as the approaching face is within 45 m of the observation point, measured horizontally. Three *stages of movement* can be distinguished in relation to the point of the face.

In the first stage the roof beds drop by a few centimetres, under the influence of the abutment pressure and initial convergence which precede the face. In the second stage, the undercutting of the point and the increasing fracture-deformation and joint-plane slip together set off a rapid movement of settlement, almost without a transitional pause. When the face, including its border zone, has passed on out of the area affecting the point, or if working is halted, the third stage begins, in which the collapsed roof beds and the packing material are gradually compressed by their own dead weight. Essentially, taking the area as a whole, this third, compactional stage acts as the impulse for subsidence in the higher strata and at the surface (cf. Fig. 258d). Any fresh working within the area affecting the point sets off a fresh movement, which again exhibits two stages – or three including the opening-out stage – up to the attainment of roof settlement amounting locally to $a_x \times M$ (Fig. 56).

From the beginning of working to when the ground comes to rest again, the field of deformation in the rock mass changes continuously as a result of constant disturbance to the equilibrium of forces, brought about by mining activity and increasing convergence in the mine excavation. Consequently, the boundaries between zones of expansion and contraction, extension and compression shift at intervals, and the extent of deformation increases or decreases locally. In the next section it will be evident how difficult it is to calculate the extent of deformation in the rock mass, and especially the periodical movement of points in the rock.

3. The Calculation of Strata Movement

3.1 General Assumptions in Calculation Procedures

The mathematical effort required to calculate movement in strata affected by underlying workings can be kept at a justifiable level only if certain simplifying assumptions are made concerning the structure, the deformation behaviour, and the extent of the rock mass (Fig. 57). Thus in many procedures the rock mass is regarded as a *continuum*, the separate constituents of which, such as granular particles, rock beds, or theoretically assumed geometric forms, are held together by cohesive forces in such a way that the rock mass deforms as a single whole and has, like some homogeneous material, identical properties throughout. The deformation behaviour of this rock mass can be regarded as either elastic or plastic. It can be isotropic – i.e., homogeneous, or uniform – in all directions or, like a shaly rock, it can be subject to differing degrees of deformation at right angles and parallel to the stratification – in other words, anisotropic. For example, a stratified rock mass corresponds to a continuum with vertical inhomogeneity and horizontal isotropy, if fault and slip planes are ignored.

By contrast, a *discontinuum* implies a system of loosely assembled elastic elements, in which the network of joints defines the paths to be followed by the separate elements as they move. This is a concept which fits a strongly faulted rock mass that has been extensively fractured by repeated working. For model purposes, an assemblage of layered beams, cubes, prisms, etc., or of incohesive rubble or sand grains, provides a good approximation.

As regards its extent, the rock mass may be compared to a *semi-infinite space*, since it is limited in one direction by the horizontal earth surface and stretches without limit in all other directions. Its deformations are nevertheless difficult to define, and a description of them is therefore confined to simplified special cases such as a rotationally symmetrical stress state or a two-dimensional deformation state applicable to a vertical plane at right angles to the working front. Furthermore, for calculation purposes a constant volume and a minimum expenditure of energy during deformation are usually assumed. In addition, only the simple case of a flat-lying stratification and the final state of movement is considered.

Many calculation procedures which rest on model assumptions have been developed only to the stage of statements. Consequently, for purposes of practical planning, deformation of the rock mass over an irregular pattern of workings, considered spatially and not in a plane, must be calculated according to simple, empirical approximation procedures resting on survey observations and dispensing with any similarity to a mechanical model. Finally, both in model

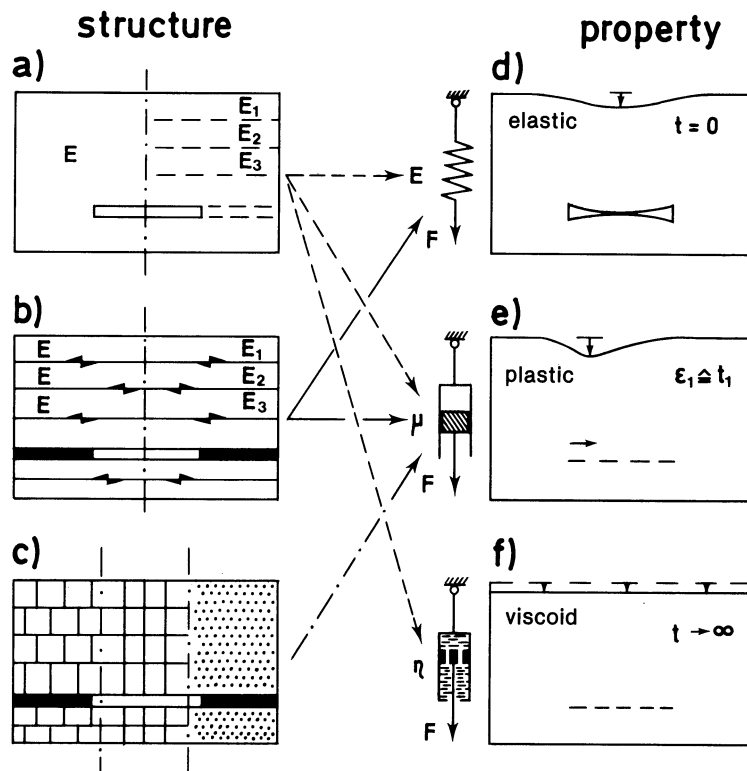


Fig. 57. Models of the rock mass, of varying structure and properties, showing extraction opening. **a** Homogeneous or inhomogeneous continuum with elastic, plastic, or viscous properties; **b** bedded homogeneous or inhomogeneous discontinuum, with elastic or plastic properties; **c** discontinuum containing laterally offset or continuous, coinciding joints, or broken up in small fragments (elastic) and having plastic properties; **d** elastic behaviour: immediate deformation so long as force applied; flexure of roof and floor; **e** plastic behaviour: deformation persists even when force no longer applied; trough over reclosed extraction opening is asymmetrical; **f** viscous behaviour: deformation is retarded in relation to the force acting (delayed subsidence); with the passage of time the trough over the reclosed extraction opening levels out

concepts and in empirical approximations, a distinction must be drawn between procedures embracing the development of movement throughout the whole rock mass and those which permit the calculation of movements only at a horizontal limit plane – say, the roof bed or the upper earth surface. Calculation procedures referring to the surface are not considered in the following paragraphs. They will be described in Chaps. 8 and 9.

3.2 Calculating Convergence in the Workings

Strata movement originates and begins at the mining horizon (see Sect. 1.2). The roof settlement which takes place at that horizon is the starting point for calculating subsidence in successively higher strata, as for example along the line 1' – 1 or

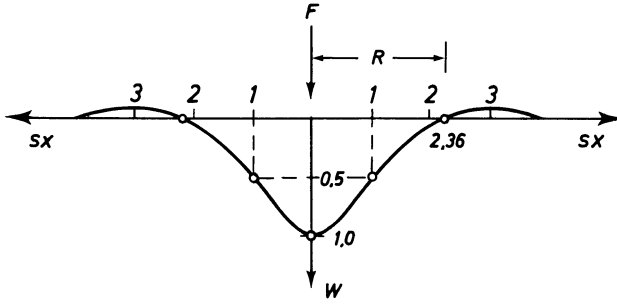


Fig. 58. The elastic bending profile of an infinitely long beam resting on an elastic underlay and with a central point-load F , as in Eq. (20). Note lifting of beam at $sx = 3$

the line 2' – 2 in Fig. 35. The amount, in volume terms, by which the immediate roof settles over a working gives the volumetric content of the subsidence trough of any given roof bed higher up, allowing for the increase in volume of the lower main roof as it breaks up and for the net resultant of elastic expansion on load relief and compaction on bending.

Settlement of the immediate roof and main roof into the mine excavation is best thought of, in model terms, either as the elastic bending of a slab or beam, or as clastic movement in a loose-rock medium, or as rheological flow in a Kelvin body.

3.2.1 The Roof as an Overlying Beam

The *slab theory* is based on the supposition that, as illustrated in Fig. 58, the main roof sags in a wavy outline like an infinitely long beam bedded on an elastic underlay and loaded at the point $x = 0$ with a concentrated load F :

$$w = k e^{-sx} (\cos sx + \sin sx). \quad (20)$$

In the factor

$$k = \frac{Fs}{2C_b b} \quad (21)$$

are indirectly incorporated, firstly, the force of reaction q of the compacted underlay, which on Winkler's hypothesis

$$w = \frac{q}{C_b} \quad (22)$$

should increase in linear response to the compression, corresponding to the bedding value C_b (the constant of resilience in N/cm^2); secondly, the width b of the beam; and thirdly, the *measure of flexibility*

$$s = \sqrt[4]{\frac{C_b b}{4EI}}. \quad (23)$$

If $sx = 3/4\pi (= 150 \text{ gr})$ is substituted for the product in Eq. (20), the expression in brackets becomes zero – i.e., at the point $sx = 3/4\pi = 2.36$ the concentrated load F has no effect on the elastic underlay ($w = 0$).

Physically, the width of a critical area (better regarded here as a critical length $2R$) is reached when the abutments at the two seam edges have no effect on the subsidence or the distribution of pressure at the centre. In accordance with the “wavelength” concept

$$\lambda = \frac{2\pi}{s} \quad (24)$$

customary in strength theory, that happens in practice at a distance of $x = R = \lambda/2 = 3.14 \div s$ – i.e., ignoring the wave line oscillating with decreasing amplitude around the abscissa, which can appear as a pressure wave in the zone in front of the face in the case of an elastic main roof of sandstone. At the origin $x = 0$ of the concentrated load F , the maximum amount of sag is

$$w_0 = \frac{Fs}{2C_b b} \quad (25)$$

Given that, with initially compacted packing material, the bedding value shows a linear increase of

$$C_b = 0.176 F \quad (26)$$

in response to increasing pressure (in N/cm^2 – see Fig. 11), Eqs. (23) and (26) together give

$$w_0 = \sqrt[4]{\frac{F}{0.35 E I b^3}} \quad (27)$$

In other words, settlement w_0 of the immediate roof at the centre of a critical area of extraction alters by only $\sqrt[4]{F}$, assuming F to be a concentrated point load. This explains the well known minimal influence of depth both on the subsidence factor and on maximum convergence in the workings, once a depth of 400 m has been exceeded.

If consideration has to be given to the difference in bedding support provided by seam or packing material, the expression that gives the pressure on the stowed

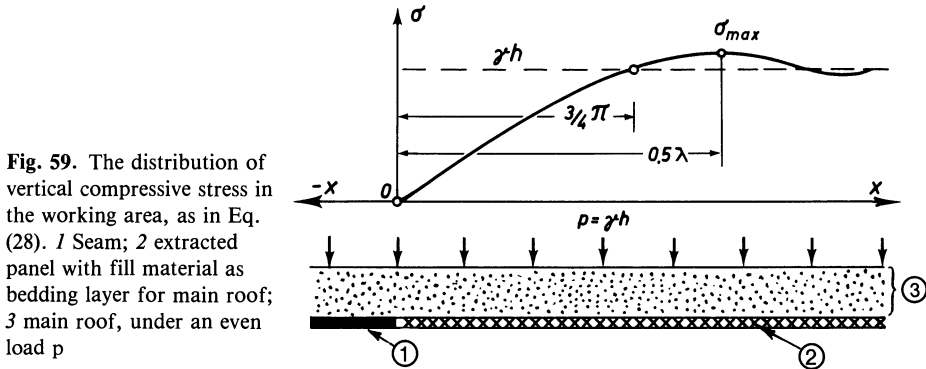


Fig. 59. The distribution of vertical compressive stress in the working area, as in Eq. (28). 1 Seam; 2 extracted panel with fill material as bedding layer for main roof; 3 main roof, under an even load p

goaf in a critical extraction area, exerted by a roof that is evenly loaded with p , is (Salustowicz, 1964):

$$\sigma = \gamma h \left[1 - e^{-sx} \left(\frac{s' - s}{s' + s} \sin sx + \cos sx \right) \right], \quad (28)$$

where s represents the measure of flexibility when the roof is bedded on packing material, and s' the same measure when it is bedded on the seam. On this basis the pressure increases, in an oscillating curve, from $\sigma = 0$ at the face edge ($x = 0$) to a final value of $p = \gamma h$ over the goaf (Fig. 59 – but it should be noted that, because of the lift in front of the face under tensile stress, as shown in Fig. 58, the value of σ at the point $x = 0$ is not absolutely zero). With the calculated local pressure, the convergence at the point x can then be derived from Eq. (22) or from the linear relation

$$\varepsilon = \frac{\sigma}{E_s} \quad (29)$$

provided the stiffness index E_s of the packing material is known.

An exponential curve also results for the line of curvature of a thick roof bed which is evenly loaded by the caprock and elastically supported by seam or packing, in cases where bending is chiefly caused by the transverse force Q (loading P and reaction of bedding layer Cw or $C'w$) and where the bending moment can be ignored (Salustowicz, 1964). The differential equation for the sagging of the roof, if the curvature d^2w/dx^2 is proportional to the angle of slip γ – i.e., to the tilt of a previously vertical cross-sectional plane – then reads:

$$\frac{d^2w}{dx^2} = \frac{3}{2GS} Q \quad (30)$$

with $\gamma = \tau/G$, $\tau = 3 Q/2 S$ for right-angled cross-sectional planes S , and $Q = C'w - p$ ahead of the face, or $Q = Cw - Cw_0 - p$ in the stowed goaf (G = shear modulus in N/m^2).

From this is obtained the line of curvature of the main roof bed over a critical extraction area ($x \geq 0$):

$$w = \frac{p}{C'} + \frac{p}{C} \left(\frac{m+n}{m} - e^{-nx} \right) \quad (31)$$

and over the unworked seam ($x \leq 0$):

$$w = \frac{p}{C'} + \frac{p}{C} \left(\frac{n}{m} e^{mx} \right). \quad (32)$$

In these equations, $p = \gamma h$ in N/cm^2 , $m = \sqrt{3 C'/2 GS}$ and $n = \sqrt{3 C/2 GS}$. C' and C represent the bedding value, or elastic resistance, of the seam or packing.

The subsidence of a roof bed resting inelastically on pillars of salt rock is dealt with in a general manner in the literature (Döring, 1964; Kittrich, 1963), and attention is drawn thereto in view of the comprehensive formula derivations.

In contrast to the model concepts mentioned above, what rests on the main roof bed in practice is neither a point load concentrated at the centre of the working nor an evenly distributed surrounding load. Nor is the main roof a beam: it is a slab, with perpendicular or diagonal fractures, and consequently displays a bending resistance N which varies from place to place. Furthermore, the packing material yields inelastically, and so the Winkler hypothesis does not apply. Apart from that, a load element dq_x at the point x not only creates a subsidence step dw_x confined to that point (theory, 1st order) but also, because of the spatially overlapping compressive stress and lateral compression of the packing material, influences the subsidence of the neighbouring steps (theory, 2nd order). The equations given for the line of curvature are therefore only rough approximations based on simplified model assumptions which are valid only for critical areas of extraction. No theoretical treatment of a subcritical area model on the hypothesis of an infinitely long, elastic beam has yet been evolved. It would have to pay regard to variations in the support provided in a subcritical excavation (i.e. to variations in the density of fill material), as well as ahead of the face, and also to the uneven loading of the main roof.

3.2.2 A Roof of Loose Rock

If one assumes, as an extreme contrast to elasticity, a clastic, unconsolidated rock mass consisting of loose rock, what acts on the extraction area is the mass γ of the column of rock ABCD, diminished by the friction along the lines AC and BD (Fig. 60). And if within the column of rock the section EFGH of height dz is considered, and an equation is constructed to describe the balance of forces acting on that section, a formula is obtained which defines the pressure per unit area of the roof over a stowed panel of width $2b$ (see Terzaghi, 1954):

$$\sigma = \frac{\gamma b}{\lambda \tan \rho} (1 - e^{-\lambda \tan \rho (h/b)}). \quad (33)$$

In this statement, ρ represents the angle of friction of the rock mass, and λ an empirical quantity approaching unity. For small panel widths, the *silo formula* applies:

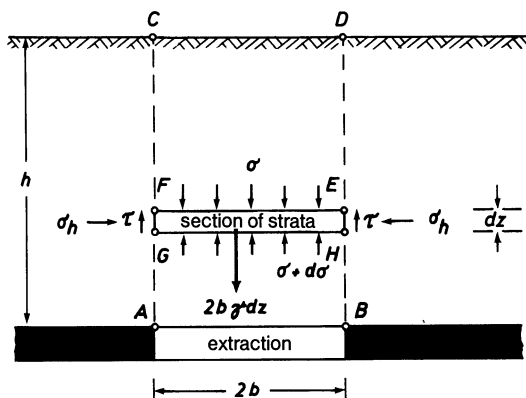


Fig. 60. The balance of forces acting on one section of a loose rock mass between two vertical slip planes, as in Eq. (33)

$$\sigma = \frac{\gamma b}{\lambda \tan \rho} . \quad (34)$$

Accordingly, on the simplifying assumption that stresses normal to the roof of a working will be evenly distributed when the rock medium is a loose one, convergence will be as follows everywhere in the stowed goaf (thus constituting a subsidence step):

$$w = \frac{1}{E_s} \frac{\gamma b}{\lambda \tan \rho} (1 - e^{-\lambda \tan \rho (h/b)}) . \quad (35)$$

It will increase gradually with increasing panel width b up to a limit value of $w_{\max} = \gamma h / E_s$. In a rock mass consisting of loose rock therefore, the fill material in the mined panel will be compacted equally throughout, thus producing not so much a line of curvature as a distinct step. The concept of a model consisting of elastic rock material provides a good illustration of the increase in roof pressure with increasing panel width.

3.2.3 The Roof as a Kelvin Body

So far only roof settlement in a working that is at a standstill, and only in its final state, has been considered; or alternatively, it has been assumed that settlement takes place immediately on loading. As is known, however, the final deformation of a piece of rock or of an abutment when under load sets in only after a certain lapse of time. A roof resting on packing material must therefore be regarded as a *rheological medium* which, in its time-dependent deformation behaviour, approaches that of, for example, a visco-elastic Kelvin body consisting of an elastic, Hookean spring and a viscous, Newtonian piston damper, or “dashpot” (Fig. 61).

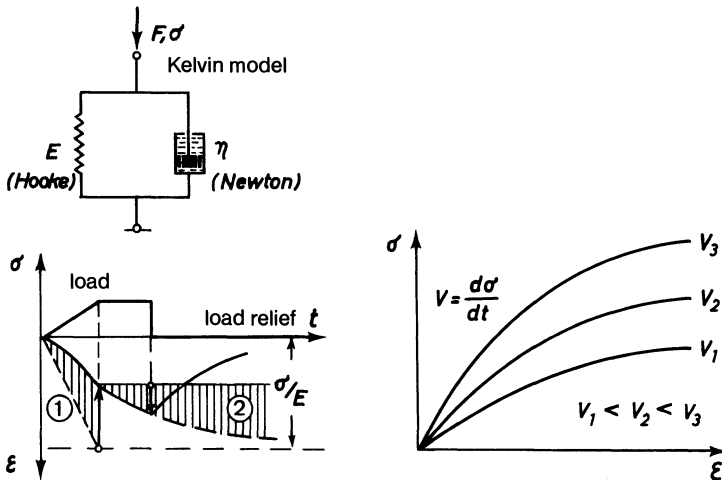


Fig. 61. A rheological model consisting of a spring (modulus of elasticity) and a damping piston device, or “dashpot” (viscosity η); also strain diagrams after Eq. (37). 1 Delay in relation to elastic deformation on increase in load; 2 gradual increase in deformation when load is constant and continuous

This model, it is true, lacks the component of plastic, permanent deformation on the relaxation of load: for example, the subsidence trough at the surface does not even out and disappear when the effects of mining cease, as suggested by this model. In the Hookeian element, stress and strain are proportional ($E \times \varepsilon$); in the Newtonian element, the relationship depends on the rate of deformation ($\eta \times d\varepsilon/dt$). In a Kelvin body, the total stress acting to produce strain is given by

$$\sigma = E\varepsilon + \eta(d\varepsilon/dt), \quad (36)$$

where η signifies viscosity. What characterizes a Kelvin body placed under a constant load from the point of time $t = 0$ onwards is a time-related deformation which starts from zero and tends asymptotically towards a final value of $\varepsilon = \sigma/E$, thus:

$$\varepsilon = \frac{\sigma}{E} (1 - e^{-(E/\eta)t}). \quad (37)$$

As a working face advances, however, compressive stresses on seam or fill material at a fixed point are not constant (Fig. 62). They increase ahead of the face, fall to zero as the face passes, and rise again behind the face to $p = \gamma h$. In this development, the distance x of the face from the point under consideration, which depends on time t and rate of advance v , can be expressed as $x = L + vt$ for the zone ahead of the face, or as $x = vt$ for the stowed panel. Ahead of the face, the length L (which is negative) signifies the width of the zone of abutment pressure. A variable compressive stress $\sigma(x)$, which depends on the distance from the face, must therefore be introduced into a modified Eq. (36) for stress in a Kelvin body. The rate of deformation in the zone ahead of the face is then given by

$$\eta \frac{d\varepsilon}{dt} = p \underbrace{\left[1 + \left(\frac{C'}{C} \right)^{1/2} e^{mx} \right]}_{\sigma(x) \rightarrow \text{Eq. (32)}} - E\varepsilon \quad \text{with } x = L + vt \quad (38)$$

and in the stowed panel by

$$\eta \frac{d\varepsilon}{dt} = p \underbrace{(1 - e^{-nx})}_{\sigma(x) \rightarrow \text{Eq. (30)}} - E\varepsilon \quad \text{with } x = vt. \quad (39)$$

The integral of Eq. (38) reads as follows (Salustowicz, 1960) for the zone in front of the face (subscript k):

$$\varepsilon_k = K e^{(E/\eta)(L-x)/v} + \frac{p}{E} \underbrace{\left[1 + \frac{E}{\eta} \left(\frac{C'}{C} \right)^{1/2} \frac{1}{\frac{E}{\eta} + mv} e^{mx} \right]}_{\sigma(v) \div E}. \quad (40)$$

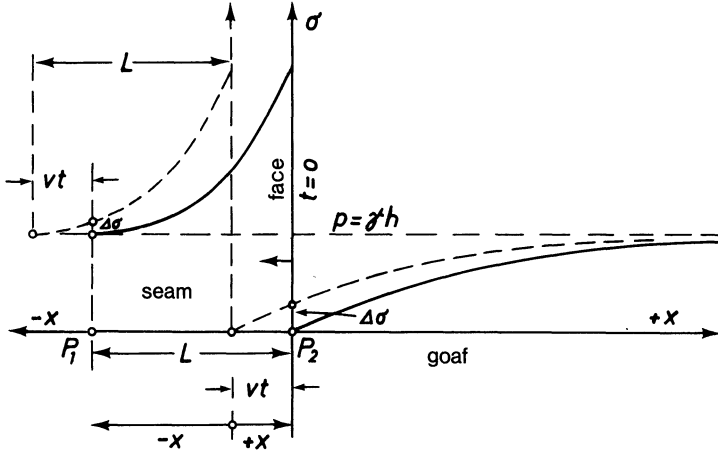


Fig. 62. Distribution of strata pressure over zone ahead of face and over working area as face advances by $x = vt$; also changes in pressure $\Delta\sigma$ at the fixed points P_1 and P_2 , in accordance with Eqs. (38) and (39)

Similarly, the expression for the stowed panel (subscript v) can be obtained from Eq. (39) by integration, introducing initial convergence $\varepsilon_{k \max}$:

$$\varepsilon_v = K e^{(E/\eta)(x/v)} + \frac{p}{E} \underbrace{\left[1 - \frac{E}{\eta} \frac{1}{\frac{E}{\eta} - nv} e^{-nx} \right]}_{\sigma(v) \div E} + \varepsilon_{k \max}. \quad (41)$$

The constant K for Eq. (40) follows from the condition that at $t = 0$ — i.e., at $x = L$ ahead of the face (point P_1) and $x = 0$ behind the face (point P_2) — time-dependent deformation ε is also zero. From

$$K e^{(E/\eta)(0/v)} = 0 \quad (42)$$

it follows that, with $e^0 = 1$, then — depending on conditions — $K = 0$. At the point $x = L$, since L represents a very large absolute value and $e^{mx} = e^{mL}$ tends to zero, Eq. (40) becomes

$$\varepsilon_k = \frac{p}{E}. \quad (43)$$

Maximum vertical deformation of the coal at the face (initial convergence) amounts — depending on the rate of advance v — to:

$$\varepsilon_{k \max} = \frac{p}{E} \left[1 + \frac{E}{\eta} \left(\frac{C'}{C} \right)^{1/2} \frac{1}{\frac{E}{\eta} + mv} \right]. \quad (44)$$

In the stowed panel, Eq. (41) gives for $x = 0$ (point P_2)

$$\varepsilon_v = \varepsilon_{k \max} \quad (45)$$

and for $x = \infty$

$$\varepsilon_v = \frac{p}{E} + \varepsilon_{k \max} \cdot \quad (46)$$

Figure 63 shows stresses and strains for various rates of face advance. The areas A_1 and A_2 enclosed by the lines of stress and of cover-load pressure ahead of and behind the face are at first not of equal size, because stress equilibrium has first to be created by the process of deformation (i.e., the overcoming of frictional and breaking resistance).

3.2.4 Empirical Formulae for Convergence in the Workings

In many mining districts, measurements of roof settlement and convergence taken in roadways and face areas are regularly investigated with the aid of correlation calculations to test their dependence on distance from the face, depth, thickness of seam extracted, and rate of face advance. In making use of the empirical formulae discussed below, it should be remembered that these subsidence movements have mostly been observed in accompanying roadways at the edge of a panel and are thus smaller than movements at the central line of working along strike or dip. They are also not always unaffected by strata movements connected with the roadways themselves. Furthermore, convergence c represents, as is known, the perpendicularly narrowing distance between two vertically opposed points and thus comprises both the drop in the roof and the heave in the floor. Consequently, convergence should be used as a starting point for subsidence calculations only where the floor heave is relatively slight.

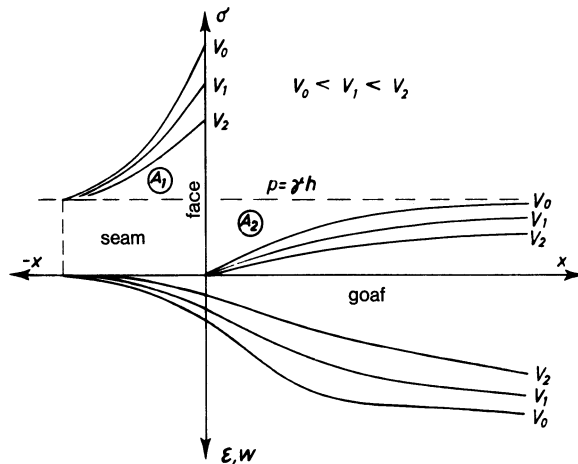


Fig. 63. Vertical strata-pressure stress, and deformation ε or settlement w at the mining horizon, for various rates of face advance, as in Eqs. (40) and (41). v_0 Face halted

In the *Lorraine field*, the dependence of convergence c (in mm) on the distance x (in m) between the measuring station and the working face can be expressed by a logarithmic curve (Schwartz, 1959):

$$c = \gamma \log(\alpha + \beta x), \quad (47)$$

in which α , β , and γ are empirical constants and c represents convergence related to the point in time when the measuring station is exposed, with convergence ahead of the face ignored. The rate of convergence, i.e. the daily increment in convergence c_d of a pair of observation points expressed in mm/d, reaches its maximum between 2 and 35 m behind the face, at the point where the convergence curve passes through its transition point (Fig. 64). The steeper the roof and the more rapid the advance, the farther back from the face is the transition point of the convergence curve.

In the *Upper Silesian coalfield*, settlement of the roof at a point $-x$ metres in advance of an approaching face can be expressed with sufficient accuracy by the equation:

$$w_0(x) = w_{00}e^{-bx} \quad (48)$$

in which w_{00} signifies roof settlement in mm at the seam edge, and b a coefficient related to the nature of the goaf. For caving, as an example, it is -0.09 , and for hydraulic stowing -0.07 (Borecki and Bilinski, 1960). Roof settlement in the goaf is calculated using the equation

$$w_2(x) = w_{00} + \frac{Mx}{a + b_1x} \quad (49)$$

in which M is the working height at the face, and a and b_1 are two coefficients for the behaviour of the roof, both in the face excavation (for example, for a central road and hydraulic stowing, $a = 0.078$) and in the goaf (e.g., $b_1 = 0.0084$).

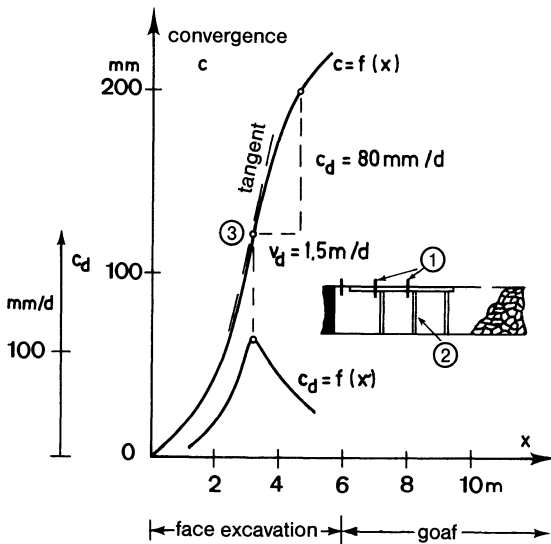


Fig. 64. Convergence c and rate of convergence c_d , measured from the exposure of the convergence points and related to distance x from the working face ($x = 0$). v_d Daily face advance; 1 convergence points in roof; 2 props in face excavation; 3 transition point

In the *Donetz field*, for roof settlement w (in mm) in the face excavation up to the edge of the caved or stowed area, as governed by the rate of advance v (in m/d), the following exponential function has been found to apply:

$$w_1(x) = \alpha M b e^{\beta v}. \quad (50)$$

On this basis, roof settlement over the face is proportional to the face height M and the face width b (in m). The dimensional values α and β work out, for example with caving and powered supports, at $\alpha = 74.2$ and $\beta = 0.13$.

In several of the coalfields of Europe, the *daily convergence* c_d (in m/d), as measured in the face excavation at the first and then the second prop row (Fig. 64), has been related to the daily face advance v_d in m/d (Chambon, 1966). The part of the convergence curve considered therefore lies in the face area, around the transition point. Here the curve is falling in almost a straight line and can thus be substituted, as an approximation, by the tangent at the transition point. The slope of this tangent corresponds approximately to the quotient

$$\tan \alpha = \frac{c_d}{v_d} = 0.2 \sqrt[4]{h} (aM)^{3/4}, \quad (51)$$

which depends on the depth h , the subsidence factor a , and the face height M , and from which the interim convergence curve in the face area for each rate of advance can be drawn.

Similar investigations in the open face areas of the *Ruhr District*, which can be up to 7 m wide, revealed an average relative convergence c_{rel} of 39 mm per metre of face advance (Leonhardt, 1969). In longwall caving, the rate of convergence reaches its maximum with a daily face advance of 1.8 m/d, as compared with 3.6 m/d with pneumatic stowing. For calculating interim strata movement it is important to know the time-related increase of convergence in a stationary longwall face (see Fig. 37), but in the *Ruhr District* it has been possible to observe this for a consecutive period of only 60 h. On this basis, convergence between a pair of points, at a distance x from the face, increases during the standing time t in accordance with the exponential function

$$c_1(x) = A t^B + C x^D \quad (52)$$

until it reaches a final, static value. The exact values of coefficients and exponents applicable to individual cases are still unknown, but they are of the order of 0.17 for A , 0.5 for B and C , and 0.6 for D . The final shape of the convergence or subsidence curve from the zone in front of the face to the centre of the panel depends on many factors. A statistical calculation of it is not possible, however, because comprehensive measurements in the inaccessible goaf are lacking.

Nevertheless, as will be shown later, with many procedures for calculating strata and ground movement it is quite adequate to replace the curve of roof settlement by steps 25 or 50 m wide, the percentage relationship of which to maximum subsidence can be read off Fig. 13.

3.3 The Pre-Calculation of Strata Movement

The importance of strata movement calculations lies chiefly in the protective measures to be taken for the shaft lining, which risks going out of true within the zone of influence of the workings and being damaged by axial forces within the rock mass. In salt-mining, deformation of the rock mass must not be allowed to lead to the fissuring of impervious beds holding back ground water. Strata movement also constitutes the link between convergence at the mining horizon (Z_4 , Fig. 65) and ground movement at the surface (Z_0). By contrast, alterations in the gradient and the general lie of mine roadways caused by strata movement present no technically insuperable problems for mine haulage and water control. Little

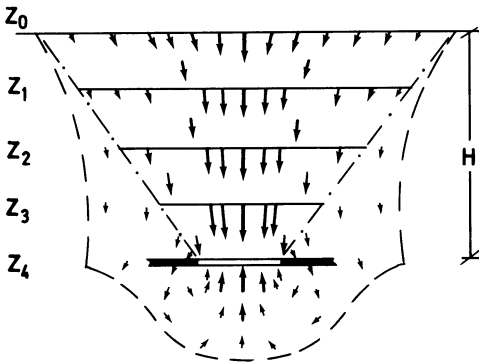


Fig. 65. Movement (change of position) of points in the rock mass coming within the influence of a working, from horizon z_0 (earth surface) to horizon z_4 (working)

Table 5. Methods of calculating convergence in workings and strata movement

Scope of calculation	Empirical methods	Conceptual models	Aim of calculation
Convergence in workings	Formulae for determining roof-bed profiles (Difficulty: not of general validity)	1) A beam bending elastically on a yielding underlay 2) A clastic rock mass (e.g. sand model) 3) A visco-elastic Kelvin body (Difficulties: fill material inelastic; local load unknown)	Convergence, or roof settlement and floor heave, at a particular location
Strata movement	Integration-grid method (Difficulty: influence of rock-mechanics characteristics unknown)	1) A continuum, for elastic and stiffly plastic materials 2) A stochastic medium 3) The finite (elastic) elements method Difficulties: only plane case considered; fracture deformation and time factor ignored; stratification considered only in the finite element method)	Vertical and horizontal components of the movement of points in the rock mass Final extent of deformation in the rock mass, and its development over time

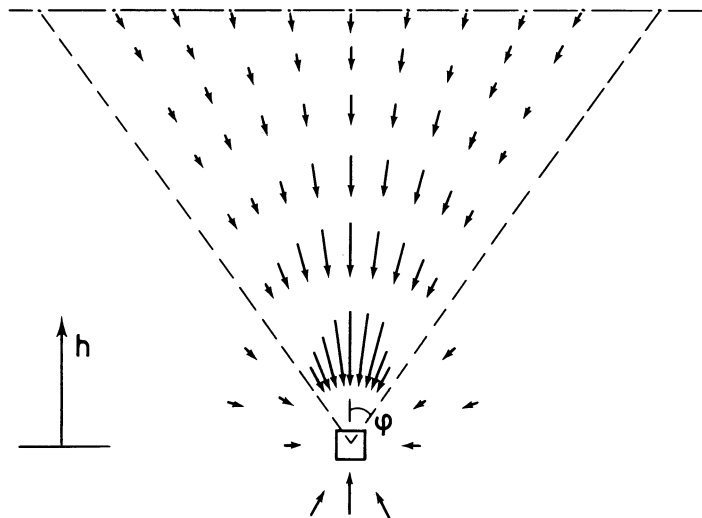
research has been done, however, into the effect of large-scale strata deformation on convergence in roadways lying within the zone of influence of mining.

As in the preceding section, methods of calculation can be divided into the empirical and the model-based (see Table 5). Their aim is to find the vertical and horizontal components of the movement of points in the rock mass (Fig. 65). From these it is possible to arrive at the likely tilt in the shaft axis and the probable deformation of the rock between two points – i.e., vertical expansion and contraction (Fig. 35), and horizontal extension and compression (Fig. 43), as already discussed and illustrated in Sect. 2.2. Bed curvature, a strain value which is of importance for crack formation and horizontal shear movement, is calculated from the downward movement of three points in the rock. The method employing influence functions and integration grids can be regarded as largely empirical. Among conceptual models there are several methods, all based on the theory of elasticity, for resolving strata movements with the aid of point forces, differential equations, or finite elements. In Poland, the rock mass has been described as a discontinuous material obeying statistical laws, in the U.S.S.R. as a stiffly plastic material.

3.3.1 The Integration-grid, or Influence-Function, Method

Graphical calculation of strata movement over flat-lying workings with the aid of integration grids is the oldest method and the only one widely applied. In this method, the rock mass lying above the point under consideration is thought of as having been removed (Fig. 65), and the movement of a point in the interior of the mass is ascertained in the same way as ground movement at the surface, namely by integrating subsidence and displacement influences emanating from elementary units of extraction (Bals, 1932). Each infinitesimal element of the mine excavation sets off, in the overlying and underlying strata, rock movements in the direction of that excavation. In Fig. 66, if one considers only the movement

Fig. 66. Movement of points within a circular cone of the overlying rock mass caused by an element of excavation in the working



of points in the overlying rock within a circular cone subtending the angle 2φ , it will be noticed that the size of the movement decreases both from the bottom up and from the central line outwards to the envelope of the cone. The amount of influence that an elementary unit of extraction (an "extraction element") has on the vertical component of movement (the subsidence) of a point at the surface (the so-called influence function) has been ascertained many times in Germany since 1925 from ground subsidence measurements over both small and large workings. By assuming that neighbouring extraction elements generate identical elementary troughs of subsidence at the surface, and by superposing the influence of all extraction elements, it becomes possible to determine the subsidence of a given point at the surface. Alternatively, for purposes of later calculation, a "critical area of extraction" can be divided into five annular zones in such a way that each zone is responsible for 20% of the complete, or "full", subsidence of the central point P (Fig. 67). This method of ground movement calculation is applied to all horizons of the rock mass. In each case, the subdivision of the critical extraction area applicable to ground movement is retained, and only the size of the grid is reduced according to the height h of the rock point concerned above the level of the working (Fig. 67).

The division of the grid area into five concentric zones, each of equal influence, is explained later, in Sects. 8.3 on ground movement. The mining of one of the zones – No. 1, 2, 3, 4, or 5 – centred on the point under consideration has the effect of lowering that point by one-fifth of the maximum possible subsidence $v_{z \text{ Max}}$ [see Eq. (16)]. To facilitate graphical assessment, each zone is divided into eight equal sectors, with the result that the whole area of influence,

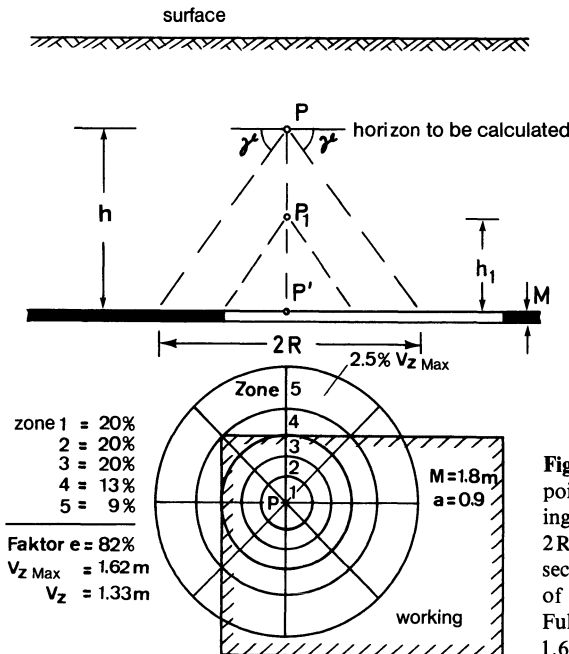


Fig. 67. Calculating the subsidence of a point P at a rock horizon h above a working, using an integration grid of diameter $2R$ corresponding to the height h . *Top*: section through rock mass. *Bottom*: plan of working, with grid centred on point P. Full subsidence $v_{z \text{ Max}} = 1.8 \times 0.9 = 1.62\text{m}$

or “critical area” is split into $5 \times 8 = 40$ grid meshes of varying size but equal subsidence-inducing influence (i.e., $2.5\% v_{z \text{ Max}}$).

The calculation is done as follows. The transparent paper on which the integration grid of diameter $2R = 2h \cot \gamma$ (the critical area) has been drawn is laid on the plan of the seam to be mined and so aligned that its central point covers the point under consideration P' , as projected vertically down to the seam level. From a count of the number of n grid meshes covering the workings, the influence factor e is obtained from

$$e = 0.025 n, \quad (53)$$

and this gives the amplitude of subsidence at the point P :

$$v_z = e v_{z \text{ Max}}. \quad (54)$$

Where only part of a particular grid mesh covers workings, the appropriate proportion of the 2.5% influence represented by each mesh is estimated. This method has the advantage that it can be used equally well for polygonal workings. If two points P_1 and P_2 are separated by a vertical distance h , the vertical expansion or contraction of the relevant section of the rock mass can be calculated from the subsidence experienced by each of the points:

$$\pm \varepsilon_z = (v_{z2} - v_{z1}) \div \Delta h \quad (\text{in mm/m}). \quad (55)$$

The subsidence value obtained by the integration method – or indeed by any other method of calculating ground movement (e.g., Knothe’s in Sect. 8.35) – is unaffected by the thickness or thinness of the cover overlying the horizon in question. Ignoring the depth of workings is contrary to experience of strata movement in practice, which varies according to the amount of load at the horizon in question and the rigidity of the whole suite of beds, which in turn depends on their thickness. It is the geometric magnitudes of the limit angle γ , the calculation height h_1 , and the mine layout itself which alone determine the position of the point P_1 , where partial subsidence v_z on the vertical line $P - P'$ becomes full subsidence $v_{z \text{ Max}}$. A division into zones by depth, permitting stratigraphical structure and cover-load pressure to be taken into consideration, cannot yet be reported, owing to the paucity of point observations.

The so-called zone of full subsidence within the limit-angle cone depending from P , in which the rock mass is said to be in theory free from vertical expansion and contraction, is probably only found over the centre of a supercritical area of extraction, and only if the fill material there is completely compacted. Incorrectly measured roof-settlement steps at the edge of a working [factor a in Eqs. (16) and (54)] falsify the calculation, particularly with low heights of under 200 m.

3.3.2 The Rock Mass as an Elastic Material

Although bedded and porous sedimentary rocks fulfil Hooke’s Law only approximately, a few simple cases of strata movement, such as the deformation of an elastic mass as the result of a circular area load at the mining horizon, or as the

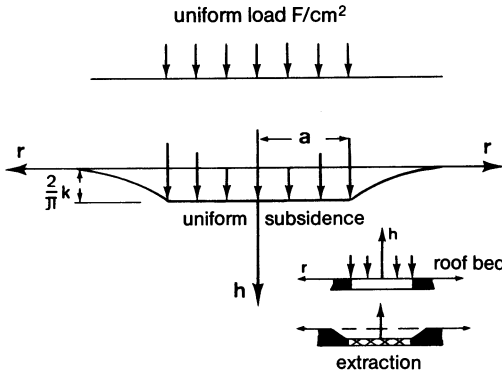


Fig. 68. *Top:* uniform loading of the surface. *Bottom:* uniform depression (subsidence) of the surface or (*inset*) of a roof bed

result of a very small excavation (an extraction element), are capable of solution with the aid of the theory of elasticity¹. Mathematical deductions in mining subsidence engineering are made more difficult by the fact that the starting point is not, as in soil mechanics, a known load but only the dead weight of the rock mass and the shape of the mine excavation.

In an incompressible, elastic semi-infinite mass (Poisson's ratio $\nu = 0.5$), the upper surface of which is uniformly loaded or compacted within a circle of diameter $2a$ (Fig. 68), settlement v_z symmetrically around the axis in terms of radial (r) and vertical (h) distance, amounts to the following, given uniform pressure distribution F/cm^2 (Boussinesq, 1885):

$$v_z = \int_0^a 2\pi r w dr = \frac{F}{2G} \int_0^a \left(\frac{r}{\sqrt{r^2 + h^2}} + \frac{h^2 r}{\sqrt{r^2 + h^2}^3} \right) dr \quad (56)$$

and to the following, given uniform depression by $2k/\pi$ (Trefftz, 1935):

$$v_z = \frac{2k}{\pi} \left(\frac{ah}{a^2 + h^2} + \int_0^\infty \frac{\sin ax}{x} I(rx) e^{-hx} dx \right), \quad (57)$$

ignoring compaction from dead weight in both cases. These formulae from soil mechanics lend themselves to use in subsidence engineering if the semi-infinite mass in Fig. 68 is thought of as inverted, and if its upper limiting surface, which will then be at the bottom, is seen as the roof bed of a working, on a circular area of which, of diameter $2a$, downward directed external forces act uniformly, or on which a cylindrical downward buckling of magnitude $2k/\pi$ is produced by unknown forces. After solution of the integrals in Eqs. (56) and (57), the subsidence of points over the centre of the working at $r = 0$ and $I(rx) = 1$ (Bessel's function) decreases from $Fa/2G$ or alternatively $2k/\pi$ at the mining horizon to zero at an infinite height above it, given a uniform pressure on the stowed goaf of

$$v_z = \frac{F}{2G} \frac{a^2}{\sqrt{a^2 + h^2}} \quad (58)$$

¹ For important formulae of the theory of elasticity (Hooke's Law, equilibrium conditions, Airy's stress and Love's displacement functions, the singularity principle, compatibility conditions, etc.) the literature should be consulted.

or alternatively a uniform roof settlement of

$$v_z = \frac{2k}{\pi} \left(\arctan \frac{a}{h} + \frac{ah}{a^2 + h^2} \right). \quad (59)$$

Up to a height $1a$ above the centre of the workings the elastic rock mass undergoes the greatest vertical expansion (Fig. 69). In Eqs. (58) and (59), just as in the integration method, the depth of the workings has no influence on the amplitude of subsidence in the rock mass.

The removal of a small mass A (an "extraction element") from the interior of a rock mass lying under its own dead weight also removes a restraint on the expansion of material in the vicinity of the cavity created, thereby freeing vertical ($\sigma_z = \gamma h$) and lateral ($\sigma_h = \sigma_z/m - 1$) dead-weight forces (Bräuner, 1959). Because of the symmetry of the system, the resultant F of this set of forces² runs down the perpendicular through A and acts on the focal point S of the roof strata, which have now come within the influence of the mine cavity and active working. If the perpendicular z -axis is taken as passing through the point of application S at a distance c from the horizontal ground surface (Fig. 70), the two variables x and y can be combined, because of rotational symmetry, into the radial variable $r^2 = x^2 + y^2$. With these coordinates, the infinitesimal movements of subsidence w and lateral displacement u , induced in an elastic semi-infinite mass of constant volume by the vertical resultant internal force $F(c)$, can be calculated for any desired points $i(r, z)$ as follows (Mindlin, 1936):

$$w = \frac{F}{8\pi G} \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{(z-c)^2}{R_1^3} + \frac{z^2 + c^2}{R_2^3} + \frac{6cz(z+c)^2}{R_2^5} \right) \quad (60)$$

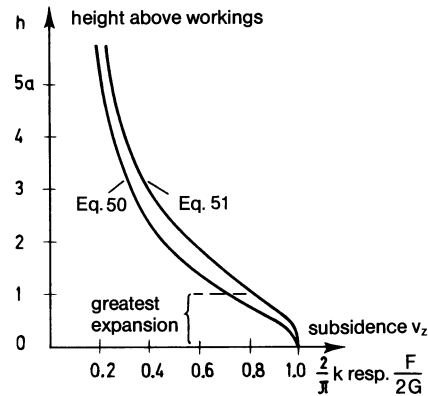


Fig. 69. Subsidence in an elastic rock body over the centre ($r = 0$) of a circular working-area of diameter $2a$

² There are now doubts whether it is admissible to assume point forces in equilibrium for the constituent elements of the mine excavation. With a point force, movements diminish in proportion to $1/R$ [see Eq. (62), with $R = h$]. With undisturbed superposition, and with maintenance of the principle that movement within the cone of influence is independent of depth (the limit-angle principle), the zone of movement would have to die away in proportion to $1/R^2$ to agree with Bals's hypothesis (Neuhaus, 1975).

$$u = \frac{Fr}{8\pi G} \left(\frac{z-c}{R_1^3} + \frac{z-c}{R_2^3} + \frac{6cz(z+c)}{R_2^5} \right). \quad (61)$$

In these equations, $R_1 = \sqrt{r^2 + (z-c)^2}$, $R_2 = \sqrt{r^2 + (z+c)^2}$, and G as in Eqs. (56) and (58) represent the modulus of shear.

Of principal interest for subsidence engineering are the infinitesimal or elementary subsidence troughs at the imaginary mining horizon $z = h$, at the level $z = c$ of the point on which strata forces bear, and at the upper surface $z = 0$ (Fig. 70). The resultant force F which replaces the withdrawn extraction element also presses the floor of the working downwards, where according to the boundary condition $w = 0$ for $z = h$ there should be no movement, owing to the interrupted lines of force in the mine excavation. To fulfil this boundary condition, the downward movement of the floor w_h is deducted from the subsidence movements in the overlying strata of the semi-infinite mass: $w'_c = w_c - w_h$ and $w'_0 = w_0 - w_h$. The infinitesimal trough w'_0 arrived at in this way at the upper surface of the semi-infinite mass fits in well with the subsidence trough expected over a subcritical area of extraction. Equations (60) and (61) for an internal point force can therefore be regarded as equally valid for an area force acting on those parts of the semi-infinite mass which are sufficiently far removed from the point of application of strata forces³.

The second boundary condition $w = 0$ for $z = 0$ and $r = h \cot \gamma$, which in subsidence engineering must be fulfilled at the surface boundary of influence, can be met in an elastic semi-infinite mass by a suitable choice of height at which forces act — e.g., $c = 0.5 h$ for $\gamma = 50^\circ$ — if the differential function $w'_0 = w_0 - w_h$ for the location $r = h \cot \gamma$ gives a value approaching zero.

Above a point force F , the infinitesimal subsidence w , both on the basis of Eq. (58) (integral term) for a force acting on the undersurface of a weightless

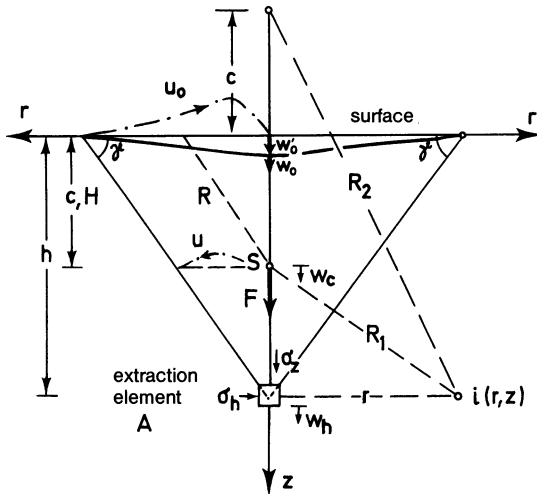


Fig. 70. The result of the action of a vertical point-force which takes the place of an infinitesimal element of extraction in an elastic semi-infinite mass; and the corresponding infinitesimal trough element for $z = 0$ at the surface

³ St. Venant's principle: at a sufficient distance from the zone of action of a system of forces, its effect no longer depends to any perceptible extent on its locational distribution but only on its static resultants (point forces and moments of force).

semi-infinite mass at $z = h$ (an interior point) and $r = 0$, and also on the basis of Eq. (60) for an internal force at $c = h$, $z = 0$ (a surface point) and $r = 0$, amounts to

$$w = \frac{F}{2\pi G} \frac{1}{h}. \quad (62)$$

The subsidence w of a central point is consequently inversely proportional to its vertical distance h from the bearing point of the force. Similarly, integration of Eq. (60) for a circular area load in the interior of a semi-infinite mass leads to the subsidence v_z , found by Eq. (58), of a central point ($r = 0$) at a load distance h , as can be shown by introducing $c = h$ and $z = 0$ into Eq. (60) and comparing with Eq. (56). The movements of eccentric points over an irregular load area or settlement area (extraction), to obtain which entails integrating Eqs. (56), (57), (60) or (61) several times, are in practice obtained as approximations with the aid of tabulated “influence values” (Love, and others) or integration grids (Newmark, 1935; Bräuner, 1959). This aspect is gone into in more detail in Sect. 8.42, on the precalculation of ground ($z = 0$) movements.

The calculation of strata movements in terms of space in a transversely isotropic body leads to harmonic functions, which can be solved by means of diagrams (Berry and Sales, 1962).

3.3.3 The Rock Mass as an Inelastic Material

The elastic properties imputed to the rock mass in the last section – namely, the capacity for spontaneous deformation in proportion to a load, and for immediate and complete recovery on removal of the load – do not quite fit an inhomogeneous suite of rock beds which is divided up by joints and slip planes into a so-called multi-body system (L. Müller, 1963). This is because such a rock mass partially evades an increased load by an irreversible, plastic slip movement along joint planes and behaves in a rheological – i.e. time-dependent – manner. The rate of change in loading $\Delta\sigma \div \Delta t$ influences the stress-strain relationship over time $\sigma \div \varepsilon(t)$, as well as the degree and duration of consequential deformation⁴. This is because the structure of the rock and of the rock mass adjusts to a new external field of forces only gradually, and even when the change in load conditions has already ended.

These inelastic properties of the rock mass – delayed deformation, persisting after-effects, influence of the rate of load application, and plasticity – can be represented in various ways, each related to the rate of load application. A *Kelvin model* (Fig. 61) will represent delayed deformation and subsequent deformation, and a *St. Venant model* (Fig. 71) permanent deformation and flow deformation. The latter model proposes a friction body sliding on an underlying surface: after overcoming friction resistance (at the flow boundary), it shifts unhindered and

⁴ In the literature, the finite, reversible after-effects are sometimes referred to as “creep elasticity”, and the slow irreversible flow of a solid as “plastic creep”. In a viscous Newtonian element, the transfer of force is proportional to the velocity of the piston, and the degree of deformation over time is dependent on the duration of the period of increased load.

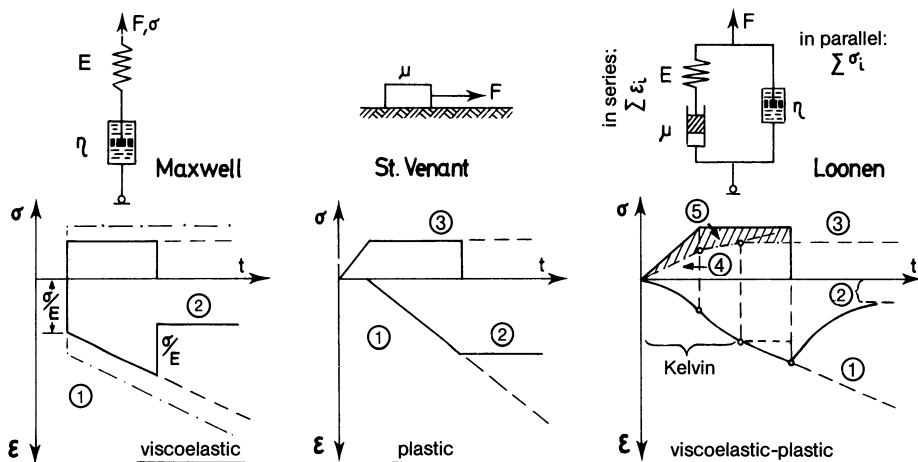


Fig. 71. Rheological models reproducing characteristics of deformation. 1 Linear flow; 2 permanent deformation; 3 limit of load for the St. Venant friction body at the point where flow behaviour begins; 4 curve of elasticity (spring); 5 take-up of additional load in the Maxwell body (*hatched*)

permanently with the passage of time, under the influence of a force of traction or of thrust which is then no longer capable of intensification. The non-linear, time related, and permanent content of the deformation in the rock mass can thus be traced to visco-elasticity and plasticity.

Visco-elastic deformation in the Kelvin model (Fig. 61), which after a certain delay reaches the final value $\varepsilon_{\text{end}} = \sigma/E$ of an elastic model, can be calculated with simple formulae of the type (Salustowicz, 1958)

$$\varepsilon = \varepsilon_{\text{end}} \left[1 - \left(1 - \frac{G}{G_0} \right) e^{-(G/\eta)t} \right]. \quad (63)$$

In this particular formula, $G_0 = \gamma/\tau$ denotes the modulus of shear immediately after the application of load, G the modulus of shear for deformation occurring later ($G < G_0$), and η the coefficient of viscosity. Deformation ε_{end} arrived at via elasticity theory needs only to be multiplied by the time function. The disadvantage of the viscoelastic Kelvin model, however, is that deformation gradually drops to zero on load relief (Fig. 61). As is known, the plastic content in deformation persists in the rock mass even when this is relieved of load.

The visco-elastic *Maxwell model*, an arrangement in series of a spring and a piston with holes through it (Fig. 71), accords well with the behaviour of the rock mass during load relief. For instance, the movement of unending flow when under continuous load corresponds in particular to the deformation of salt and of rocks at great depth, while with strain held constant, stress abates with time (signifying relaxation). On the other hand, when under load this model shows no degressive, elastic recovery, and no flow boundary. Final and permanent deformation sets in immediately on the relief of load.

One medium which comes near to reproducing the true conditions in a rock mass, as well as combining the characteristics of the Kelvin and St. Venant

models, is the visco-elastic-plastic *model of Loonen* (also in Fig. 71). In this model it is assumed that the threshold of flow behaviour in the rocks surrounding an underground working is overstepped in the process of stress redistribution, and that, as a result, a plastic zone forms in them, which develops in time to a balance between elastic and plastic zones, depending on the relative stresses. In this process, not only the local strains but also the stresses developing around the excavation are a function of time. The curve of strain against time, the shape of which is determined by viscosity and the modulus of shear, has no horizontal asymptote, and the visco-elastic, delayed initial deformation (Kelvin model) changes into linear flow once friction resistance in the friction body (St. Venant model) has been overcome. If in the course of deformation the stress at any point drops back to the flow-boundary value, the rock mass at that point will again behave as a Kelvin body. On complete removal of load, the model will be only partially re-deformed by the elastic energy stored in the spring up to the flow boundary. The degree of permanent deformation will correspond to the energy consumed in the viscous and plastic medium.

The flow condition of a visco-elastic-plastic rock in a semi-infinite plane, for example, can take the following form (Cernyi, 1966):

$$\tau - \tau_{Br} - \eta \frac{dv_x}{dy} = 0 \quad (64)$$

where τ is the shear stress existing at any point, τ_{Br} the shear-stress limit after Coulomb⁵

$$|\tau| = c + \sigma_n \tan \rho, \quad (65)$$

η the kinematic viscosity, and dv_x/dy the gradient of the curve of flow velocity (c = cohesion). The shear-stress limit in a slip plane is given by

$$\sigma_n + \frac{d\sigma_n}{dt} \geq \sigma_0 \quad (66)$$

that is to say, when static normal stress σ_n , increased by the dynamic value $d\sigma_n/dt$, exceeds an experimentally established value σ_0 at which the rock begins to flow. Flow behaviour in rocks, unlike that in fluids, is thus determined not by the totality of tangential stresses but only by that part of them lying beyond the boundary of elastic deformation ($\tau - \tau_{Br} = \Delta \tau$). Movement in the model material must cease when cavities in it are filled (see Kelvin and Loonen models) and not, as in a purely viscous or plastic material, when the surface has been evened out and the stratigraphic stress field has been restored.

If the zone of brittle fracture directly over the working is disregarded, the process of visco-plastic deformation of the rock mass yields a system of three partial-differentiation equations, derived from the equilibrium conditions at the extraction element for vertical and horizontal deformation components (Cernyi, 1966):

⁵ For other flow states related to the principal stress – as examined, for example, by Tresca, Mises, and Huber-Mises-Hencky – and also for Mohr's stress circle, see the literature. Displacement and subsidence in an elastic-plastic rock mass have been calculated by W. Neuhaus (1975)

$$\begin{aligned}
\frac{\partial v_x}{\partial t} + v_y \frac{\partial v_x}{\partial x} + v_x \frac{\partial v_x}{\partial y} &= X - \frac{1}{\gamma} \frac{\partial F}{\partial x} + \frac{\eta}{\gamma} \nabla^2 v_x \\
\frac{\partial v_y}{\partial t} + v_y \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} &= Y - \frac{1}{\gamma} \frac{\partial F}{\partial y} + \frac{\eta}{\gamma} \nabla^2 v_y \\
\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} &= 0.
\end{aligned} \tag{67}$$

Here X and Y denote the projection of mass forces on to the coordinate axes, F the all-round local pressure, Δ^2 the Laplace operator $\partial^2/\partial x^2 + \partial^2/\partial y^2$, v_x and v_y the components of point velocities, η the viscosity, and γ the density (Fig. 72). So far these equations have been used to solve the problem only for the approximate final values of surface deformation, in a semi-infinite plane lying under its own dead weight and having only a narrow slit as mine excavation (see Sect. 8.43).

The theory covering visco-elastic to plastic media proposes a deformation process giving a good approximation to reality and reflecting the influence of time and of the rate and direction of face advance. It therefore has great prospects of application in the study of strata movement, once suitable formulae are to hand.

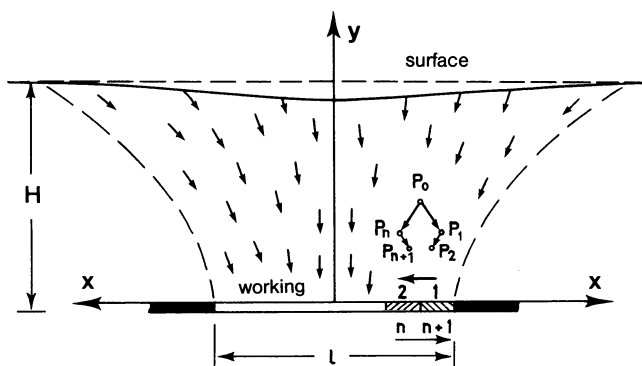


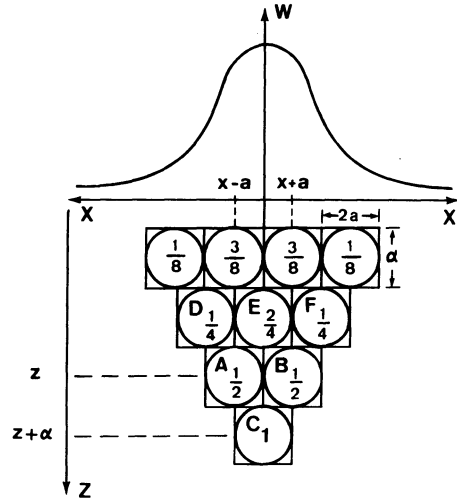
Fig. 72. Stiffly plastic flow in a rock mass influenced by underlying workings. The final position P_2 or P_{n+1} in the movement of the rock point P_0 after the mining of Strip No. 1 depends on whether the direction of mining is from 1 to 2 or from n to $n+1$

3.3.4 The Rock Mass as a Stochastic Medium

A rock mass that has been worked through many times can be regarded in an extreme case – if, for example, it is riddled with joints and fissures – as an accumulation of more or less loose blocks of stone. The blocks within such a large-scale and “mobile” deposit of rubble, so to speak, have such a degree of freedom that it seems obvious to use statistical methods in calculating the movement of such loose elements of the rock mass. This is because the laws to which the incohesive fragments of the rock mass are subject in their movements will depend only to a small extent on the properties of those fragments: stochastic, that is to say chance, movement processes will have a far more important influence.

The *development of movement* in a stochastic medium may be explained in terms of a rhombic packing of spheres as in Fig. 73. If the sphere in field C is re-

Fig. 73. Development of movement in a stochastic medium consisting of spheres, and the probabilities w governing changes of place into fields A, B, C ...



moved, the vacant space can only be filled by a neighbouring higher sphere from fields A or B. The probability as between sphere A, with centre-point coordinates $x - a, z$, and sphere B ($x + a, z$) occupying the empty field C($x, z + \alpha$) satisfies the coordinates

$$\frac{1}{2}P(x - a, z) + \frac{1}{2}P(x + a, z) = P(x, z + \alpha). \quad (68)$$

If we subtract the quantity $P(x, z)$ from both sides and expand by a^2 or α , we obtain a differential equation in the form

$$\frac{a^2}{2\alpha} \left[\frac{P(x - a, z) - 2P(x, z) + P(x + a, z)}{a^2} \right] = \frac{P(x, z + \alpha) - P(x, z)}{\alpha}. \quad (69)$$

From this, after the limiting process $a \rightarrow 0$ and $\alpha \rightarrow 0$, in which a^2/α approaches 1, there arises a parabolic differential equation:

$$\frac{1}{2} \frac{\partial^2 P(x, z)}{\partial x^2} = \frac{\partial P(x, z)}{\partial z}, \quad (70)$$

which describes a stochastic process such as that of diffusion (J. Litwiniszyn, 1957).

The change of places among spheres is repeated at higher levels. For example, the move of a sphere from A to C causes another sphere to migrate from D or E to A; likewise from E or F to B, if field B rather than field A has become vacant. At the third horizon the probabilities governing a change of place are 1/4, 2/4, and 1/4 respectively. The distribution of probabilities governing changes of place in all the various fields, as set off by the removal of one sphere at the lowest horizon (i.e., one extraction element), is shown in Fig. 73. At the highest horizons, the graph of probability values – that is, the frequency of sphere exchanges, representing in fact infinitesimal subsidence movements w – resembles the bell-shaped Gaussian curve.

Eq. (70) when rewritten thus

$$B(z) \frac{\partial^2 w}{\partial x^2} = \frac{\partial w}{\partial z} \quad (71)$$

is valid also for the relation between infinitesimal differences in vertical subsidence $\partial w / \partial z$ (vertical expansion, compaction) and horizontal curvature $w'' = \partial^2 w / \partial x^2$ in the plane case. Both quantities are thus proportional to the factor $B(z)$. Equation (71) is the basis even of that interpretation of the propagation of strata movement from a working to the surface which sees it in terms of the diffusion of gas or the distribution of heat (Buchheim, 1969; Avershin, 1969). The fundamental *stochastic equation* for the infinitesimal subsidence w of a point of space (x, y, z) in flat-lying measures therefore runs as follows, in Cartesian coordinates:

$$\frac{\partial w}{\partial z} = B(z) \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \quad (72)$$

or alternatively, owing to the rotational symmetry of the deformation, in cylindrical coordinates (r, z, δ) :

$$\frac{\partial w}{\partial z} = \frac{B(z)}{r} \frac{\partial}{\partial r} \left(r \frac{\partial w}{\partial r} \right). \quad (73)$$

$B(z)$ is dependent on z and is a quantity in metres characterizing the deformation properties of the stratum found at the level z . It is obtained from Eq. (71) or (72) by dividing the vertical expansion or contraction measured at one place in this horizontal stratum by the sum of the curvatures in the x - and y -axis.

For a particular horizon $z = h$, the solution of Eq. (73) for an *elementary trough* reads (Bräuner, 1961):

$$w(r) = \frac{aM}{4\pi\varphi(h)} \exp\left(-\frac{r^2}{4\varphi(h)}\right), \quad (74)$$

where a is the subsidence factor, M the thickness of seam worked, and

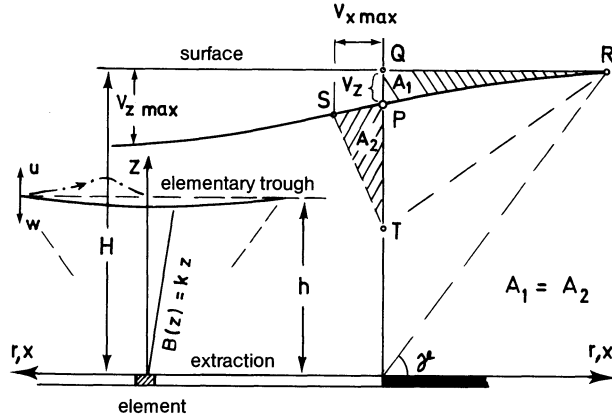
$$\varphi(h) = \int_0^h B(z) dz. \quad (75)$$

Infinitesimal subsidence w over an extraction element thus decreases, according to Eq. (74), with height h and radial distance r (Fig. 74). An analytical solution for the final subsidence v_z of a point on the ground ($z = H$) above an irregularly shaped working is dealt with in Sect. 8.41.

There are as yet no observational data for the quantity B which would enable the integral $\varphi(h)$ in square metres to be obtained for all strata between the extraction level $z = 0$ and the horizon in question $z = h$. In homogeneous rock the degree of deformation⁶ B will increase almost linearly with z up to the value at

⁶ For $B = 0$ at the extraction level there is assumed a roof bed which is lying under the full cover-load pressure (vertical expansion = 0) and which has settled uniformly (bending = 0). In Eq. (74) the exponential function e^x is shown as $\exp(x)$, which is easier to print.

Fig. 74. Deriving the structural function $B(z)$ from the volumetric constancy of areas A_1 and A_2 . Also shown is an extraction element with matching curves of elementary subsidence w and displacement u



the surface $B(H)$, thus: $B(z) = kz = z/H B(H)$ (Avershin, 1958). In theory, the *structural characteristic* $B(H)$ of the rock mass can be derived from the incompressibility condition for the section QRT of that mass lying over the unworked seam (Fig. 74). This block of the mass does not alter in volume during the period of influence of the working, if the sum of all the subsidence along the line PR is equal to the sum of all the lateral displacement down the vertical line PT, i.e. if $A_1 = A_2$. Down this line PT, maximum displacement over the ribside should decrease linearly from $v_{x \max}$ at the surface to zero at the point T, the depth of which is for the present still unknown. With a critical area of extraction it will lie at approximately half the depth of the workings (0.56 H), if in the formulae for the areas of A_1 and A_2 there are inserted the values $v_{x \max} = 0.4 \text{ aM}$, $v_z = 0.5 \text{ aM}$, and $\overline{QR} = H \cot 61.5^\circ$. From the constancy of volume there is obtained for $B(z)$ not a linear increase but an exponential function of z (Budryk, 1953) with the maximum values

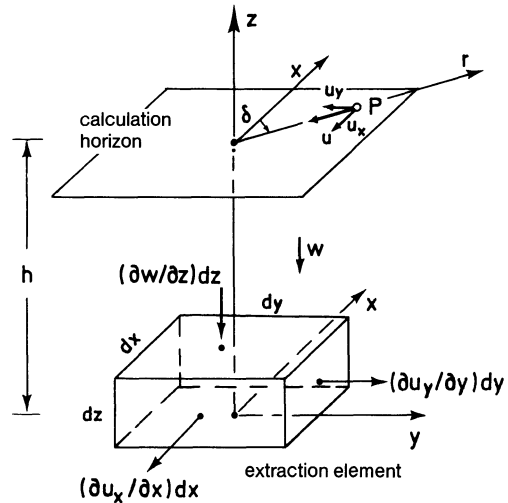


Fig. 75. Deriving the equation of continuity (77) at the extraction element, and the infinitesimal displacement of a rock point P in the horizon $z = h$

$$B(H) = \frac{H}{\sqrt{2\pi} \tan \gamma} \quad \text{and} \quad \phi(H) = \frac{H^2}{4\pi \tan^2 \gamma} \quad (76)$$

at the surface $z = H$.

In stochastic theory, *horizontal displacement* of a rock point is derived indirectly from the *equation of continuity*, which states that, in deformation, the volume $V = dx dy dz$ of a prismatic particle of the rock mass (Fig. 75) alters by dV if the prism is vertically expanded or contracted by $(\partial w / \partial z) dz$, and horizontally extended or compressed by $(\partial u_x / \partial x) dx$ or $(\partial u_y / \partial y) dy$. If, for example, in the zone of vertical expansion of the particle, subsidence w increases towards the bottom by $\partial w / \partial z$ per unit of measurement (mm/m), the height dz of the prism will be greater by $(\partial w / \partial z) dz$ (in mm). The volume alters in consequence by $dx dy (\partial w / \partial z) dz$ (i.e. area of base times change in height) or, since $dx dy dz$ is equal to V , by $V (\partial w / \partial z)$. This leads, on a linear change in all three prism axes, to the equation of continuity:

$$V \frac{\partial u_x}{\partial x} + V \frac{\partial u_y}{\partial y} + V \frac{\partial w}{\partial z} = dV. \quad (77)$$

In the case of an incompressible rock mass ($dV = 0$), for which Eq. (77) after reduction with V becomes

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (78)$$

vertical expansion or contraction necessarily entails a horizontal compression or extension of the rock particle – i.e., a difference in subsidence results in a difference in displacement, and vice versa.

In cylindrical coordinates, the equation of continuity reads:

$$\frac{1}{r} \frac{\partial}{\partial r} (ur) + \frac{\partial w}{\partial z} = 0, \quad (79)$$

from which can be determined the radial displacement $u = f(r)$ of a rock point P at horizon z towards the coordinate origin or extraction element, if for $\partial w / \partial z$ the fundamental stochastic Eq. (73) is introduced and integration is carried out (Bräuner, 1961):

$$u(r) = -B(z) \frac{\partial w(r)}{\partial r}. \quad (80)$$

On this basis, the infinitesimal displacement u of a rock point above an extraction element is proportional to the infinitesimal tilt $\partial w / \partial r$. The value for $\partial w / \partial r$ is obtained by differentiating the elementary trough Eq. (74) for r . Consequently, the *elementary displacement function* for the horizon $z = h$ reads:

$$u(r) = \frac{aMB(h)}{8\pi\phi^2(h)} r \exp\left(-\frac{r^2}{4\phi(h)}\right). \quad (81)$$

Final displacement v_x is discussed in Sect. 8.41.

a) In a two-dimensional structure model⁷ of a strata sequence (Fig. 76), the vertical and horizontal movement components w_1 and u_1 of nodal points in both roof and floor of working and seam are given. What is sought initially as an interim result is the nodal-point forces f_1 arising from the enforced movement $\delta(u, w)$, which are combined together to form a column vector $\{f\}$. In such a case, where the subsidence of a horizon of elements is given, the term commonly used is “*movement model*”, and for this the system stiffness matrix $[S]$, a numerical notation arranged by lines and columns, needs to be known:

$$\{f\} = [S] \{\delta\}. \quad (82)$$

From the calculated nodal-point forces the movements of the remaining nodal points can then be determined, using the inverse stiffness matrix:

$$\{\delta\} = [H] \{f\}. \quad (83)$$

b) The given dead weights of the separate elements, which act as a vertical external force F through, for example, the nodal points formed above or below by the extremities of those elements, add together in the model to constitute an overlying pressure. Above the extraction opening, this dead load leads to movement and deformation of the rock elements, the extent of which is what requires to be found. At the mining horizon, nodal point subsidence is counteracted by resistance from the layer of fill – a resistance which increases with increasing sag in the roof. For purposes of this “*load model*” a system resilience matrix (symbol $[H]$, after Hooke) is set up, and with this the required movement $\{\delta\}$ of all nodal points can then be determined from the nodal point forces $\{f\}$:

$$\{\delta\} = [S]^{-1} \{f\} \quad (84)$$

$[H]$ is thus the inverse matrix of $[S]$. Both matrices express the movement-force relationship.

How true to reality a model and the selected elasticity values are in practice can be tested by comparing, in the case of the movement model, the subsidence trough observed at the surface, and additionally, in the case of the load model, the more or less known bending profile of the roof bed, with the amount of subsidence calculated for the top or bottom layer of elements. In the movement model there is no need to reproduce the inelastic reaction of the fill material.

The calculation proceeds essentially in four stages:

- a) Idealization of the strata sequence in the plane case by means of a system of triangles and rectangles based on a longitudinal and a transverse section at the centre of the workings. Since stresses and strains within an element are regarded as virtually constant, the elements network in the area of high strain gradients – i.e., in the main roof – must be sufficiently fine-meshed.
- b) Evaluation of the kinetic properties of elements by means of a stiffness matrix. In the computer programme, matrices for the stiffness and load of elements are formed from the numerical description of their structure (e.g., co-

⁷ A three-dimensional solution founders on present-day computer capacities in the case of large objects.

In conclusion it can be said that stochastic theory certainly offers interesting formulations for resolving strata movement calculations, but also that the necessary observational data on rock-mass characteristics and their mechanical interpretation are still lacking.

3.3.5 The Rock Mass as Finite Elements

The concept underlying the new “finite-elements method” is that an elastic continuum or discontinuum be regarded not as a single block but as a collection of separate “elements”, notionally bound to each other only at nodal points, where they interact through their stresses and displacements. This approach enables conclusions to be drawn on the state of the whole from that of the individual elements (Argyris, 1957; Zienkiewicz and Cheung, 1968). It also facilitates the solution of problems concerning irregular and heterogeneous bodies. For example, the stratified structure of the rock mass might be represented by a network of plane triangles and rectangles or three-dimensional tetrahedra (Fig. 76). By “nodal points” are meant the intersections of the dividing lines. The finite nature of such a structure enables stress-strain relationships in a rock mass affected by underlying workings to be solved with linear equations based on matrix algebra, provided high-capacity digital computers are available (John and Malina, 1967).

Applied to the rock mass, the elements method offers two routes for the calculation:

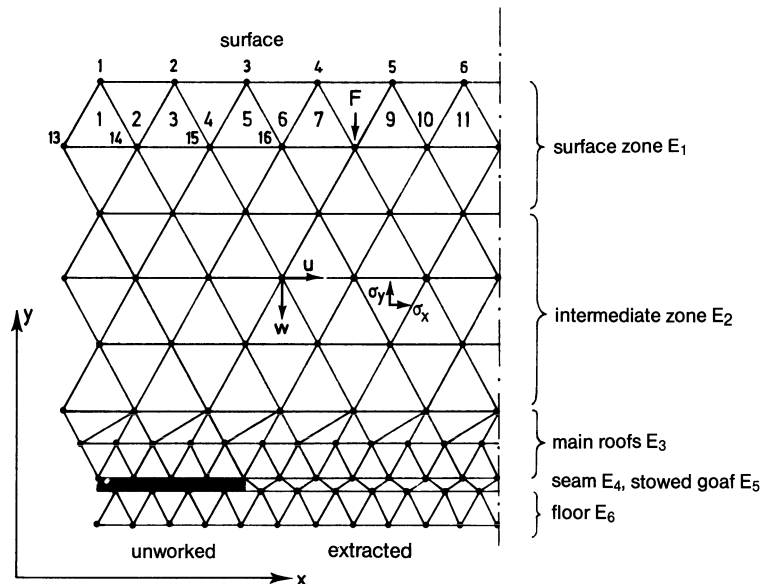


Fig. 76. A rock mass divided up into triangular elements, as seen in cross-section. Required are the lateral and vertical movements u and w of the nodal points. Each zone or layer is evaluated with a specific Young's modulus E

ordinates, elasticity values, density). The stiffness matrix for the system as a whole is obtained by combining those for the elements.

- c) Iterative calculation of nodal-point movement (rock-point movement) with the aid of separately itemized loads, by means of inversion of the system stiffness matrix as in Eq. (83), concluding with interpolation of points lying aside from the plane of the section. In doing this, three conditions have to be satisfied at the same time:
1. equilibrium between internal and external forces;
 2. compatibility in the deformation of elements at element boundaries;
 3. maintenance of the dependence of internal forces and deformations in each element on its geometry and material properties.
- d) In addition, for rock mechanics investigations, the stress components σ of each element can be calculated by means of a stress-transformation matrix $[M]$ from the movements δ obtained for nodal points:

$$\{\sigma\} = [M] \{\delta\}. \quad (85)$$

Equation (85) gives, inter alia, the distribution of strata pressures at the mining horizon. (In view of the mathematical derivations, attention is drawn to the by now extensive literature on this matter – including Kratzsch, 1975.)

For calculation in data-processing installations, all nodal points and elements in the strata section are numbered consecutively, as a means of codifying the structure of the strata sequence, and the data are recorded on punched cards. When calculated, the nodal-point movements, together with stresses and strains in the elements, are printed out.

The finite-elements method establishes for the first time a connection between subsidence, displacement, stress, and strain, but also between developing movements in all strata from the extraction level to the surface, and it does so plausibly, in accordance with mechanical laws. The nature of the stratification can be taken into account in constructing the network of elements, since different material values can be selected for each element, and even slip and fault planes can be represented by approximation as thin and weak features. Improved rock-mechanics values can be introduced into the final calculation without having to alter the structure matrix. As disadvantages of the method, mention must be made of the limitation at present to the plane case and the omission of the rheological and plastic factors in deformation, although solutions for both have now been proposed.

4. Strata Movement at the Shaft

4.1 Subsidence Damage to the Shaft Lining

Mining causes damage not only at the surface but also to mine structures in the interior of the rock mass. In mine roadways and working areas, it is true, this damage is caused principally by rock pressures which belong in the domain of rock mechanics. At the shaft, however, it is caused solely by the strata movements discussed in the last chapter. The mining subsidence engineer must therefore concern himself also with the kinematics of shaft damage, on which there is already a considerable literature. Protection of the shaft, the prime haulage and ventilation route linking the working levels with the surface installations, is after all of the highest importance to the maintenance of the mine undertaking.

Up to the turn of the century, a comparatively short distance between the shallow workings of that time and the shaft was sufficient to keep the latter out of the zone of strata movement; but with the much greater working depths of today, a cylindrical *shaft safety-pillar* with a measured radius of 50 or even 100 m is no longer adequate (Fig. 77). In the 1920s therefore, a cone-shaped safety pillar, defined by the angle of break or the limit angle, was increasingly adopted in the coal- and ore-mining industries of Europe. The large areas of perfectly good coal or ore contained within such pillars seldom remained untouched, however, and the shaft, which might be up to 1000 m deep, still found itself within the zone of influence of nearby workings. The exception was salt-mining, where, to avoid all risk of an inrush of water, no extraction is permitted within the shaft safety-area.

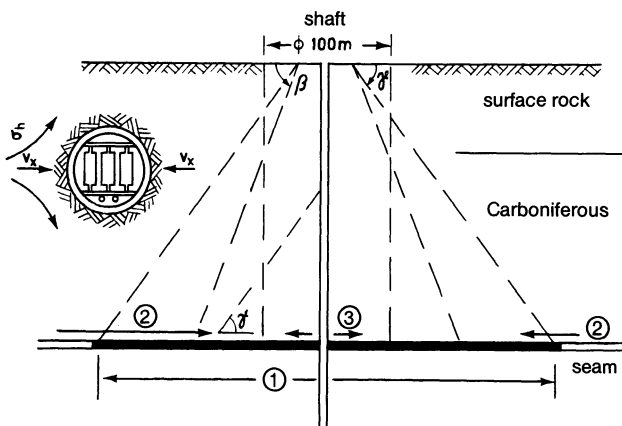


Fig. 77. Cylindrical or cone-shaped safety pillar for the shaft. The upper section of the shaft, which stands in water-bearing surface rock, is particularly at risk and in need of protection. *Left:* shaft section with built-in structures. β Angle of break; γ limit angle; 1 safety pillar at seam depth; 2 working from outer edge of pillar, showing increasing influence on shaft from top downwards; 3 central working, from shaft outwards

When working takes place in the safety zone, the outer cylinder of the shaft wall, enclosed as it is by the rock mass, is exposed to the highly varied movements of the strata through which it passes (Fig. 39). If the inner safety zone is worked, the lower part of the shaft is forced somewhat upwards into the mine excavation by the footwall beds, whereas the hanging wall or roof, to which the upper section of the shaft is bonded by mortar, concrete, or some general interlocking action, tends to drag it down with itself. In this process considerable compaction damage to the wall and internal structures of the shaft often results at the working horizon. Repairs are expensive and have to be conducted without interruption to the hoisting of the mine product. Higher up, as workings pass round the shaft, it is axially stretched and lengthened by the strata as they become perpendicularly relaxed (Fig. 35). If mining takes place at the outer boundary of the shaft safety-pillar, the part of the shaft nearest to the surface falls within the zone of vertical compression ahead of the face — a zone in which higher points in the rock mass subside more than lower ones. At the dividing line between the sandy surface layers, which slip down the shaft, and the solid rock beneath (e.g. in the Carboniferous), which acts as a firm support for the shaft, a peak of axial compressive stress can build up in the shaft lining. It is precisely there that a fracture of the shaft wall is most feared in water-bearing surface layers.

The horizontal components of strata movement have the effect of displacing the shaft outwards — i.e., away from the centre of the workings — both in the floor and in the main roof, and inwards — towards the centre — in the higher strata. The result is that the shaft structure, from the extraction level to the surface, leans towards the goaf and becomes bent. This inclined position of the shaft starts to develop ahead of the approaching face and is at its maximum just after the passage of the face, near the line $P_2 - P'_2$ in Fig. 43, gradually reducing towards the centre line $P_1 - P'_1$, where the shaft is once again vertical. In an *inclined shaft* a somewhat higher rate of wear is noticeable in the hoisting installations, but a rupture of the shaft is unusual given a regular pattern of strata displacement, since even a horizontal compressive stress of 2 mm/m implies that the walls of a 6 m diam shaft well close in by only 12 mm. This degree of horizontal compression is largely taken up by the pressure arch in the rock mass encircling the shaft section and by the surrounding porous layer (Fig. 77, left). If, however, the surrounding concrete has been tamped down and compacted, the rock pressure can produce a strain of 1 – 2 mm/m of diameter in the shaft section itself (Skutta, 1979). Bending of the shaft axis is mostly slight, but possible bending stresses have to be taken into account in the construction of rigid linings. Horizontal shear movements in severely bent strata (Fig. 38) are always associated with fractures in shaft linings.

The method followed in Germany since the 1930s for calculating the amount of lengthening and shortening in mine shafts (Bals, 1939) starts from the assumption that each section of the shaft, all down its length, sinks to exactly the same extent as the subsiding rock mass around it. Axial deformation of the shaft is therefore equated with vertical deformation of the rock mass. In the type of shaft in which the lining is bonded tightly to the rock mass by an outer layer of concrete (as distinct from some of the more modern shafts incorporating an enveloping layer of asphalt to act as a slip plane) prediction of the effects of mining is

Table 6. The effects of mining on the shaft

Effects and damage	Cause	Manner of transmission	Critical factors	To be determined
Vertical compression and extension of shaft	Subsidence of rock mass – of varying amplitude at depth	Frictional and adhesive shear stress	Vertical compression and extension of strata; shear force (wall area, effective length); deformation resistance (cross-section, linear changes, E modulus)	Transmission relationship of rock mass and shaft
Tilting, bending	Horizontal strata displacement, varying at depth	Close fit between shaft section and rock face	Siting of shaft in relation to workings	Extent of strata displacement
Shearing fracture	Intense bending of strata	Shear force parallel to stratification	Degree of bending, resistance of bedding planes to shear	Shear path
Horizontal pressure	Compaction of a weak bed	Transverse expansion	Vertical compressive pressure, degree of yield in strata, compression	Degree of pressure on shaft

Axial shear force: 100 N/cm² to break bond; 10 N/cm² kinetic friction in firm rock; 3 N/cm² in loose rock.

E modulus: Brickwork 1.2×10^5 ; concrete walling 2.5×10^6 ; cast iron 1.0×10^7 N/cm².

Slope of shaft deformation line on 100 m: tubbing in surface rock, 0.15 mm/m; concrete in surface rock, 0.21 mm/m; brickwork in Coal Measures, 1.8 mm/m.

thus related to calculations of strata subsidence. With an integration grid (Fig. 67), as used for predicting ground movements, subsidence is calculated at several points down the shaft 50 m apart, and from the difference in subsidence between neighbouring points lying one above the other the lengthening or shortening of the section of shaft concerned is obtained as a linear change in mm/m (see Table 6).

Although this simple method of calculation has served well for over three decades in the prediction of axial shaft stresses, the assumption that “shaft deformation equals strata deformation” is in fact valid only for the middle sections of the shaft. It does not hold good for the top and bottom sections, those nearest the surface and the workings. Measurements show that the shaft head frequently appears to “grow” out of the ground: which indicates that there is relative movement as between the upper shaft structure and the surface zone of strata. When the inner shaft safety-pillar is mined, a shortening of the shaft structure has been observed both above and below the mining horizon where, according to calculations, only a lengthening or, as in the footwall, no effects at all should occur. Ahead of the face the structure has been shortened even below – i.e., outside – the limit of influence of the workings, whereas by calculation it should be unaffected. In one shaft, subsidence of 14 cm was measured as much as 300 m below extraction at the edge of the pillar (Skutta, 1979). A shaft structure bound to the rock mass thus reacts to the influence of extraction differently from the

rock mass itself. In the following pages, therefore, a new method of calculation is proposed which starts from the transmission of frictional forces to the shaft (Kratzsch, 1969 and 1970).

4.2 Bases of the Calculation Procedure

As extraction begins to exert an influence, the tight fit of the tubbing key-rims and masonry footings, as well as the interlocking and cohesive action of the back-filled mortar (adhesive shear stress), have the effect that the shaft wall is at first firmly bound to the rock face surrounding it. In this initial phase the subsiding rock mass compacts or stretches the shaft lining to the full extent of the strata deformation. Here the lining cylinder behaves like a helical spring acting within its elastic limit and exerts a resistance to this deformation which increases linearly in proportion to the change in its axial length. At a certain magnitude of compaction or extension, the limit of the strength of bonding to the rock face by reason of the tightness of fit and the interlocking action is reached, and a rupture surface forms between the outer wall of the shaft and the rock face. The rock mass then slides down the shaft. During this relative movement a frictional force is transmitted to the shaft structure, which builds up over the extent of the slip plane and deforms the lining cylinder further, up to a limit of compaction or extension at which the lining either shears off inwards (shear fracture) or pulls apart (rupture). From these considerations it is clear that, up to the breaking-point of the lining, the shaft shares in the deformation of the rock mass only to a certain definite extent, which is determined by its deformation resistance and the amount of frictional force, and also that the build-up of frictional force leading to the deformation is influenced by the amount of frictional area engaged – i.e., by the shaft length¹.

The breaking strength of the shaft's close fit in the rock and its interlocking with it is of the order of $\tau_{Br} = 100 \text{ N/cm}^2 = 1 \text{ MN/m}^2$ (Sitz, 1968). A break between the shaft wall and the rock face will accordingly occur almost as soon as the effects of extraction begin to be felt, i.e. when axial deformation of the shaft lining amounts [Eq. (29)] to 0.05 mm/m in the case of concrete bonding, or to 0.1 mm/m with mortar bonding. This means that a 1-m section of shaft can be stretched out or closed up by the action of the subsiding rock mass by only 0.1 mm without the bond between shaft and rock rupturing and the deformation resistance of the shaft cylinder exceeding the limit of shear strength. Once a rupture plane has been formed in this way, the local force transmissible to the shaft wall drops to an adhesive shear or frictional stress of around 10 N/cm² in solid rock, and 3 N/cm² in loose ground (the measure of slip resistance).

For the frictional or *tangential shear force* F_t acting on the shaft as the rock strata slide down it, the following equation can be formulated:

$$F_t \leq \tau A l \quad (86)$$

¹ In Sects. 11.4 and 12.4 this process is demonstrated with reference to models (Figs. 286, 294, 307, and 308).

with

$$\tau = c + \mu p. \quad (87)$$

Here τ denotes the static shear stress in N/cm^2 , A the wall area per metre of shaft length (in sq cm per running metre), l the effective length (i. e., depth) of the shaft in metres, c the cohesion in N/cm^2 , μ the coefficient of friction, and p the horizontal strata pressure, in N/cm^2 , acting normal to the shaft wall.

The transmitted shear force is met, by way of force of reaction, by the elastic deformation resistance D of the shaft lining:

$$D = \epsilon ES 10^{-3}. \quad (88)$$

This is determined by the amount of shaft deformation ϵ in mm/m, the Young's modulus of elasticity E of the lining in N/cm^2 , and the horizontal cross-sectional area S of the lining cylinder in cm^2 .

The frictional force which builds up and is transmitted to the shaft thus increases linearly with shaft length, in accordance with Eq. (86). Correspondingly, the graph of shaft deformation will rise in a straight line until it reaches the magnitude of the strata deformation. Finally, there is obtained from the equated right-hand sides of Eqs. (86) and (88), after restatement, the required slope $\tan \alpha$ of this line of shaft deformation (shown in Fig. 78), in mm/m per running metre of shaft length:

$$\frac{\epsilon}{l} = \tan \alpha = \frac{\tau A 1000}{ES}. \quad (89)$$

In this way a simple connection is found between the shear force and the degree of vertical extension or compression ϵ in relation to shaft length. At the same time however (through this is not completely true), the coefficient of friction and the strata pressure normal to the shaft section l remain constant, while elastic, through not plastic, deformations after fracture of the lining cylinder are taken into account.

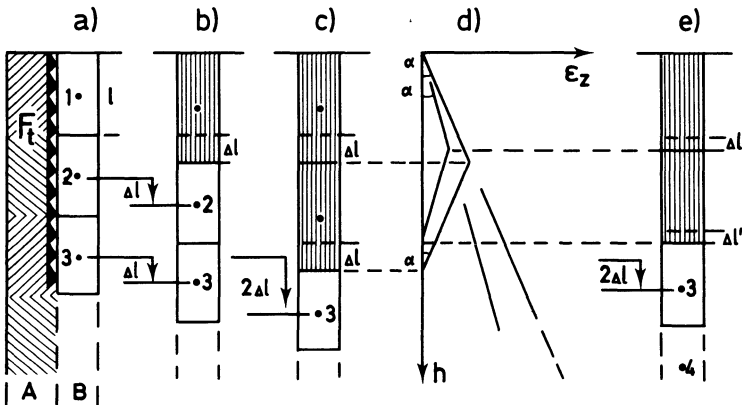


Fig. 78. Axial deformation of shaft lining through friction as strata subside. *A* rock face; *B* cross-section through shaft wall

If this process is looked at in terms of separate sections of the shaft structure of the short length l (Fig. 78), then a friction force F_t , which acts in the first place only on section 1 of the shaft, should lengthen that section by Δl (Fig. 78 b). This will have the effect of displacing the neighbouring sections 2 and 3 of the shaft by Δl also. The amount of elongation Δl is determined by the magnitude of the friction force and the deformation resistance D of the shaft lining cylinder, which in turn depends not only on the nature of the material and the strength of the shaft wall but above all, within the zone of elasticity – just as with an extended helical spring – on the amount of strain $\Delta l = \epsilon l$ reached at the time. The greatest possible strain Δl is reached if the adjoining elements 2 and 3 move away unresistingly by the same amount Δl (Fig. 78 b). Then F_t will be equal to D , and the given coefficient of friction μ can be said to have been exhausted. Elements 2 and 3 of the shaft lining will necessarily in that case have gone down by at least Δl . If now element 2, which has been displaced by Δl , is itself lengthened by the maximum possible amount Δl , then element 3 will be pushed down by $2 \Delta l$ (Fig. 78 c). In such a case, deformation ϵ_z of the shaft lining will increase linearly with the depth z and at the maximum possible angle α (Fig. 78 d). If, however, elements 3 and 4 provide a resistance to the displacement because they have not been pushed down far enough by the subsiding rock mass, then elements 1 and 2 can lengthen by only $\Delta l' < \Delta l$ (Fig. 78 e); and the slope of the line of shaft deformation will thus have become less steep: $\alpha' < \alpha$. To exhaust the coefficient of friction completely therefore, a degree of subsidence or deformation of the rock mass corresponding to the strain gradient is necessary – e.g. of Δl in horizon 1, $2 \Delta l$ in horizon 2, $3 \Delta l$ in horizon 3, and so on.

The straining Δl of any one element of shaft lining is caused by the frictional or adhesive shear stresses engendered between the shaft wall and the rock face if the rock mass subsides more than the shaft at the place in question, or if it expands or contracts perpendicularly to the stratification in the course of subsiding. This expansion or contraction of the rock mass is transmitted in whole or in part to the shaft lining cylinder by those frictional or adhesive shear stresses. It is transmitted only in part in those depth zones where the increase in strata deformation is greater than that of shaft deformation, because the latter is limited by the fact that $F_t = D$; and in full where downward movement of the lowest elements of the lining is hampered by insufficient strata deformation – i.e., usually at intermediate depths (Fig. 79). It is thus only at intermediate depths that the assumption “shaft deformation equals strata deformation” holds good. Such are the rock-mechanics considerations on which the new graphical method of calculation is based.

Strata deformation ϵ_z down the side of the shaft is calculated for a given state of extraction – e.g., with an integration grid – and entered as a curve in a depth-against-deformation diagram. Into this diagram the line of shaft deformation, rising at an angle α_i to the z -axis, is then entered according to rules which are dealt with in the next chapter (see also Sect. 12.4).

4.3 Calculation of Shaft Stress

In constructing the line of deformation for the shaft, three cases must be distinguished:

1. the shaft is in the zone ahead of the face: in this case the shaft cylinder will be axially compressed in its middle and upper reaches;
2. the shaft stands within the worked panel, and its lining has been removed ("slit") up to seam height at the working level before the start of extraction (see Fig. 99): in this case the shaft cylinder will be stretched throughout, from the mining horizon to the surface, since it can sink into the mine excavation by the same amount aM as the rock mass;
3. the shaft stands within the worked panel, but its lining is not removed at the working level: in this case it will be stretched only in its middle and upper reaches; at the bottom, near the workings, it will be strongly compressed, because the cylinder is firmly held in the mine floor and the sections of it in the roof cannot move down, like the rock mass, by aM .

a) The shaft in the vertically compressed zone ahead of the face. In the zone ahead of the face, the rock body above – i.e., within – the limit of influence of the workings is compacted as shown by the solid line in Fig. 79. According to past methods of calculation, vertical compression of the shaft would be equal to vertical contraction of the rock mass from point 4, well down its depth, up to point 1 at the top. Under the new method, the line of shaft compression rises at the angle α_1 from the pit-head to point 2, follows the line of strata compaction as far as point 3, where the lower line of shaft compression, rising at the angle α_5 , meets the line of strata compaction after crossing it on the way and delineating two areas A_2 and A_3 , which are equal². The shaft will therefore undergo com-

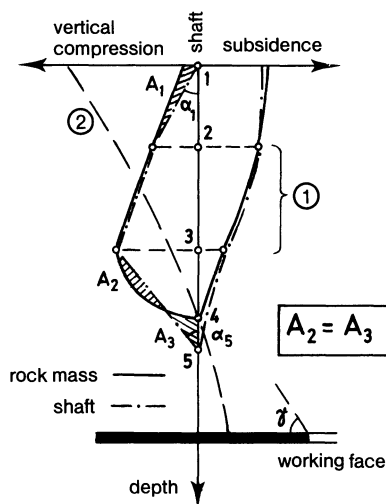


Fig. 79. Shaft in vertically compressed zone ahead of face. Working of safety pillar from outer edge inwards. 1 Equal subsidence and vertical compression of strata and shaft; 2 limit of influence of the working

² See Sect. 12.4. The areas A_1 show the difference in subsidence as between the rock mass and the shaft wall.

pression even below the limit of influence, up to point 5. The short stretch of adhesive shear stress below point 3 and above point 5, before the formation of a fracture surface at 0.1 mm/m relative compaction, is ignored both here and in Fig. 80. Above point 2 the shaft is compressed less than, and therefore subsides less than, the rock mass. The shaft head rises out of the ground surface to an extent corresponding to the relative movement represented by area A_1 . Below point 3, subsidence of the shaft is greater than that of the rock mass.

b) The shaft in the vertically expanded roof of the workings. With a shaft lining “slit” at the working horizon, and with extraction of the shaft safety-pillar proceeding from the shaft outwards, the graph of shaft elongation rises from the top and bottom of the upper shaft structure at an angle of α_1 or α_4 respectively until it meets the graph of rock-mass expansion (Fig. 80). In the middle zone of the shaft, from point 2 to point 3, there is no relative movement between shaft wall and rock face. Above point 2 and below point 3, axial lengthening of the shaft is less than vertical expansion of the rock mass by the amount of area A_1 or A_3 respectively, and the shaft head sinks into the ground.

A very different situation from that in Fig. 80 arises if the shaft lining is *not slit*: it cannot move down into the mine excavation, like the subsiding rock mass, but remains firmly bedded, as a continuous structure, in the floor beds beneath. As a result, below point 4 the shaft lining starts to undergo compression, which reaches a peak at the working horizon (point 5) and comes to an end only at point 6, below the workings. The shaft is stretched therefore only in its middle and upper sections. The compaction observed over and under the workings in an unslit shaft, or in one that is slit but has an insufficiently yielding wood filling in the opening, is confirmed by the graphical calculation method.

The *shaft deformation line* in Fig. 81 must be so drawn that, whilst its angle of slope is correctly reflected, shaft subsidence above an, at first, unknown depth

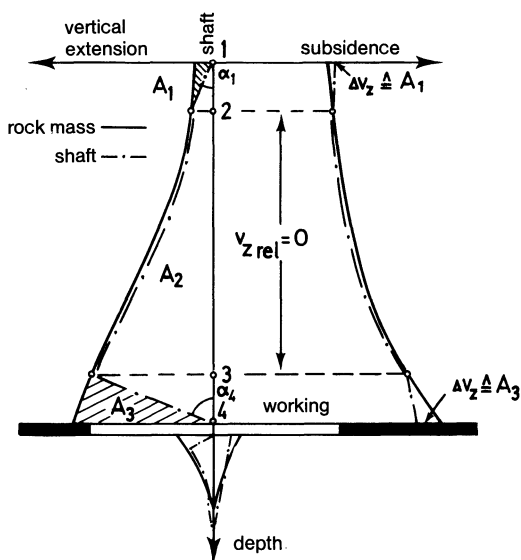


Fig. 80. Influence on shaft lining, “slit” at working horizon, exerted by overlying strata vertically expanded on working the shaft safety pillar from the centre. Below the workings the mine floor heaves upwards

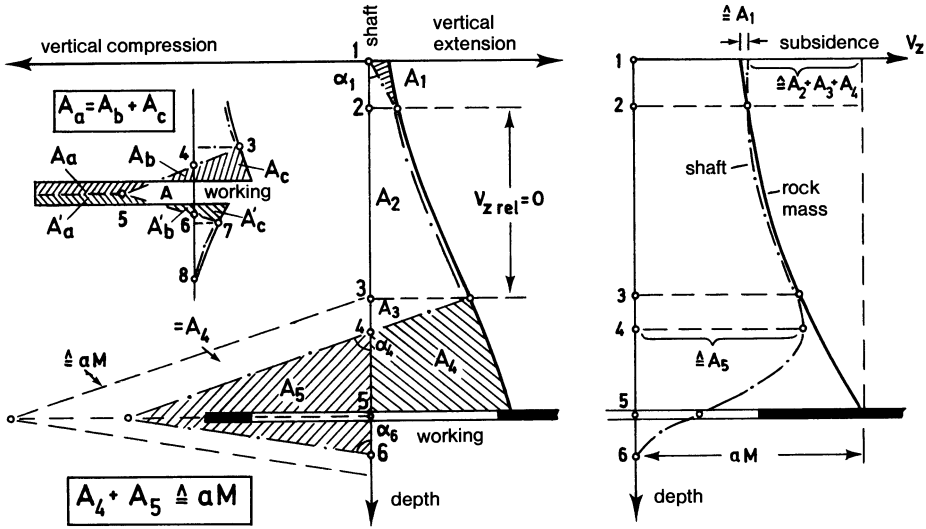


Fig. 81. Unslit shaft lining seen during central working of shaft safety pillar. Points 3 and 5: maximum axial extension and compression correspond to transition points in subsidence curve. Point 4: axial extension = 0, and subsidence of shaft lining at maximum

point 3 agrees with strata subsidence. For subsidence at strata point 5 in the working horizon (roof bed), more or less full subsidence aM can be assumed. As a result of vertical expansion, strata subsidence diminishes as it goes higher, as far as point 3, by an amount Δv_z , to which the areas A_3 and A_4 in Fig. 81 correspond. The areas A_i in fact represent the sum of all expansions and contractions above a vertical section Δh – i. e., the difference in subsidence Δv_z between two points of depth. The subsidence of strata point 3 accordingly amounts to:

$$v_{z3} \triangleq aM - (A_3 + A_4). \quad (90)$$

Since the shaft from point 6 to point 3 must also subside by v_{z3} , this lower shaft section must be axially compressed (A_5) or stretched (A_3) by

$$v_{z3} \triangleq A_5 - A_3. \quad (91)$$

From this there arises the condition

$$A_4 + A_5 \triangleq aM \quad (92)$$

for constructing the line of shaft deformation. On the vertical axis a triangle with angles α_4 and α_6 is erected, the apex $\epsilon_{\max}$ of which lies at the working horizon in the field of vertical compression, and the area $\frac{1}{2}(\epsilon_{\max} \Delta h)$ of which corresponds to full subsidence aM (dashed line). To satisfy Eq. (92) ($A_5 \triangleq aM - A_4$), the area of this triangle is then, by parallel displacement of the shaft deformation line, reduced along its outer edge by the amount of the area A_4 , where this is intersected by a projection of the upper side of the triangle. The apex of the triangle remains on the working horizon.

In the inset to Fig. 81 (top left), the shaft deformation line is shown superimposed (due account being taken of the area condition $A_a = A_b + A_c$) on an

enlargement of the rectangular compaction area (corresponding to full subsidence aM) of the yielding stowed or caved goaf. The shaft, which above point 5 is compacted less than the rock mass by $A_a - A_b$, must – up to point 3, where it subsides with the rock mass, being firmly bound to it – be axially extended by the same amount A_c less than the rock mass, since $A_a - A_b = A_c$. In the area where the mine floor is vertically expanded, the quantity for reconciling subsidence at point 7 is similarly obtained: $A'_a = A'_b + A'_c$. Since $A_a + A'_a$ is equal in area both to $aM - A$ and to $A_b + A'_b + A_c + A'_c$, the correctness of the graphical solution $aM - A_4 \triangleq A_5$ derived from Eq. (92) can be proved with this inset diagram, after restatement as $aM - (A_c + A'_c) \triangleq A + (A_b + A'_b)$.

Below depth point 3 and above point 2 there is relative movement between the rock face and the shaft wall. The shaft head sinks into the ground to an extent corresponding to area A_1 and amounting, in comparison with its starting height, to

$$v_{z1} = aM - (A_2 + A_3 + A_4) . \quad (93)$$

The shaft experiences its greatest subsidence at point 4, above the area of vertical compaction A_5 . At intermediate depth, between points 2 and 3, it is bound to the rock mass and subsides with it. With the differences in subsidence deducible from the size of these areas, it becomes possible to trace the rarely needed shaft subsidence curve for the cases represented in Figs. 79 and 80 as from points 2 and 3, in which shaft subsidence and predicted strata subsidence agree, and also for the example in Fig. 81 as from the subsidence-free point 6.

If in constructing the line of shaft deformation the assumption is made that shear stress τ , after the strength of the tight fit and interlocking with the rock face has been overcome, will be 10 N/cm^2 (0.1 MN/m^2) in Carboniferous rock and 3 N/cm^2 in loose surface rock, certain conclusions concerning linear changes in the shaft axis can be drawn from Eq. (89). For a shaft lining with walls 50 cm thick, irrespective of shaft diameter, the axial change in length will be of the order of 0.018 mm/m in a brickwork lining ($E = 2,500,000 \text{ N/cm}^2$) in Carboniferous rock, and between 0.0025 and 0.005 mm/m in loose rock. For 10 cm thick tubing with an additional 30 cm of concrete, the line of shaft deformation will have a slope of 0.0015 mm/m. In employing the graphical calculation method for axial linear changes it has been found convenient to relate 100 m of depth to a 2 cm length of ordinate, and to use a 4 cm abscissa for linear changes of 1 mm/m.

4.4 Superposed Mining Influences; and Tilt

With the new method, the influence on a shaft of several workings should not, as in the past, be added together to find the total influence. Instead, the strata deformation line resulting from the vertical expansion and contraction produced by each separate working should first be drawn, and the line representing the total of shaft deformation should then be inserted in accordance with the rules described.

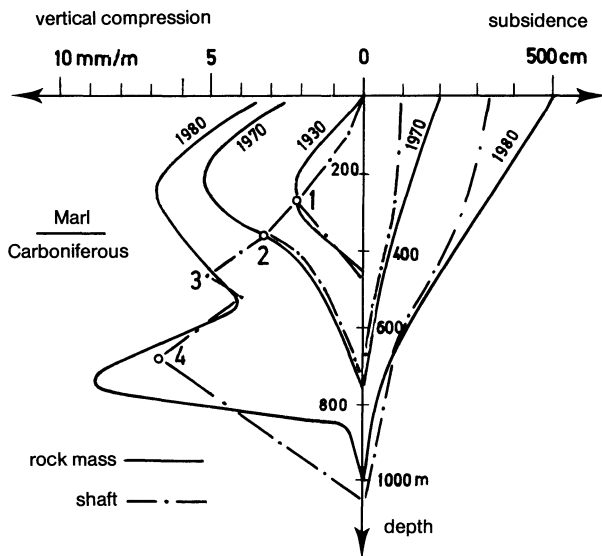


Fig. 82. Axial compression and subsidence of a shaft with brick lining (tubbing down to 300 m), on working the safety pillar from the edge up to the angle of break (4 seams at 400–500 m depth from 1900 to 1930, and 2 seams at 700–800 m depth from 1964 to 1970); then working up to within 200 m of the shaft (4 seams at 900–1000 m depth from 1970 to 1980)

For example, the slope of the diagrammatic line showing deformation at the top and bottom of a slit shaft does not alter if further workings in the same or a neighbouring seam exert an influence and accentuate strata deformation at the depth concerned. Once the coefficient of friction has been used up over the whole depth of working and the lines of shaft deformation rising at angles of α_1 and α_3 meet at a point P on or within the line of strata deformation, any further increase of expansionary or contractional strata deformation has no effect on the elastic lengthening or shortening of the shaft. That is a very favourable finding for mining, bearing in mind that it implies that, essentially, only the first workings force the sensitive tubbing apart or crush it together in the water-bearing surface rock, and that later workings are less dangerous to this section of the shaft.

For example, in Fig. 82, the further workings carried out after 1930 at the outer edge of the pillar, which produced further vertical compression, exerted no more compressive stress on the upper part of the shaft, with its tubbing down to a depth of 300 m, than had already occurred up to 1930, even though vertical strata compression increased from 2 to 5 mm/m and later even to 7 mm/m. The chronological peaks of compaction, 1 to 4, in the shaft curve are smaller and at greater depth than those in the strata curves. The lines with a year show total strata compaction and subsidence since the first mining effects in 1900. Adding together the shaft compaction effects ascertained separately for the periods 1900–30, 1930–70, and 1970–80 would, as stated above, lead to faulty results.

The shaft, being tightly embedded in the rock mass, has to follow the latter's horizontal movements completely: and if the shaft happens to stand at the edge of a working, its originally vertical axis will become tilted towards the centre of that working. Its deviation from plumb, the horizontal direction of which can vary greatly down its depth depending on how extraction develops, is obtained by calculation from the horizontal displacement of rock points near the shaft (usually taken at a vertical distance apart Δz of 50 m – see Fig. 83):

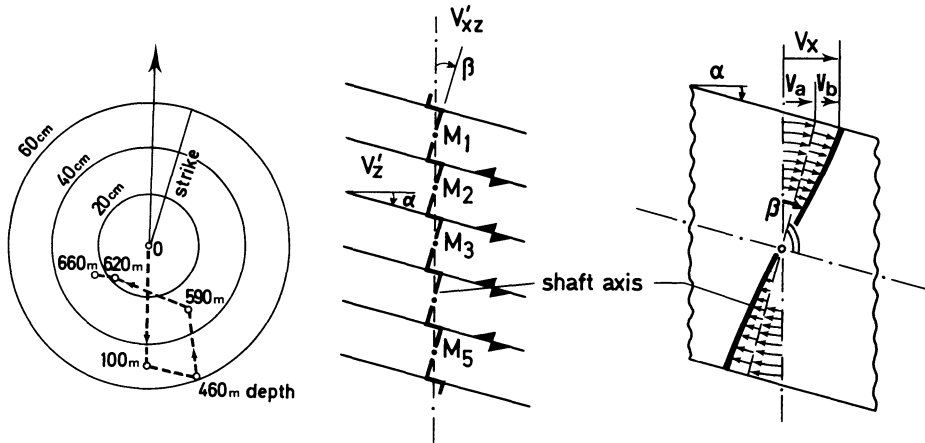


Fig. 83. Displacement of shaft axis (seen in plan) on working two seams dipping 20° to the west at depths of 610 m and 650 m. *Centre:* the shearing of individual shaft rings in loosely bedded strata – tilt of shaft rings corresponds to inclination of strata. *Right:* tilting and bending of shaft axis in a compact rock mass

$$v'_{xz(1-2)} = \frac{v_{x2} - v_{x1}}{\Delta z} . \quad (94)$$

The shaft head may be displaced by 1 m or more, but in general the deviation from plumb brought about by working a seam is not more than 1 or 2 mm/m of depth, except in the zone up to 150 m above the edge of a working. If in an extreme case of massive bending the roof beds undergo a shearing displacement in each bedding plane, the shaft lining will be tilted in each of the beds, while the line joining its centre points M remains vertical and unbent. In a different extreme case where a sequence of beds does not become loosened on bending, the shaft axis is displaced by v_a , at right angles to the beds, which slope at an angle of α and by a further v_b as a consequence of the internal transverse deformation of thick slabs, making a total tilting and bending of an angle of β (Fig. 83). The amount of bending v''_{xz} or v''_{yz} is calculated from the horizontal displacement of three points in the shaft axis (Sect. 7.2).

4.5 Extent of Vertical Expansion and Contraction in Strata

Earlier it was mentioned that vertical strata deformation can be ascertained from the subsidence of points in the rock mass calculated with an integration grid, by dividing the difference in subsidence (in mm) by the vertical distance between points (in m). Special integration grids for the direct calculation of expansion and contraction have also been designed, but they offer no great saving in labour or time as compared with deriving strata deformation from subsidence values (Sauer, 1961; Kochmanski and Magdziorz, 1972).

What appears very promising for future shaft investigations is the finite-elements method, if the shaft-lining cylinder is regarded as a vertical column of

infinitesimal elements within the total mesh, to which is assigned a substitute E modulus for tubbing, brickwork and concrete, related to the shaft section as a whole (Köse, 1978). The adhesive shear bond between the exterior wall and the rock face can be represented in model terms by a weak boundary layer. A gradual diminution of the E modulus in those elements of the boundary layer which have reached a critical shear stress enables the transition to kinetic friction to be allowed for.

In *Poland* they calculate chiefly the situation at depth and the maximum extent of vertical strata expansion or contraction at the shaft and equate this value to axial shaft deformation³ (Knothe, 1969). With circularly symmetrical working of the inner or outer shaft safety-pillar to a radius of r , maximum vertical deformation amounts, in its final extent (time factor = 1) to:

$$\pm \varepsilon_{z\max} = 0.462 aM \frac{R}{rH} \quad (95)$$

at a height of

$$z_m = 1.447 \frac{rH}{R} \quad (96)$$

above the workings (Fig. 84). At a horizon z , a maximum possible vertical expansion or contraction of

$$\pm \varepsilon_{z\max} = aM \frac{2}{2.718 z} \quad (97)$$

may be expected if the shaft safety pillar⁴ is mined from the centre outwards to a radius of

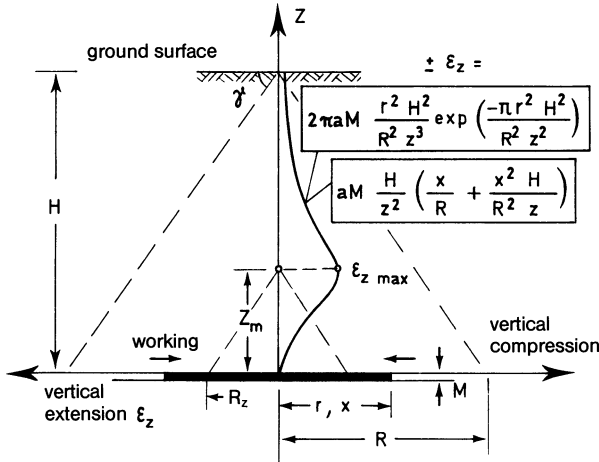


Fig. 84. Calculation of axial deformation in a shaft, using Knothe's functions, for a given state of progress in working, either concentrically (r), or from one flank (x) of safety pillar

³ Derived from Eq. (187) by introducing $R(z) = Rz/H$ for R (Fig. 84). To be consistent with Polish practice, z here signifies not depth but height above workings.

⁴ Instead of a straight line for the limit of influence in the rock mass (Fig. 84), a curved limit line is often assumed today, with a radius of influence $R_z = R(z/H)^n$ and with $n = 0.5$ to 0.65 (see also Fig. 95).

$$r = \frac{R}{\sqrt{\pi}} \frac{z}{H} . \quad (98)$$

From the functions of Eqs. (97) and (98) represented in Fig. 85 it is possible to estimate in advance the maximum vertical expansion or contraction at the horizons z for the given values aM (full subsidence), r and H . For example, for the relation $z/H = 0.2$, the nomogram gives the values $r/R = 0.115$ (i.e. $r = 0.115 R$) and $\varepsilon_{z\text{Max}} H/aM = 3.65$ (i.e. $\varepsilon_{z\text{Max}} = 3.65 aM/H$). The findings must be reduced in proportion to the working area if only one circular or annular sector is worked. With working from only one flank, the formula

$$\pm \varepsilon_{z\text{Max}} = aM \frac{1}{4z} \quad (99)$$

gives the maximum possible axial deformation which arises, for a distance between working face and shaft axis of

$$x = \frac{R}{2} \frac{z}{H} = \frac{R_z}{2} . \quad (100)$$

As the broken-line curve in Fig. 95 shows, vertical deformation with mining of the shaft safety pillar from a single flank amounts to one-third of that with concentric working, at every stage. The functional equations for the whole expansion or contraction curve are inset in Fig. 84.

During working, *dynamic* contraction reaches only 30% of its final value, whereas dynamic expansion is 60% greater than final expansion (Pielok and Stroka, 1979). With a more rapid face advance, dynamic expansion becomes greater and dynamic contraction smaller. For a stochastic medium (Sect. 3.34), vertical deformation is obtained by derivation from the subsidence equation [Eq. (220)] for the variable z (Szpetkowski, 1974). For example, expansion over the centre of a square working of side length q is by this theory:

$$\varepsilon_z = 244 aMz^{-\frac{3}{2}} q \exp(-0.5 \kappa^2 p^2) \Theta(\kappa p) \quad (101)$$

$$p = \frac{q}{2\sqrt{z}} - 1$$

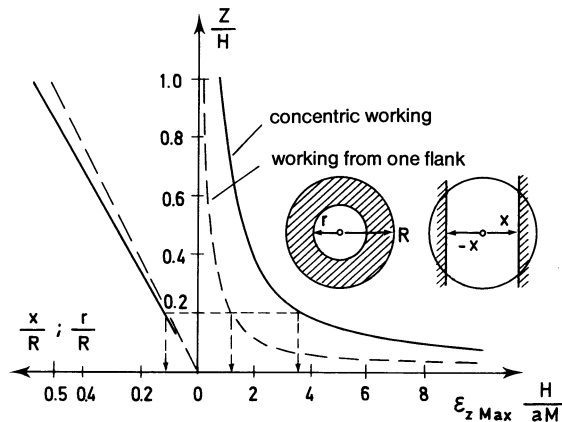


Fig. 85. Maximum possible axial extension or compression of shaft on basis of Eqs. (97) to (100), at a particular horizon and depending on progress in working

with $\kappa = 0.3$ and $p = \frac{q}{2\sqrt{z}} - 1$ (for the function Θ see Sect. 8.41).

On this basis, with workings of small area, very considerable vertical expansion will occur – e.g., 16 mm/m at $z = 20$ m over a working with $q = 60$ m – which, however, rapidly becomes less higher up. In workings of larger area, maximum expansion is in fact found higher up – e.g., 3 mm/m at $z = 120$ m with $q = 140$ m. A diminution of vertical expansion or contraction over a working on the basis of an e-function has also been observed in equivalent models (Awerschin, 1971).

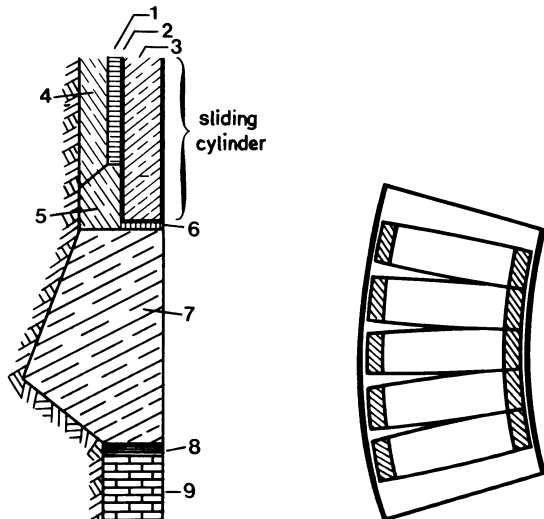
5. Systematic Mining of the Shaft Safety-pillar

5.1 Measures to Protect the Shaft

The proper functioning of the shaft must be safeguarded for dozens of years until the closing down of the pit. If mining of the shaft pillar is planned at any time during this period, measures of shaft support and general mine engineering must be undertaken to avoid damage to the shaft and consequent dislocation of production. The section of the shaft passing through water-bearing surface rock is in particular need of protection. In this area in recent shafts, the transmission of friction forces from strata to inner shaft structure is inhibited by a bitumen-filled *slip joint* inserted between the outer layer of concrete and the actual shaft lining (Fig. 86). If all strata movement is to be kept well away from the shaft, then working can only be permitted outside the limit-angle cone (Fig. 77). An extraction-free zone marked out on the basis of the angle of break is insufficient for this purpose, as the upper shaft structure will still be affected by movement ahead of the face when working from the edge of the shaft pillar.

The preconditions for complete protection of the shaft are, however, seldom fulfilled. On the one hand, a slip joint cannot be introduced into old shafts as an afterthought. On the other, mining deep seams can become uneconomic if the working of well-placed parts of deposits, offering short haulage and ventilation lines because near a shaft, has to be ruled out, particularly when the surrounding area remaining to be mined is geologically faulted or is narrowly limited by mine

Fig. 86. Section through a rigid shaft-lining which is able to slide (*left*), and through a flexible lining of loose concrete rings between two steel walls (*right*). 1 15–20 cm sliding joint; 2 steel plate, 1–3 cm thick; 3 concrete, 40–60 cm thick; 4 35 cm outer layer of concrete, against rock face; 5 concrete apron; 6 lead course; 7 reinforced concrete foundation; 8 crusher timber; 9 50 cm of masonry



boundaries. At a depth of 700 m the diameter of a shaft safety-pillar, given a limit angle of 61 grade, is already as much as 1 km. The surface installations of the mine, which stand on the untouched shaft pillar, rise up out of the subsiding terrain by several metres and soon find themselves in a zone of tensile stress. The gradient of approach lines in the mine railway can in time become unacceptably steep (Fig. 87), and unsubsidised portions of the mine area can become barriers to streams and waste-water trenches.

In view of these drawbacks in shaft safety-pillars, the concept of complete protection of the shaft has had to be abandoned with increasing working depth, and the fact of mining influence on the shaft accepted. Experience in coal mining has shown that by observing certain basic rules it is possible to maintain shaft winding even during mining near the shaft and, to a large extent, to protect the section of the shaft standing in water-bearing surface strata. These *rules* cover:

balancing out axial extension and compression stresses on the shaft by the action of two or more workings;

ensuring *symmetrical working* around the shaft to minimize tilting and shearing;

minimizing strata subsidence at the shaft through special mining measures such as stowing the goaf or leaving a central zone standing;

constructional measures affecting the shaft lining itself, such as "slits", layers of crusher timber, and timber cribs (see Table 7); in the case of new shafts, sliding and flexible linings;

regular observation of axial deformation, tilting, and bending of the shaft, by mine survey means.

Mining of the shaft safety-pillar is planned in careful stages, both on a large and on a small scale, and from the point of view both of the final total situation and of detailed individual operations. First the mine engineer, in collaboration with the mine surveyor and with the aid of existing records of the deposit, fixes the maximum scope of the operation in the light of the local geology, the development undertaken, and assessments of workability. Severely faulted, unworkable, or prematurely worked sectors of the deposit reduce the area of seam within the shaft safety-pillar which can be considered for mining. The mine survey division then calculates the strata movements and the changes in shaft length to be expected at the final stage of the operation as a whole. If the investigation shows that, in the light of material tests, the breaking strength of the shaft lining

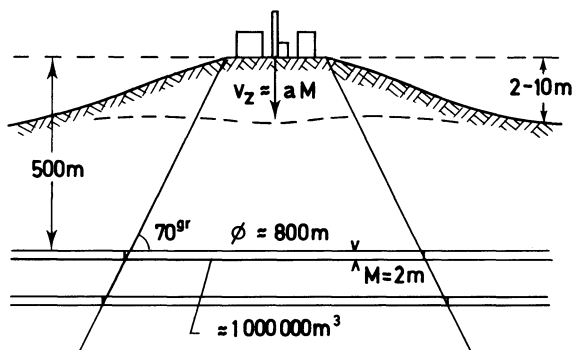


Fig. 87. Area to be protected against effects of mining appears to rise out of surrounding terrain. When seams in protection zone are worked later, ridges remain at surface, and central zone becomes compressed

Table 7. Measures to protect shaft lining when extracting in shaft safety-pillar

Protection through	Nature of measure	Effects	Remarks
Mining plan	Working from edge of pillar	Axial compression of upper shaft structure	Drawback: shaft thrown out of plumb
	Working from centre of pillar	Axial extension of lower shaft structure	Drawback: severe axial extension at bottom, if no core zone
	Edge and centre working combined	Axial compression and extension balanced out, both in time and in magnitude	Complicates underground operations; precise time-tabling essential
	Partial extraction only	Reduced stress — only axial compression or extension, not both	Sacrifice of mineral; unequal subsidence of shaft installations
	Symmetrical working	No deviation from plumb; no shearing of lining	Extensive and unwieldy extraction operation
	Stowing (core zone)	Halves strata movement	Raises cost of operation
Securing lining	Sliding joint	Reduced friction	Suited to water-bearing surface strata
	Expansion slit in lining	Shaft can evade axial compression	At mining horizon or over loading stations
	Crusher courses and joints	Shaft can yield and bend	Vertical interval 30–50 m, but not watertight
	Weak mortar	(see above)	Facilitates repair of masonry
	Annular cushion of ash	Absorbs lateral expansion of seam	At mining horizon if risk of vertical compression
	Outer ring of concrete	Seals shattered zone	Counters shear movements from intersected faults
	Close monitoring	Timely recognition of danger	Measurements and inspection reports quarterly

will be exceeded, or that the shaft will risk becoming sheared off by being tilted too far, then further areas of seam will have to be excluded or stowing methods of mining prescribed.

Only when fresh calculations of the final values for axial extension or compression and for tilt have been reconciled with the safety of the shaft, will proposals for the direction of working, daily progress, and sequence of operations be worked out, in the first instance only from the mining engineering standpoint. This provisional mining plan serves as the basis for a detailed subsidence-engineering calculation and examination of shaft stresses foreseen at the time of the quarterly or half-yearly stages of progress selected. In carrying this out the time factor z is introduced into the formulae, for simplicity's sake, as unit $z = 1$, and earlier stresses on the shaft lining are also taken into consideration. For example, freeze-bored shafts are subjected to axial compression shortly after completion if the sandy surface strata subside on thawing. By altering the

direction and sequence of mining in the light of mine-engineering, subsidence-engineering, and shaft-safety considerations, it is possible to bring an initially excessive increase in shaft deformation down to an acceptable level (a matter that is examined in later pages).

In the final phases of planning the operation, the shaft engineer proposes suitable measures of shaft-lining technique to minimize the load on the lining in the sections which are at risk. In addition, a plan is drawn up for the mine survey division to keep the axial length and plumb of the shaft, the width of joints, and radial deformation of the lining rings under constant observation by inspection and measurements. If there are sensitive surface installations near the shaft, these must also be brought within the subsidence-engineering investigation (see Sect. 14.2 and K. Lehmann, 1938).

5.2 The Mining Plan for the Shaft Safety Zone

The engineer can plan mining within the limit-angle cone in such a way that the section of the shaft to be protected – for example, the tubbing structure in water-bearing surface strata – is axially extended or compressed, tilted or bent only to an acceptable degree and, throughout the whole operation, is exposed only to the *type of deformation* – whether extension or compression, and whether with or without bending – which is least dangerous to the shaft lining in the light of its construction and materials. At the beginning of the mine-planning exercise therefore, the sections of the shaft to be protected, and the permissible degrees of deformation, must be firmly decided. The degree of stress will be determined by the amplitude of subsidence in the workings (in turn depending on area of working, convergence, etc.) and initially also by the added influence of deformation forces acting on the section of shaft concerned. The type of deformation will depend on the siting of the shaft in relation to the workings in horizontal terms.

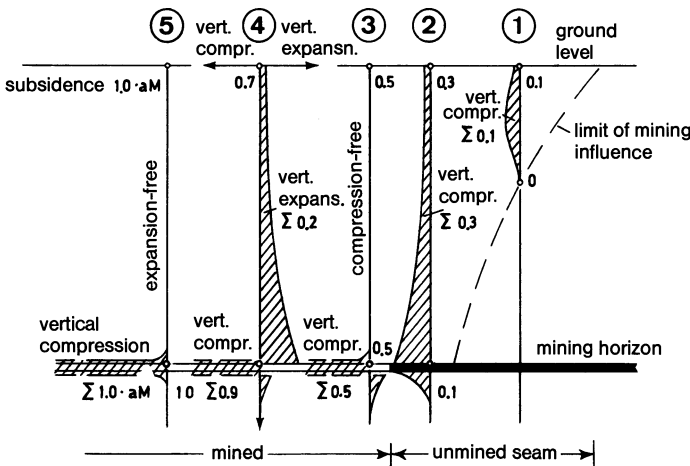


Fig. 88. Deformation of rock mass beside shaft, for different positions (1) to (5) of shaft (critical area of extraction; final situation)

In the zone ahead of the face the shaft is exposed, down to the limit of influence of the working, to vertical compression in the rock mass (position 1 in Fig. 88). This vertical compression increases, in a column of the rock mass which is displaced towards the working, by an amount in total matching the increase in subsidence of the relevant surface point, and the point of maximum compression travels downwards. From position 2 onwards, by which time the compression maximum has reached the mining horizon, vertical compression in the column of overlying rock is declining again, and the increase in subsidence from, say, 0.3 to 0.5 aM up to shaft position 3 results from the yielding of the soft seam or fill material. In this compression-free peripheral position 3, the rock column has subsided along with the roof of the working by around 0.5 aM. Deeper into the panel the overlying rock mass is vertically extended, and since the ground surface subsides further, the stowing material at cross-section 4 must be forced down by the combined amount of 0.2 aM vertical expansion and 0.7 aM surface subsidence, i.e. by 0.9 aM. At the centre of a critical area of extraction, vertical expansion also declines, while vertical compression at the mining horizon increases to the maximum possible value of absolute aM (position 5). The floor of the working expands upwards at positions 3 and 4, near the edge of the panel.

Interim stress on the shaft can differ greatly from this final stage of strata deformation, depending on the conduct of the mining operation and the treatment of the shaft lining at the mining horizon (whether it is slit and provided with a timber crib, etc.). A working which proceeds at critical extraction width $2R$

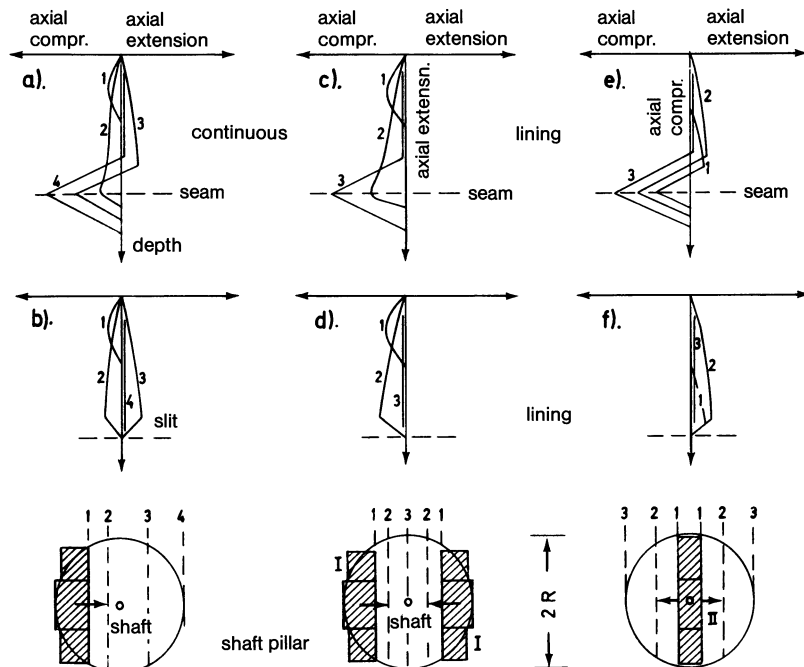


Fig. 89. Stresses affecting a continuous shaft (*top*) and a shaft which is "slit" at working horizon (*centre*), with extraction proceeding from one side of shaft pillar, from both sides symmetrically, and from centre of pillar symmetrically outwards (*bottom*)

from the edge of the shaft pillar towards the shaft, and from one side only, has the effect at first of axially compressing the shaft head (Fig. 89a). As the face advances, the zone of axial compression in the shaft lining extends to the mining horizon (curve 2). After the face has passed round the shaft, the middle and upper reaches of the shaft become extended (curve 3). The compression maximum which develops at the mining horizon, up to the area of extension-free shaft at the centre of a critical area (curve 4), damages the shaft lining if it has not, as is more usual, been opened up to seam height and had a yielding timber crib inserted (Fig. 89b). On this basis, a face carried inwards from one side stresses the shaft structure, first by compression and then by extension. Furthermore, the shaft becomes tilted in the meantime. With a face of less than $2R$ (critical width), a slight axial extension remains in the upper shaft lining after the shaft safety zone has been worked through, instead of curve 4.

This interim (dynamic) vertical extension and compression has not yet yielded to calculation with the desired degree of accuracy, because the time-related deformation process in the rock mass could only be measured, up to now, in a few places and at long intervals of time. As a first attempt at converting the calculated end-values into dynamic extension and compression, an exponential function $f(u)$ dependent on the dynamic variable u (Sect. 10.4) is proposed:

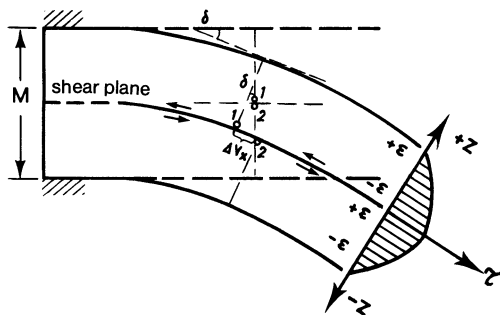
$$\varepsilon_{\text{dyn}} = \varepsilon_{\text{end}} f(u) . \quad (102)$$

This takes into account the rate of face advance, the time factor applicable to the calculation horizon, and an empirical radius of influence for the level of the calculation point $[R(z)]$ in Fig. 84; Pielok and Sroka, 1979].

The alternating stress on the shaft and its deviation from plumb can be avoided by working the shaft safety zone *symmetrically* and from the outer edge inwards. Compression of the upper section of the shaft, down to just below the limit of mining influence, is greater than shown at Fig. 89a (curve 1) because of the doubled mining influence. At position 2 of the face, the lower section of the shaft is also axially compressed. After that the upper shaft structure becomes unstressed, but high compression remains above and below the workings (curve 3), unless the shaft lining has been slit (diagram d). Consequently, with symmetrical working from the edge of the pillar inwards, at no time does the dreaded stretching effect arise in the upper shaft structure, which consists of segments of tubbing to seal off the water-soaked sand beds of the overburden layer (see also Fig. 82). With this mining procedure, stretching can be avoided throughout the length of a concrete shaft (which is vulnerable to tensile stress), provided it is slit at the mining horizon.

If the shaft pillar is mined symmetrically from the shaft outwards – and there are many good mining reasons for doing this, not least the early uncovering of the shaft walling at the mining horizon, which is at risk – then either the upper section of a continuous lining (diagram e) or the whole shaft structure above a slit (diagram f) remains free of axial compression. Working on both sides has the effect of acting first on the lower part of the shaft (curve 1), which is axially compressed and extended (diagram e) or only extended (diagram f). In position 2 of the face, the extending action is affecting the shaft lining right up to the pit-bank level. With a critical extraction area, a slit shaft shows no linear

Fig. 90. The shear travel Δv_x after formation of a shear plane in a severely bent group of beds



change on completion of mining (see Fig. 55). A continuous lining, on the other hand, shows (as in diagrams a and c) a dangerously high degree of axial compression at the working level. Since rupture cracks in the lining cannot be ruled out, this mining plan can only be followed in dry strata.

A comparison between the deformation lines of a continuous and of a slit shaft lining clearly shows, through the absence of axial compression at the mining horizon, the advantages of interrupting the lining at that level. Further consideration of means of reducing interim shaft deformation by overlapping the opposing effects of mining influences therefore starts from a slit shaft lining. Furthermore, working which is distributed symmetrically to the shaft in its safety zone is called for, not only to avoid tilting and bending it but above all also to prevent an often centimetres-wide *shearing fracture* of the lining at the bedding planes between strata. Shear fracture occurs at the shaft if the beds which it intersects are so severely bent by one-sided working (e. g., position 2 of the face in Fig. 89b) that the bending shear stress τ rising parabolically at the centre of a group of beds overcomes the adhesive shear stress, or limiting friction, at a bedding plane. Two points, 1 and 2, standing on opposite sides of the slip plane so formed, are then displaced by a shear travel amounting to (Fig. 90):

$$\Delta v_x \approx \frac{M}{2} \tan \delta. \quad (103)$$

The angle δ through which the beds will have moved can then be deduced from the subsidence calculated for three strata points near the shaft. No shaft lining, however strong, can withstand the shear movement of two beds. In inclined to steep-lying measures, however, shear movements can occur without causing bending — for example, at the interface with an unworked seam (Sljacheckij, 1974).

In the initial phases of mining it is not possible to achieve a complete *balancing-out of deformation* at the shaft by causing axial extension and compression to coincide, both in time and in place. The compression resulting from working at the pillar edge develops at the shaft head (Fig. 89d). Axial extension over a central working, on the other hand, starts in the bottom section of the shaft (Fig. 89f). Simultaneous opening-up of symmetrical edge workings I and central working II therefore leads to compressing the shaft lining at the top and extending it at the bottom. It is only from position 2 onwards that the extending

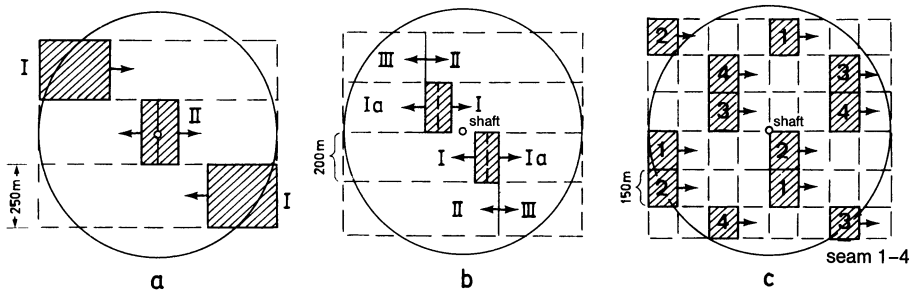


Fig. 91. Examples of combined peripheral and central working of shaft's protective zone, in one seam or in several seams

and compressing effects on the shaft structure cancel themselves out. All the overlying strata then subside equally at the shaft side and take the lining cylinder down, without linear change, by the amount of the roof settlement.

For a face advance which spares the shaft, it is usually sufficient to mine only the inner shaft protective zone from the point of view of balancing out expansions and contractions, in time and place, as much as possible. The protective zone in a seam, which at 700 m depth measures around 1000 m in diameter, is divided into three or four sections with a face length down the dip of 250 or 200 m, as the case may be. The outer segments of the circle, which will have only a slight compressional effect on the shaft head when mined later, are ignored. Where the protective area is very large, a limited approach such as this is all the more necessary to keep the operation within a manageable size and, by concentrating on the sectors having the strongest influence, to minimize the time during which the shaft is endangered.

The mining plans represented in Fig. 91 envisage, in ground-plan "a", perimeter workings I starting first, followed slightly later by central working II in both directions; in ground-plan "b", central working I in both directions from eccentrically offset faces, followed by perimeter working II; and in ground-plan "c", extracting the shaft pillar on a chequerboard pattern (*harmonically*¹), which leaves the shaft practically stress-free. In a water-bearing surface rock, the initial compression of the shaft head expected under mining plans "a" and "b" is a desirable "handicap" to offset its elongation that will come later. The expansion in the lower roof strata caused by central working represents no threat to the operational security of the shaft in dry rock and does less damage to the loading stations than compression.

As regards the rate of face advance, rapid peripheral working will (as shown in Sect. 4.5) reduce dynamic contraction, and rapid central working will reduce dynamic expansion. Rapid caving causes rapid subsidence and, consequently, since the stiffness of the bitumen depends on the rate of movement [Eq. (64)], an increase in the axial force transmitted to the lining cylinder by the slip layer.

¹ Harmonic working was originally developed with a view to protecting the surface installations of a mine (by balancing out horizontal compression and extension; K. Lehmann, 1938) and only later widened in scope to include shaft protection (by balancing out vertical compression and extension; Bals, 1939).

A mining plan such as this, entailing symmetrically combined peripheral and central working to cancel out stresses, and capable of being enlarged to take in several seams, often runs into serious difficulties in practice. For example, it may not be possible to keep to the timetable, or to effect stowing, or some unforeseen tectonic fault may be encountered. This applies particularly to robbing the shaft pillar on a chequerboard pattern. Quite apart from the cost of equipping several faces for simultaneous working, and the haulage and ventilation problems entailed, strict adherence to timetable is essential for success.

For the pre-calculation of shaft deformation, a clear knowledge of the *time factor* and the limit of movement in the rock mass is necessary if the balance of opposing forces from mining, in terms of time, place, and magnitude, is to be determined correctly. In the sections of the shaft which have sheared off at the rock face over the workings or in the upper surface strata, there is no fear of a further increase in forces – a new perception which can be of importance with peripheral or central workings acting in the same direction on a tubbing structure (Fig. 82). The shape of the shaft protection zone is determined by the structural geology and the workable areas of seam, and it demands in each case a special mining plan taking account of subsidence-engineering, mining-engineering and mining-economics aspects.

A further mining technique that can be mentioned for protecting the shaft lining is *stowing the goaf*. With stowing – which should be considered for at least the inner shaft-protection zone – subsidence of the roof beds is only half as great as with caving. Vertical contraction and expansion of the rock mass, and axial deformation of the shaft cylinder, both also decrease to a similar extent. Figure 92 shows the maximum values for shaft compression and extension decreasing from curve 1 to curve 1a, and from curve 2 to curve 2a, respectively on solid stowing – i.e., to half the value with caving (with hydraulic stowing in fact they drop to 15%). In other words, stowing protects the shaft lining very

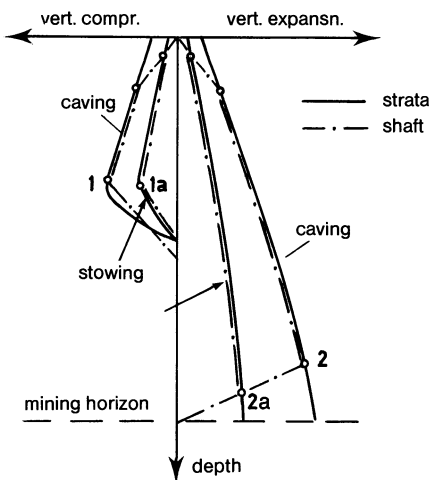


Fig. 92. Comparison of strata and shaft deformation in caving and stowing procedures

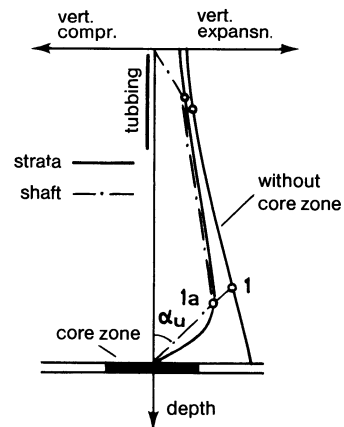


Fig. 93. Strata and shaft deformation with and without core zone. Shaft lining is "slit" at seam level

effectively from linear changes, and this applies equally to “low-subsidence” chamber-and-pillar working.

Over the goaf, the lower strata become significantly more relaxed than the upper ones. When subsidence is rapid and bending is severe, the bonding between roof beds can become loosened up to a height of 200 m, causing the lower shaft structure to be pulled apart in the early stages of central working. It has been suggested that this expansion in the main roof be kept within bounds by a “core zone”, formed either by leaving the shaft pillar unworked within a radius of 50 to 100 m of the shaft, or by stowing the same area with a very hard, unyielding material (Bals, 1956). As Fig. 93 shows, a core zone such as this certainly reduces expansion of the strata in the lower main roof but has less effect on deformation of the shaft, because in that area the rock mass slides down the shaft wall – always assuming the shaft lining has the necessary tensile strength. It is only when the shaft lining is not slit, or when in a slit lining the crib inserted is of a particularly unyielding timber, that the core zones often used in Polish coal-mining have a marked effect in reducing axial compression in the lower sections of the shaft. Under the new theory, the line of shaft deformation for a slit lining always rises from the mining horizon at an angle of α_u , and it is immaterial whether strata expansion falls to zero over a core zone or rises to a maximum immediately above a caved panel. On this basis a core zone reduces maximum extension of the shaft only slightly – i.e., from point 1 to point 1a in Fig. 93; and even in the upper tubbing sections, a core zone brings no significant reduction in shaft extension, unless the diameter of the zone is widened to several hundred metres. It can even prove harmful, like a residual pillar, if a loading station happens to be within the area of its abutment pressure. On the other hand, in an area where the main roof rests on a core zone, it does prolong the life of a shaft lining which is already fractured at the joints.

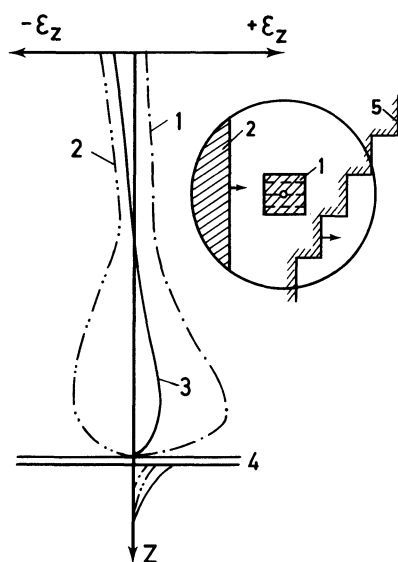
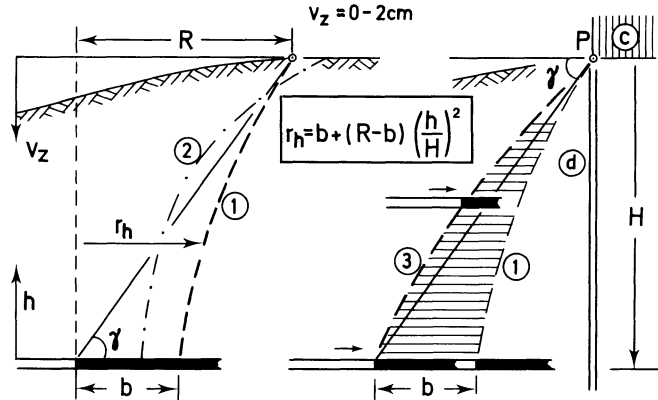


Fig. 94. Extraction of shaft's protective zone in Polish coal-mining practice. 1 First of all, core zone is extracted and stowed; 2 extraction from pillar edge; 3 combined effects of 1 and 2; 4 seam; 5 staggered working face (stepped-face method). (After Kowalczyk, Janusz, and Pielok, 1976)

Fig. 95. Limit of strata deformation in zone ahead of face (b). 1 Limit of influence normally; 2 limit in repeatedly mined strata; 3 safe limit of working for protection of building c and shaft d



When the core zone itself (1 in Fig. 94) is extracted, axial tensional stresses (curve 1) are set up in the shaft lining which reduce the compressive effect (curve 2) of extraction starting at the edge of the protective zone (curve 3). In Polish coal-mining, the core zone is stowed with flushed-in sand or with concrete, and this lessens strata subsidence down the length of the shaft cylinder. A staggered face is said to minimize the dynamic effects of extraction.

The *limit of deformation* begins in the zone ahead of the face and runs up through the rock mass in a shallow arc to the edge of the surface trough (Fig. 95, curve 1). Its distance r_h from the face increases from b to R , on the basis, for example, of the function shown in Fig. 95. If it is assumed that the measurement b decreases in an upwards direction in proportion to the cover-load and abutment pressures, then a line (curve 3) can be drawn which represents the limit of permissible working in the upper horizons if the shaft and important surface installations are to be fully protected. The amount of mineral lost can be reduced by working up to line 1, provided moderate mining influences are accepted, because in a rock mass that has been heavily worked it can be assumed that the limit of deformation will run along line 2.

5.3 Protecting the Shaft Lining Against Mining Damage

With an eye to later mining influences, the shaft lining cylinder can be so constructed that it is firmly bound to the rock mass or can slide, is rigid or can bend, is axially yielding or stiff. In dry rock, a shaft lining of brick, concrete block, or monolithic concrete is sufficient, but in water-bearing rock it must be completely watertight. Formerly only cast-iron segments, bolted together in rings and sealed with lead, were used – so-called tubbing, which was bonded to the rock face with a 20–50 cm thick layer of mortar or concrete (adhesion, form fit). Since 1950, steel segments have also been bolted together to form double-walled lining cylinders in water-bearing surface rock, or corrosion-proofed steel plate has been welded together to form a watertight casing (Fig. 96). The load-bearing and buckle-resistant constructional element is a wall of concrete or of concrete

shapes. Horizontal seams or special impact joints give the cylinder flexibility to accommodate later bending (one-sided working) and axial compression (peripheral working). In modern shafts, the bitumen joint mentioned earlier substitutes a sliding joint for the tight-fitting bond between the lining cylinder and the rock face. Throwing the shaft cylinder out of plumb as it “swims” in bitumen is to be avoided, as also is fracturing the outer layer of concrete and so allowing the bitumen to flow out into the rock mass.

Experience shows that, when working is near the shaft, the lining receives most damage where coal-seams or clay beds are intersected at the point of transition from loose to firm rock (e.g. to the Coal Measures), along geological faults and at loading stations (Fritz, 1971). In other sections of the shaft, lengthening or shortening by not more than 3 mm/m or 1 mm/m respectively generally does not cause sufficient damage to threaten the safety of shaft operations. Clay beds, coal-seams, and stowed areas which have become vertically compressed ahead of the face expand parallel to the stratification and exert a radial pressure on the shaft lining, which can cause it to bulge and shatter in places. This kind of sudden fracture, like temperature-dependent movements of tubbing joints in a downcast shaft, are often mistaken for axial mining influences. Along geological fault planes within the zone of mining influence, overlying blocks of the rock mass slide downwards, and the shaft lining becomes detached from them. To avoid a major relative displacement of blocks of strata, working within the shaft protective zone should proceed inwards from both sides of the disturbance line simultaneously. *Loading stations* constitute the weakest link in the shaft owing to their close fit with the rock mass, their more rupture-prone domed shape as compared with the shaft cylinder, and their sheer size. An annular *cushion of ash* or slag between the shaft cylinder and the 2–3 m deep layer of shattered rock will afford protection against the horizontal pressure of weak beds by taking up their

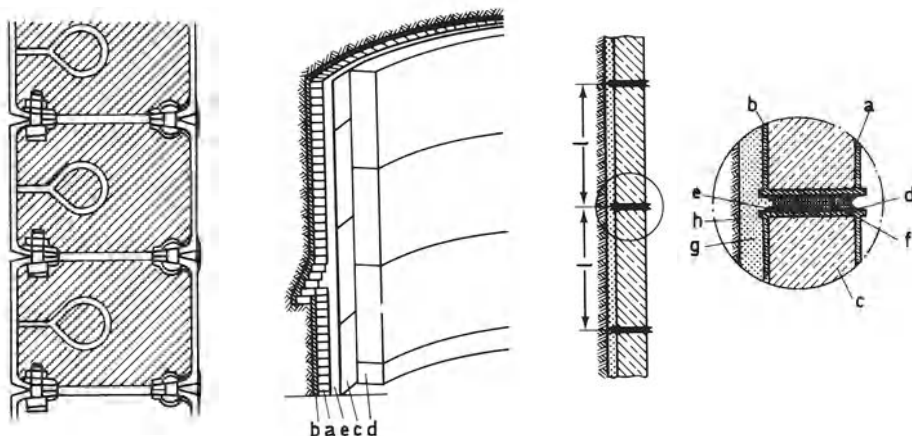


Fig. 96. Modern shaft linings. *Left:* flexible lining of bolted steel U-segments enclosing rings of concrete with horizontal joints. *Centre:* sliding, flexible lining. *a* Masonry; *b* rock face; *c* walling of welded steel plate; *d* reinforced concrete rings with sliding outer film to permit bending; *e* asphalt joint. (After Helfferich, 1976). *Right:* flexible, double-walled lining. *a* and *b* inner and outer walling of steel plate; *c* concrete; *d* and *e* articulated steel joints between lining rings; *f* elastic filler; *g* outer layer of concrete with sliding film; *h* rock face. (After Link, 1978)

lateral expansion. Shaft fractures at fault planes, although they cannot be prevented, can be sealed against an inrush of water by means of an *external apron ring*. Axial compressional forces on a loading station can be taken up by inserts of timber or *plastic material*, which enable the shaft lining above the loading station to “give”. And the loading station can be isolated from horizontal rock pressure by means of an open space behind its masonry wall or a porous outer layer.

Crusher courses of timber or plastic also provide “give” when there is axial compression on a shaft cylinder which is continuous from loading station to loading station. The desired reduction of axial pressure on the shaft lining depends on the thickness, vertical distance apart, and elastic modulus of the crusher courses. Even more effective are open, horizontal *joints* in a lining that is bound to the rock face, as they shorten the effective length of the frictional surface in the intervening sections of shaft lining (Fig. 97, right). The open slit interrupts the transfer and build-up of frictional force in the lining. The rise in the shaft deformation line begins afresh on each side of each joint, and deformation at the middle of each shaft section remains less than deformation in the rock mass. The use of such an open joint in the shaft lining is, however, confined to dry strata. The width of the slit is reckoned on the basis of the pre-calculated difference in subsidence Δv_z of the rock points a and b in the middle of the shaft section, as reduced by the amount of axial shortening in the shaft ends adjoining the slit (area $2A_2$, or $A_2 + A_3$ in Fig. 97). In the graphical representation, the width of this slit – on either side of which the shaft sections move vertically in relation to each other as mining proceeds – corresponds to the area A_1 delineated by the shaft and the line of strata deformation between the depths a and b.

In the event of axial extension of the shaft, layers of roofing felt have proved to be less effective in reducing damage than open compression joints, where the

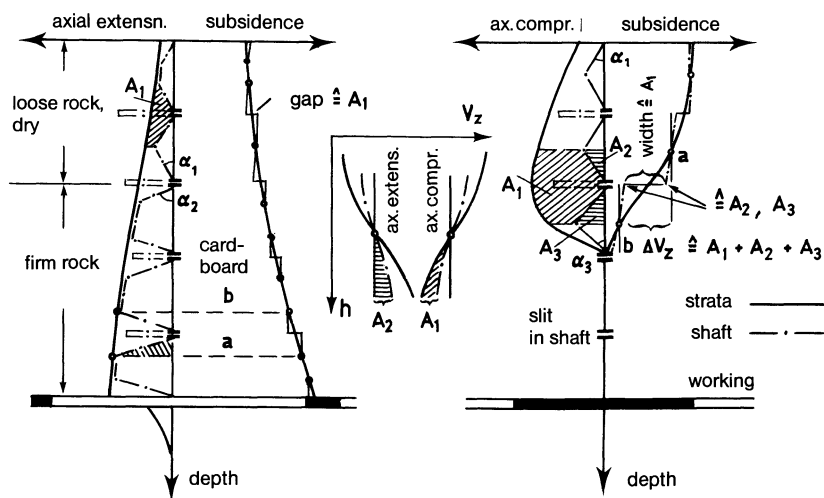


Fig. 97. The effects of a separation joint of roofing felt in an axially extended shaft lining (left), and of an open slit in an axially compressed lining (right). Centre: enlarged detail

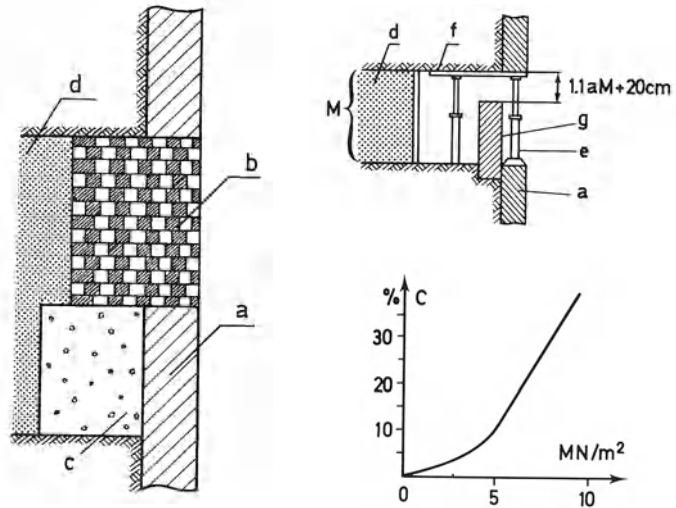


Fig. 99. Support of upper shaft structure by layers of timber (*left*) or by yielding props (*right*), both placed in slit in lining; and *graph* showing compression of timber packing (*bottom right*). *a* Shaft walling; *b* timber packing (wooden chocks); *c* concrete apron; *d* fill material stowed in mine excavation; *e* yielding hydraulic prop; *f* bar; *g* masonry walling as barrier to mine excavation after upper shaft structure has subsided. *C* convergence. (After Kowalczyk, Janusz, and Pielok, 1976)

Nevertheless, movement of the shaft ends is usually restricted by ring-shaped timber packing (wooden chocks) of seam thickness M , inserted in the slit opening in the form of planks laid crosswise (Fig. 99). The result is compaction of the *timber packing* and axial compression of the shaft lining. The shaft structure above the workings can also be supported by yielding hydraulic props (Fig. 99). In compacting timber packing by 15%, 40%, and 45%, compressive stresses of 4, 8, and 12 MN/m² respectively have been measured (Brinkmann and Neveling, 1969). If a compressive stress of $\sigma = 10 \text{ MN/m}^2$ is assumed in the timber packing, then the part of the shaft lining which acts as the “anvil” ($E = 12,000 \text{ MN/m}^2$) undergoes a maximum axial compression, in the neighbourhood of the open slit, of

$$\varepsilon = \frac{\sigma}{E} 1000 = 0.8 \text{ mm/m} . \quad (105)$$

This compression rises at the shaft deformation angle of α_1 but decreases linearly to nil at top and bottom (Fig. 100). At the level of the yielding timber packing, the shaft deformation line rises at the steep angle of α_2 , derived for a vertical compression of the packing by 45% M . In comparison with a shaft lining with an open slit, the area of axial extension is decreased by the amount of the strip A . In an exact representation of the shaft deformation line, the cross-hatched rectangular and triangular areas should be roughly equal, as in the inset to Fig. 81.

To safeguard the disturbed masonry above the seam from collapse, it is sometimes anchored to the rock face. As can be seen from Fig. 98b, however, a horizontal anchor prevents relative movement between the rock face and the shaft

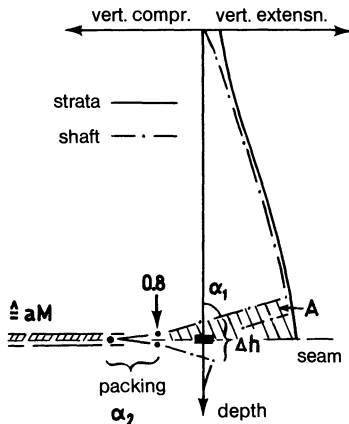


Fig. 100. Vertical compression of timber packing in slit of shaft lining, and axial compression of lining itself, in the depth zone Δh

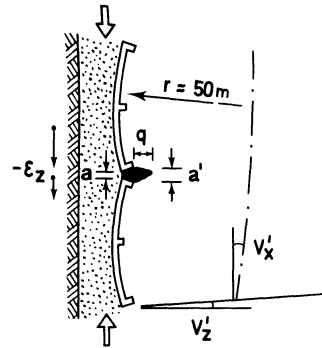


Fig. 101. Cast-iron tubing segments bent by axial compression of shaft. (After Skutta, 1979). Deviation from plumb v'_x of shaft axis does not correspond to tilt v'_z of tubing rings. Lead is squeezed out of horizontal joints up to $q = 120$ mm when $a = 1 - 2$ mm and $a' = 4$ mm

lining and so exposes the latter to shear forces. A better solution is to anchor the end of the shaft vertically to the lining higher up, or to cover the inside wall of the shaft with wire netting.

As regards the choice of *materials for the shaft lining*, the following considerations are relevant. A hard lining of monolithic concrete, bonded to the rock face, is less subject to deformation than a weaker construction of masonry, and the slope of its shaft deformation line is at a correspondingly smaller angle α . With a weaker lining, axial extension and compression in the upper and lower portions of the shaft are greater than with a concrete lining, as the shaft walling becomes detached from the rock face over a short distance only, but this does not mean that greater damage results: for the breaking-point of masonry under compression, at 3 mm/m, is twice as great as that of concrete at 1.5 mm/m. Under tensile stress, the fracture point in concrete is reached at as little as 0.15 to 0.30 mm/m, whereas in masonry the first cracks along the horizontal mortar joints do not become visible until tensile strain reaches 1 mm/m. Thus a concrete lining withstands axial compression best, whilst under axial tension a shaft lining of brick and weak mortar, which can easily be repaired, is advantageous. From this point of view masonry lining is particularly well suited to working the shaft pillar from the centre outwards, and in firm, dry rock, because breaks in the lining can be quickly replaced with new brickwork, and because proceeding in this way avoids exposing the mortar to a destructive alternation of compressive and tensile stresses. In wet rock a concrete or reinforced concrete lining as well as working from the pillar edge are to be preferred, since concrete is less vulnerable to compression than to tension. The vertical compression resulting from peripheral working in fact causes no separation cracks in the lining. In one instance, cast-iron tubing rings 1.4 m high have withstood axial compression of 2.8 mm/m in the middle of the ring and curvature of the wall surface down to radii of 50 m

without being damaged. The shortening of the tubbing structure measured was the sum of axial compression and bending of the individual rings (Fig. 101) and reduction in the width of joints through the squeezing-out of lead. The latter occurred when axial compression reached 1.3 mm/m (Skutta, 1979). The best protection against mining influences, of course, is a slip cylinder surrounded by asphalt (Fig. 86).

A welded steel casing has a high tensile strength and is also proof against buckling under axial compression, since it is radially supported outside by the bitumen or concrete layer and inside by rings of reinforced concrete. A casing of cast-iron tubbing ruptures at the bolted horizontal flanges under high tensile stress.

The shaft's *internal structures* are also exposed to vertical forces and to distortion. Before starting extraction near the shaft, hoisting guides and piping conduits should have sections inserted which can telescope. For the heavy downward pressures on hoisting guides at the seam level, sliding sections incorporating a loop have been developed. Wall recesses prepared for bearers and wall-plates should be filled with weak lime mortar. Adjustable brackets enable movements in internal shaft structures to be evened out. Yet even in a new shaft, axial stresses quite apart from dead-weight pressures can arise. This applies particularly to freeze-bored shafts. During the freezing process the rock face heaves by several centimetres and subsides again when thawing out, while the lining expands as it warms up. This interplay of temperature and friction forces can produce considerable axial compression or extension of the lining, amounting in places to 0.5 mm/m, and this, when added to the various mining influences, can significantly affect the estimation of later damage.

The prediction of shaft stresses when mining moderately to steeply inclined measures is complicated by the lack of data on strata movement. Since movement is predominantly perpendicular to the stratification, the shaft tends to be thrown out of plumb and bent even in the middle of a steep-lying working (Fig. 46).

6. The Influence of Previous Workings and Mining Practice on Strata Movement

6.1 The Reciprocal Influence of Workings

Discussion so far has assumed a virgin rock mass that is being worked through for the first time on a large scale in the course of extracting a seam. In most cases, however, the subsidence engineer is concerned with strata that have been mined repeatedly – as, for example, when there have been previous workings in the same seam, or when a deposit consisting of several seams has been worked through from top to bottom, seam by seam, for decades. In a rock mass that has been fragmented to a greater or lesser degree in this way, the strata overlying a working will undoubtedly move differently – more intensely, and also more rapidly – than over a first working, for which these pre-calculation methods were devised (Fig. 102). The degree of previous working thus affects both the development of movement and the field of stress in the rock mass as a whole. This “*degree of previous working*” is determined by the number and area of seams worked, the amount of convergence in the panels, the perpendicular distance between workings, and the spatial distribution of unmined parts of the

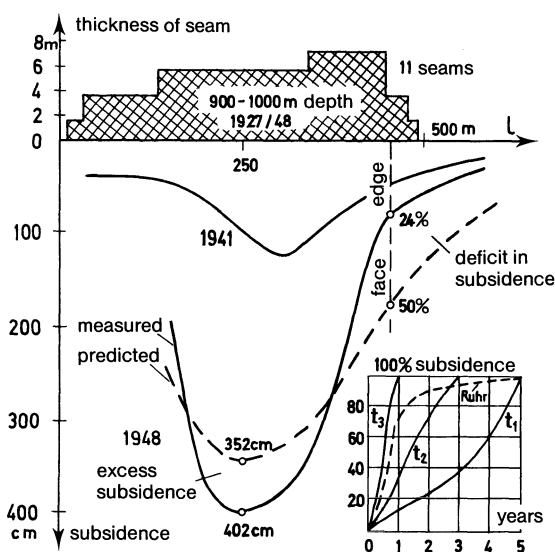


Fig. 102. Section through a subsidence trough – and time factors – on working through a rock mass several times. (After Baron, 1951). Thickness of seam worked: 3m(t_1), 3–7m(t_2), and over 7m(t_3). Panel approx. 400 m long

deposit (peripheral zones, panel ribs, residual coal pillars). With increasing mining, the structure of the rock mass is altered and the overlying strata weakened and broken up. Furthermore, this structure is affected by the *mining practices* adopted: a rapid sequence of workings concentrated on one block of strata, a rapid face advance, and caving are all conducive to fragmentation of the rock mass, and all accelerate the development of movement.

It was early recognized that a neighbouring working, or a working higher up, influences the shape of the subsidence trough, and that if a shaft safety-pillar is extracted at several levels, an unexpectedly deep and steep trough is formed (Baron, 1951). A rock mass that has been mined a number of times, in different seams, loses its mechanical continuity – its bond strength – and subsides almost silo-fashion on to the active face beneath. And the more it is worked through, the earlier the final stage of deformation develops.

The differences between calculated and measured strata and ground movement which have been observed in the European coal-mining industry since the change from solid stowing to the caving procedures prevalent today, and since the concentration of mining activity on a few coal districts (Lautsch, 1971), must be ascribed in large part to the high degree of holing through the rock body. This alters the rigidity of the groups of beds, increases the number of slip planes and joints, and renders the field of stress in the rock mass increasingly irregular. Another factor is the progress made by longwall caving as an accepted mining practice. This has meant that the daily face advance in coal-mining has increased from 1 m to 5 or 7 m, with the result that today a seven times greater area of roof strata – say $7\text{ m} \times 200\text{ m} = 1,400\text{ m}^2$ – is undermined and brought to collapse daily, leading to severe fissuring of higher strata at the same time. Since these structural changes in the strata influence the development of movement in the rock body and of convergence in the working area, and since delayed subsidence occurs in old workings subjected to renewed movement, these remote effects of mining cannot be left out of account in a precise calculation of subsidence at points in the rock. It is therefore an urgent task of subsidence engineering to investigate the relative dependence of rock kinematics on new and old workings and on the rigidity of overlying strata, in order to take quantitative account of them in the modern computing procedures.

A mining operation, by the strata movement and stress redistribution it causes, can affect old workings, chambers, and pillars, not to mention shafts and access roadways, in higher and lower and adjoining panels within the rock body, setting off secondary convergence, shifts of pressure, and renewed fissuring. Of particular importance is the mutual influence of

- two neighbouring workings at the same depth;
- two or more workings at different depths;
- a coal pillar's influence on a panel.

The nature of this interaction is examined in the following pages (see Table 8). The influence on strata movement of blasting practice and of pillar dimensions in chamber working is considered in Sect. 10.212 in relation to the time factor.

Table 8. The interaction of underground workings

Phenomenon	Influence on	Result	Cause	Effective range
Panel border effect	Within the same seam, the adjoining goaf	Delayed subsidence in goaf	Doubled length of span	50 m into goaf
Protective seam effect	Undermined and overmined panel	Little convergence	Perpendicular load relief in goaf	100 m perpendicularly, a further 20% load relief
Ribside effect	Undermined and overmined panel	Considerable convergence	Abutment pressure on ribside	100 m perpendicularly, a further 50% abutment pressure
Residual coal-pillar effect	Undermined and overmined panel	Very severe convergence	Overlapping of abutment pressure on two ribsides	100 m perpendicularly, a further 100% abutment pressure

6.2 The Border Effects of Neighbouring Workings

Over the face edge, as is well known, the roof does not drop all at once, in a single step, by the maximum amount aM (Fig. 103). It bends instead in a gentle and continuous curve over the face working and the waste edge, as shown in Figs. 13 and 28–30. The result is that the stowing material bordering the face, although compacted somewhat by the convergence of 20%–30%, is nothing like so consolidated as at the centre of the panel, where convergence is perhaps 50%. In longwall caving also, this *zone bordering the face* is spanned by the main roof, and the layer of large-sized waste rubble which forms there is under only a slight load. For example, lumps the size of a flagstone, which made up the goaf between strip packs at intervals of 20 m, could still be pulled out by hand 3 years after mining (H. F. Schulte, 1952). On this basis, the zones along the inside edges of an extracted area contain some reserve of subsidence. This is either used up by inelastic delayed subsidence over a long period of time or converted into strata

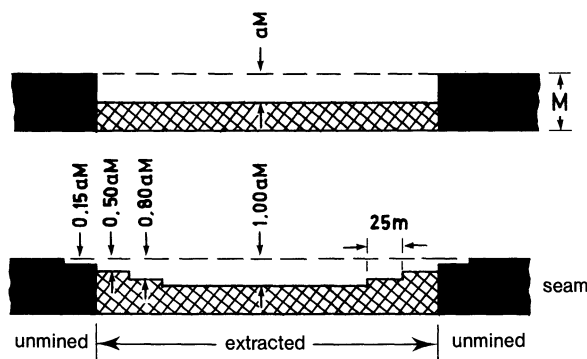


Fig. 103. Approximate representation of roof settlement over an extracted panel, in a single step (*top*) and in four steps (*bottom*)

movement by extraction of the remaining seam, which removes support from the overlying strata and doubles the length of roof span to the new ribside.

If a new working is opened along the edge of a previous working, and if there is no roadside pack still standing there, then there will be delayed subsidence in the goaf, to a sum of 100% aM , and full subsidence along the inside edge of the new panel (Fig. 104). The two separate workings will become a single, broad panel, with a continuous roof sag from one end of the seam to the other. In the border area, roof pressure on the stowing material rises until it is equal to the cover-load pressure γh , provided the panel area is sufficiently large. The abutment pressure A_0 , which previously bore on the seam beside the dividing line, is largely transferred to the newly introduced stowing material and to the still not completely compacted material in the old panel. The proportion A_2 not taken up by the stowing material appears as an increase in pressure A_5 over the old seam edge. Under the high pressure acting on the stowed goaf at the new panel centre, the floor heave of around 10% aM at what was the edge of the old stowed goaf recedes (and moreover does not reappear at all at the edge of the new panel) and in doing so contributes to increased settlement of the roof.

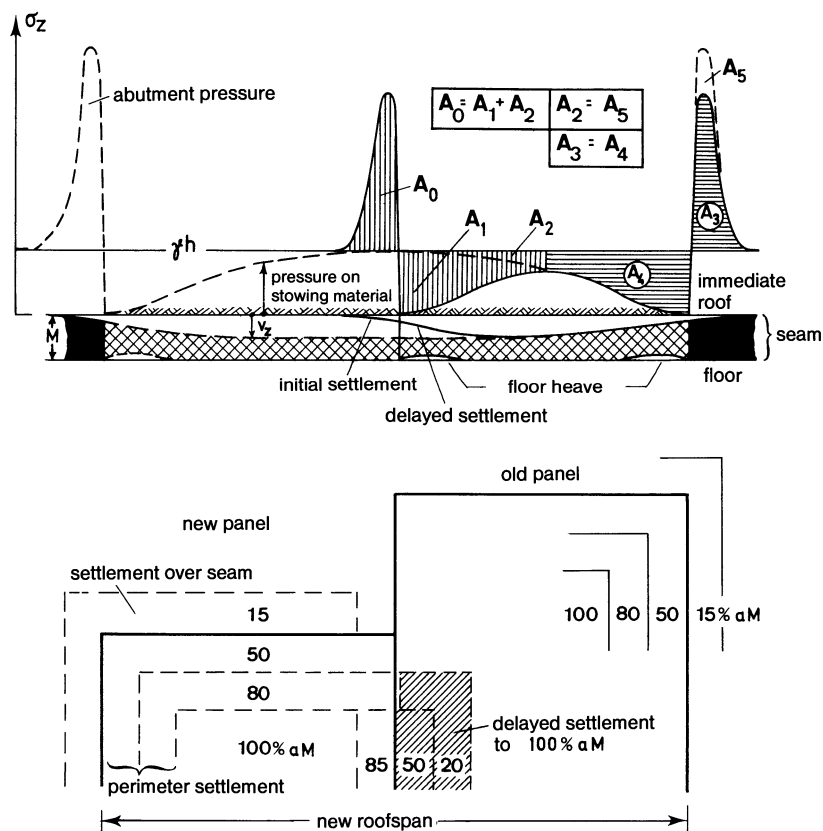


Fig. 104. Border effects of neighbouring panels. *Top:* distribution of vertical strata pressure over an old and a new working; also roof settlement. *Bottom:* panels and settlement zones seen in plan

If in place of a sandstone roof's settlement in a continuous line at the mining horizon there are substituted *four steps* of, say, 15%, 50%, 80%, and 100% of maximum settlement at the centre of the stowed goaf (with the 15% strip lying over the unmined seam), then the three outer zones can be taken as being, according to observations in the Ruhr, each about 25 m wide (Figs. 103 and 104). With a stowed goaf measuring about 250×500 m and upwards, roof settlement at the centre of the panel, even with subcritical extraction, can be equated to approximately full subsidence aM. The lines of the steps, seen in plan, run parallel to the ribside. If the exact roof settlement happens to be known, the steps over the rib where the face was opened out, and over the halted face, can be variously measured or estimated. The sag of a less than stiff shale roof can be represented by 25 m wide steps of 7% (over the unmined seam), 70%, and 90% of the settlement reached at the centre of the panel (Fig. 13). On the plan of the new working the step lines are drawn to run round the new abutments. In the (shaded) border area of the old working, settlement to 100% aM becomes complete; this delayed settlement can be shown in steps of 20% and 50%, as in Fig. 104. The roof over the unmined seam, which has already settled by some 15%, goes down a further 85% at the edge of the new panel.

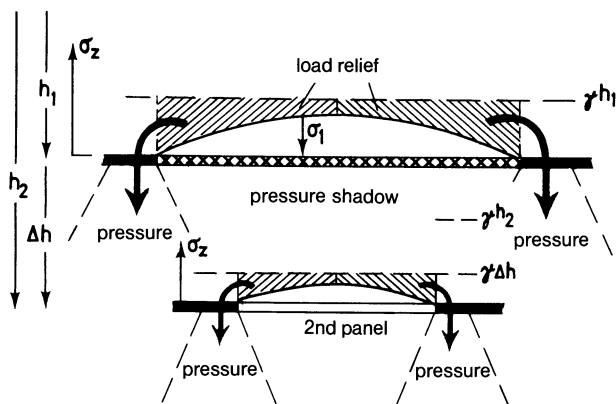
Special considerations regarding the amount of roof settlement apply if there is a supporting roadside pack still standing between the two panels.

In a *chamber working*, the vertical pressure on the pillar is always greater than the cover-load pressure γH on the overlying strata. If the salt or ore deposit represented in Fig. 14 is mined from a to b, and in the end to c, the vertical pressure on the narrowing barrier pillar, and in the adjoining zone (2) at the edge of the old panel, increases – as indicated by curves a, b, and c.

6.3 Changes in Convergence with Multilevel Workings

The perimeter effects of workings in two neighbouring seams depend on the delayed settlement on the adjoining goaf of a roof deprived of its support over an unworked seam. By contrast, where workings are one above the other (multi-seam mining), it is the gradient in perpendicular strata pressure created by the first excavation which influences the amount of convergence in a new working; and through the long-range influence of mining-induced strata movements, convergence in a new working lying up to 100 m higher or lower may be either diminished or intensified. If, for example, a second seam is mined in floor strata which have expanded upwards under an area of goaf, the “pressure shadow” of the upper stowed goaf will initially be evident in the new panel in the form of weaker strata pressure and reduced convergence; and if the face area comes within the zone of abutment pressure extending down from an old ribside, the increase in pressure will be evident through increased convergence. Conversely, the second working exerts a return influence on the old panel, which starts to move again and becomes flexed and further broken up in its roof strata as the mining under it proceeds. These roof strata, weakened as a consequence, give

Fig. 105. Mining in the pressure shadow of an old panel working, showing reduced convergence in second panel



way round the perimeter of the goaf under the strata pressure, with the result that delayed subsidence occurs even over the upper stowed goaf.

As mentioned above, where further mining takes place under a goaf area, convergence in the new panel is somewhat less than on a first penetration of the rock mass, because the upper panel isolates the lower from the full cover-load pressure $p = \gamma h_2$ [see Eq. (4)]. On the newly developed section of seam there rests only the belt of beds between it and the upper goaf. In addition to its own weight force $\gamma \Delta h$, this belt is loaded at the upper mining horizon h_1 with the pressure σ_1 of higher strata acting on the stowed goaf, which rises to a peak at the panel centre. The weight force of the intervening layer and the pressure on the stowed goaf, however, are together less than the total cover-load pressure $\gamma h_2 > \gamma \Delta h + \sigma_1$ (Fig. 105). The upper panel thus constitutes a *protective shield* against too high a loading of the lower seam – a factor which is turned to good account in combating rock bursts.

As also mentioned above, a working which advances under an upper ribside enters an underlying zone of abutment pressure and becomes vertically compressed. The magnitude and the perpendicular extent of this *ribside abutment pressure* depend partly on the mining depth and area of the goaf, from which the load exerted by the undermined strata can be calculated using Eq. (1), and partly on the bending resistance of the roof and the reaction force of the supporting layer, from which the pressure distribution can be determined using Eq. (28). If it is assumed that, on extraction of a critical area, approximately 75% of the strata load is borne by the stowing material and 25% is transferred to the two seam ends as abutment pressure (Fig. 106), then the vertical abutment pressure equally distributed to a seam end of width $b = 30$ m, given $R = 0.7 h$ (limit angle of 60 gr), will amount to a maximum of

$$\sigma_{z\max} = \frac{0.7 h}{4 b} p. \quad (106)$$

At $h = 800$ m depth and with $\sigma_z = 94$ MPa, this exceeds the cover-load pressure of $p = h \times 0.025 \text{ MN/m}^3 = 20$ MPa by almost five times. With subcritical extraction, the absolute abutment pressure in MPa is smaller than when extracting a critical area.

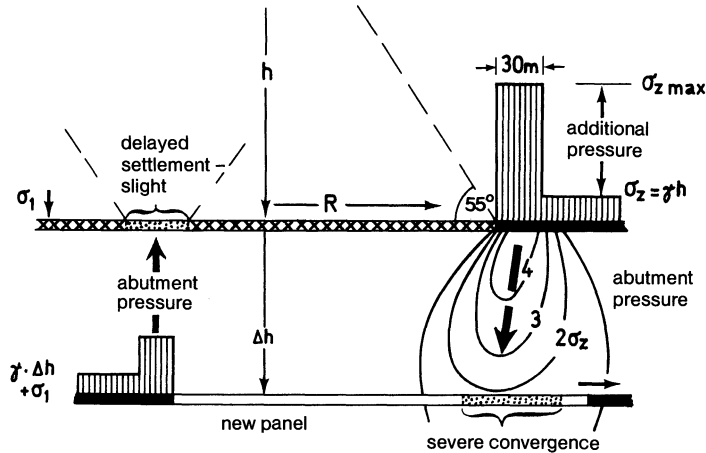


Fig. 106. Perpendicular pressure from the ribsides of an old panel on to a new working. Increased convergence in the area of abutment pressure, and delayed subsidence in the upper, older panel

The lines of equal abutment pressure extend like lines of magnetic force far into the strata overlying and underlying the unworked seam and also encroach on the strata underlying the stowed goaf (Fig. 106). A new working being carried through the abutment-pressure zone is intersected once or perhaps twice by pressure lines extending to this depth. If the value of the pressure lines intersecting the new working is known, it can be converted approximately, with the aid of the graph for the stowing material and of Eq. (20) as applied to intermediate strata regarded as a beam, into the required localized roof settlement. With brittle strata as the immediate and main roof, the localized settlement can be estimated directly from the relationship between abutment pressure and goaf convergence.

A sandstone roof that has been broken up and rendered brittle and slack by tensile stress from repeated bending can in time resemble a weak bed of shale in deformation behaviour. By Fig. 13 this can mean a delayed settlement of 4% – 6% in the centre of the panel, and of even 20% at the perimeter of the goaf. For example, a dome-shaped gap of width b (Fig. 107) remaining open at the

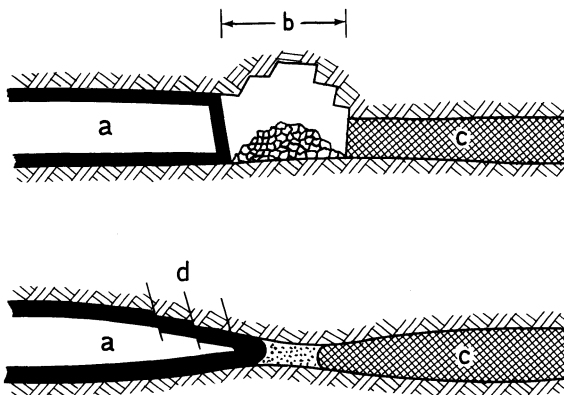


Fig. 107. Perimeter zone of a panel before (*top*) and after (*bottom*) undermining or over-mining. *a* Seam; *b* perimeter zone, open excavation, caved waste still uncompacted; *c* old panel with stowing material; *d* atectonic shear detachment. (After Lautsch, 1974)

edge of a panel can start to move and collapse when a deeper seam is mined. Abutment pressure created by the lower working will also compact this caved zone, and the roof will undergo delayed settlement. This rheological *delayed settlement* in upper panels offsets the reduction in convergence in lower workings, if they lie within the pressure shadow of roof strata that are still capable of bearing a load, and it exceeds that reduction in convergence when several seams are mined. Consequently, subsidence values calculated for the centre of a trough will be too small if the delayed settlement of roof beds that have become slack through bending is not taken into account. Thus, actual subsidence of 303 cm in a shaft head, observed after subcritical extraction of six coal seams of a combined thickness of 9 m, was significantly greater than the calculated value of 220 cm (Baron, 1951; with stowing, subsidence factor $a = 0.47$).

A further reason for delayed convergence in a repeatedly undermined goaf is the smaller amount of subsidence at the edges of a panel than at its centre, resulting from the redistribution of vertical strata pressure in the trough shape formed by the old panel as it subsides (Fig. 108). At the centre of the panel, which can be likened to a trough centre that has been bridged over, a part of the support for the roof is withdrawn, and so it presses down even more strongly on the perimeter zone – the rim of the trough – and further compresses the stowing material there. The opposite is the case with points in the strata or the ground above the ribside and over the unmined seam. Here the amount of subsidence arrived at by calculation is often too great, because in the perimeter zone, instead of subsidence in steps, the full subsidence value aM is for simplicity usually introduced into Eq. (54). At the ribside this error can amount, depending on the seam, to $50\% - 24\% = 26\%$ aM , as shown in Fig. 102. In addition, the change in the limit angle, which becomes steeper in a fragmented rock mass but is probably not taken into account in the calculations, contributes to the divergence between calculated and actual point movements. A multi-seam operation on several levels is one occasion when it is particularly important not to overlook neighbouring influences on the impact of mining, because of their cumulative effect.

From heave and pressure phenomena measured in floor strata, the perpendicular *range of influence* of the load-relieving protective-shield effect of old panels, and of the abutment-pressure effect of their ribsides, can be taken as being 250 m at least. Further, it can be calculated from Eq. (56) (Boussinesq) that, for a circular area load or load relief and an elastic medium, the relief and abutment-pressure stresses approach the cover-load pressure asymptotically with

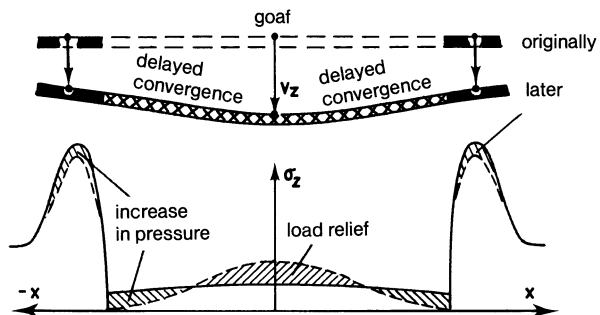


Fig. 108. Delayed convergence in an old panel affected by underlying workings, showing trough-shaped subsidence and new distribution of vertical strata pressure

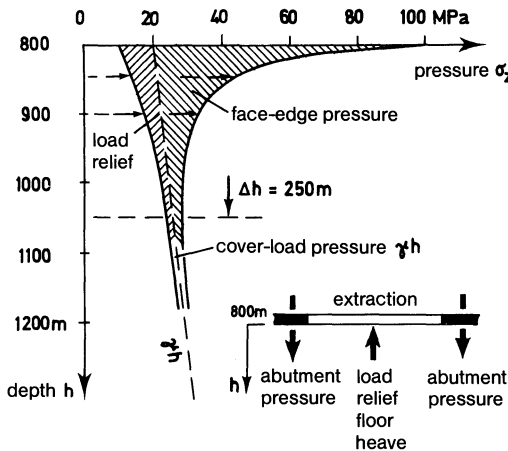


Fig. 109. Decrease in load relief and in face-edge pressure under a panel at 800 m depth

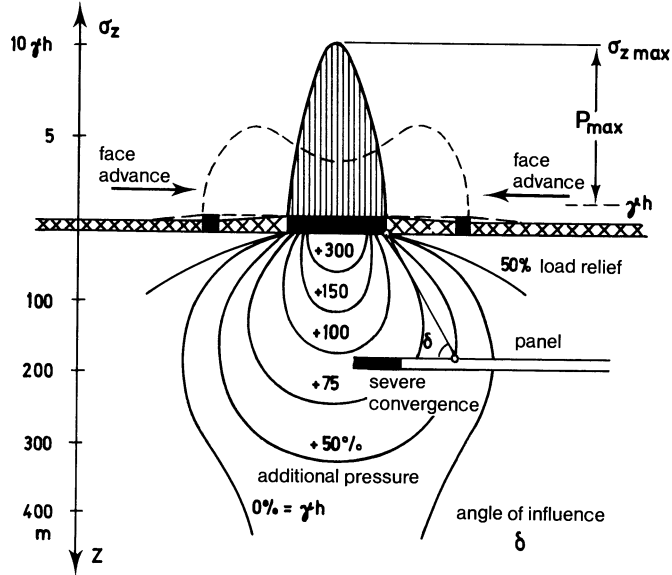
increasing depth (Kormann, 1957). The presumed shape of the curve, starting from a depth of 800 m, is shown in Fig. 109. The values taken for the mining horizon are a pressure on the stowed goaf of 50% $\gamma h = 10$ MPa and an abutment pressure peak of 100 MPa, as against a cover-load pressure of 20 MPa. On this basis, at 50 m below the panel the abutment pressure still amounts to 20 MPa and the load relief to 7 MPa. At 100 m difference in depth, the pressure difference has narrowed to 10 MPa of abutment pressure and 4 MPa of load relief. Convergence in a new subcritical extraction 50 m below an existing goaf at 800 m depth will therefore be (from Fig. 11) around 3% less in the pressure shadow (pressure on goaf 50% of 13 MPa instead of 50% of 20 MPa) and under the old ribside approximately 4% greater (pressure on goaf 50% of 40 MPa instead of 50% of 20 MPa) than on a first penetration of the rock mass.

6.4 Coal-pillar Pressure in a Panel

If an “island” of coal is left between the panels in a seam, this residual portion of the deposit supports the undermined strata as a pillar standing on a small area and transfers the resulting high load to the strata beneath. The perpendicular compressive force in the strata above and below the residual pillar increases convergence in any working in them. The special feature of a residual coal pillar is that the abutment pressure zones on each side of it overlap and can even form a single pressure maximum in the centre of the pillar, if it is narrow enough (Fig. 110; see also Fig. 21). The coal-pillar effect, feared because of the danger of rock falls, thus constitutes a special case of the overlapping of abutment pressure as two workings approach each other. Residual-pillar pressure exceeds abutment pressure both in magnitude and in range of influence.

The magnitude of the strata load bearing on the coal pillar is governed by the area and the load uptake of the surrounding panel. Any load from the undermined strata not taken up by the perimeter of the adjoining stowed or caved area is passed as an additional load to the strip or block of remaining coal pillar. The

Fig. 110. Distribution of vertical pressure in a residual strip-pillar of coal and in the underlying strata. Severe convergence in a lower panel



smaller this coal pillar is, the greater will be the pressure peak at its centre. The pillar's absorption of pressure is limited by the strength of the coal, which depends to a great extent on the seam's transverse-expansion resistance and friction resistance. At the centre of the pillar the coal exhibits a load-bearing capacity, assuming that it is constrained on all sides, of something over 200 MPa as compared with a single-axis compressive strength of 5 to 40 MN/m² at the edge of the pillar (Bräuner, 1964). This limit loading of 200 MPa can be expected with an assumed parabolic load distribution over a 40–50 m wide strip of residual coal pillar at a depth of 800 m, if it is enclosed on both sides by extracted critical areas. This conclusion can be drawn from the average loading [calculated from Eq. (106)] of $94 + 20 = 114$ MPa on a 30 m wide seam edge. A single pressure maximum of 200 MPa can be assumed at 800 m depth over the centre of a coal pillar block of 60 m diameter, given a subcritical extraction of 200 m diameter (with strata load = limit-loading of the coal pillar). With the coal pillar as a strip of width $2a$, the peak of additional pressure at the centre $p_{\max}$ decreases, in an elastic body, with the square of the depth z (Kögler and Scheidig, 1944) as follows:

$$\sigma_z = \frac{2p_{\max}}{\pi a} \left[z - \frac{1}{2}(z^2 - a^2) \arctan \frac{a}{z} \right]. \quad (107)$$

For a circular coal pillar, the equation which applies is

$$\sigma_z = p_{\max} \left[1 - \frac{2z^2}{a^2} \left(1 - \frac{z}{\sqrt{a^2 + z^2}} \right) \right], \quad (108)$$

if load relief in the direction of the lateral expansion zone of the goaf is ignored. The peak additional pressure $\sigma_{z\max}$ of $10\gamma h = 200$ MPa in the middle of a strip

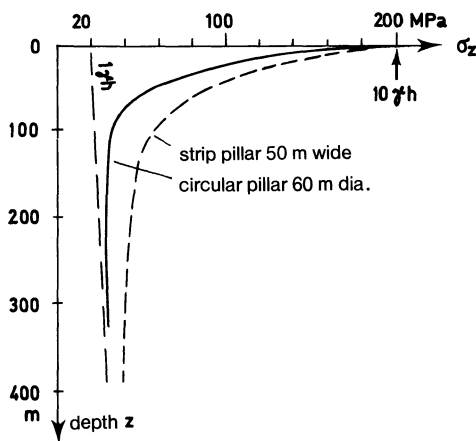


Fig. 111. Decrease in maximum strata pressure under the centre of a coal pillar at 800 m depth. Limit of load-bearing capacity at the centre of the pillar: 200 MPa = 20,000 N/cm². (After Bräuner, 1964)

pillar at $h = 800$ m depth drops, by a point 250 m down in the floor strata, to a value of only double the cover-load pressure, i. e. to $2\gamma h = 40$ MPa, as indicated in Fig. 111.

Although the pressure under a residual coal pillar is significantly greater than under a panel ribside — i. e. 90 as against 40 MPa at a point roughly 50 m under a pillar or ribside respectively — convergence in a lower panel caused by coal-pillar pressure is little greater than that resulting from a ribside effect, since convergence in a stowed goaf increases only slightly at pressures over 20 MPa. The localized convergence steps of 4%–5% aM indicated for ribside pressure can therefore be adopted also for the influence of coal pillars. The zone of increased or delayed convergence will nevertheless generally be broader under a coal pillar than under a panel ribside. For simplicity, however, it will in this discussion be limited to the 50% additional-pressure line (Fig. 110).

6.5 The Influence of Mining Procedures on the Break-up of the Rock Fabric

The stress field above and below a mine working alters very rapidly during the face advance, both in location and in intensity (cf. Fig. 56, phases 1 and 2). It is only after suspension of mining operations that a stationary stress field forms, which reaches a temporary equilibrium (phase 3) only on the cessation of strata and ground movement 1 to 5 years after the face has stopped. Then, as fracturing and deformation slowly progress, pressure and load relief further cancel each other out. In theory, this rheological evening out of stresses into the final stratigraphic stress field lasts an infinitely long time. In practice, however, it ceases when the mine excavation, including edge and corner zones, is completely reclosed and the fractured rock mass is sufficiently compacted.

During the face advance, *pressure on the rib* where the face was opened out, or on a coal pillar lying between two panels, steadily increases, because the load to be borne by the stowing material and the stationary seam edge grows in pro-

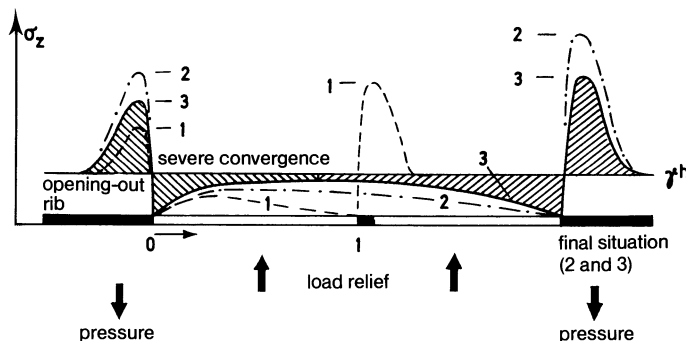


Fig. 112. Strata pressure developing over time above a panel being worked from left to right

portion to the extent of undermining, until the critical area is reached. The increase in pressure over time thus depends chiefly on the progress of the face. In two panels advancing at the same pace and at the same depth, abutment pressure on the ribside and relaxation of the load on the floor will be greater where the roof is more bend-resistant, the seam harder, the stowing material more yielding, and the thickness of seam worked greater. By contrast, after the suspension of working, the final phase of the stationary stress field will be reached first in the panel with the less bend-resistant roof, the weaker seam, the denser stowing material, and the smaller seam thickness worked. Over the opening-out rib or a coal pillar, pressure during this 1–5 years after the suspension of working diminishes somewhat as the share of the load borne by the stowed goaf on the bending of the roof beds and the compaction of the stowing material increases (Fig. 112). Load relief in the groups of strata above and below the goaf diminishes to a similar extent.

Over the advancing face, pressure when the panel is first opened out increases up to the point where a size of panel, depending on deformation of the roof and of the stowing material, is reached at which the take-up of load by the stowed goaf within a given period equals the increase in load represented by the section of strata newly undermined in the same period. The additional load on the advancing face then remains more or less constant. After the suspension of working, the pressure over this face falls off somewhat if the roof beds at the edge of the panel sag markedly and become increasingly carried by the stowing material or the caved waste.

Thus the *mining procedures* which have an influence on the stress field in the strata and the fracturing of the roof are the rate of daily face advance, the mining rhythm, and the nature of working (caving or stowing). With a rapid face advance of, for example, 5 m/d in coal-mining, the application and removal of load by the strata over the face edge also proceed rapidly. This favours the tendency of the rock to fracture, as also do rapid and delayed sagging of roof beds over a stowed or caved goaf at week-ends, after a continuous, five-day coal-getting operation. Caving encourages the fissuring and break-up of the strata, as the main roof subsides and bends almost twice as much as with solid stowing. Other mining procedures having a critical influence on the fabric of the rock

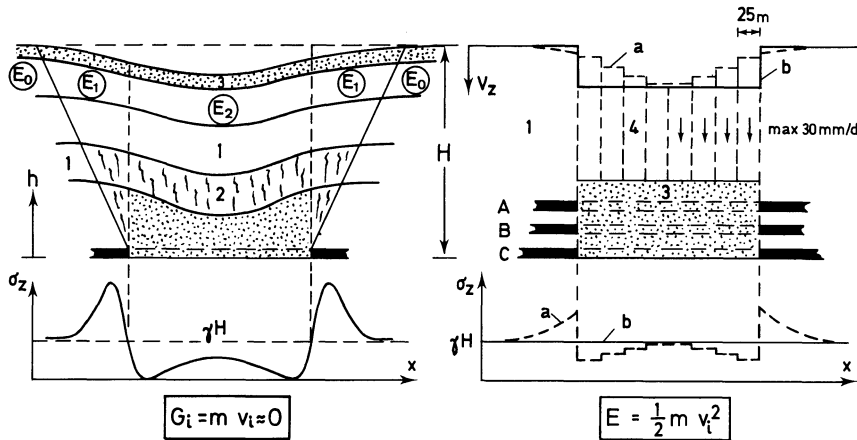


Fig. 113. Extreme states of the strata affected by underlying workings. *Left:* trough-shaped bending of elastic roof beds. *Right:* roof strata undermined several times and subsiding as blocks down vertical joints. *Below:* vertical strata pressure on seam floor. Modulus of elasticity $E_2 < E_1 < E_0$

mass are the order in which the individual seams are mined, and the spatial concentration of workings (Table 9).

The influence that the *concentration of workings* has can be appreciated from Fig. 113 (left). On extraction of the first seam, deformation of the upper strata (1) is still within the field of action of elasticity (flexure model). These strata sag without fracturing and settle on the beds of the main roof (2), which rest on the caved zone (3) and have already experienced a weakening of their rigidity through the opening-up of joints. Sagging of the beds over the caved zone follows slowly as a constrained movement of strata particles, which do not become detached from their strata bond. Compaction of the caved zone likewise proceeds slowly, until a bending profile is reached such that the sum of local supporting forces on the underside of the curved main roof (the dividing line between zones 2 and 3) is equal to the sum of the bending forces in that area, which are distributed along the x -axis according to the known σ_z curve. Because of the very slow rate of subsidence v_i , the amount of movement (impulse)

$$G_i = m v_i \quad (109)$$

of each separate particle m (mass in kg) in the overlying strata is almost nil.

If now the operations planned for a particular period and for several seams are concentrated within a block of strata which is geologically delineated by downfolds, upfolds, and faults, as is frequently the case in European coal-mining districts today, then the working front will pass repeatedly under the same rock body, advancing from Monday to Friday at 5 m/d and standing stationary each week-end after progressing 25 m. During this two days' standstill the main roof will break behind the extraction front, as illustrated in Fig. 114, left, in a clay-bed model for a stationary face edge, and diagrammatically in Fig. 115 for an active, caving face working. Not only do the beds separate from each other at their bedding planes but the individual beds are also broken up into blocks by the joints. These perpendicular main-roof breaks, which with their 25 m interval cor-

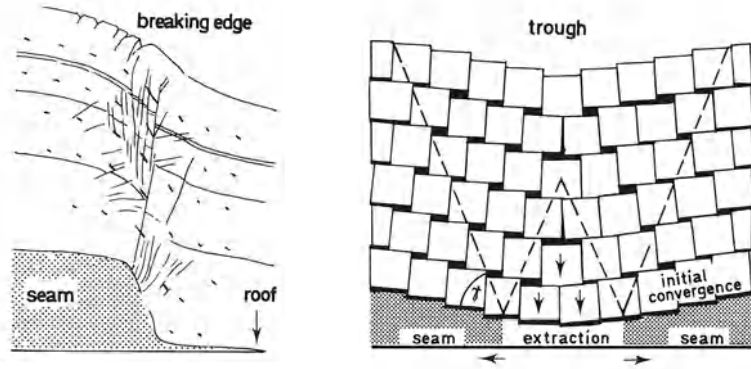


Fig. 114. Experiments on a clay model (*left*, after Cloos, 1930) and on a cubes model (*right*, after Trollope and Brown, 1965) to explain the “steep” sides of a trough above fractured roof strata

respond to the mining rhythm, can in an extreme case, if the rock body is undermined a number of times at short intervals by caving procedures, break through to the ground level, as shown in Fig. 114, left. The effect of this is that, instead of the flexure model in Fig. 113, left, a *punch-effect model* (Fig. 113, right) develops. Each undermined block of strata (4) of 25 m width drops almost with the kinetic energy of free fall, and held back only by friction along the vertical fracture surfaces, according to

$$E_{\text{kinet.}} = \frac{1}{2} m v_i^2 \quad (110)$$

on to the caved zone and compacts it like a punching press, with greater force than under the constrained movement of the flexure model. The ground surface becomes stepped from block to block as in curve a. Once the friction has been completely neutralized by the extent of the breaking-up process, the uniform cover-load pressure γH [see Eq. (4)] becomes re-established at the lower working horizon. Each block presses with its weight load on to the caved area (3). The ground surface over the panel has then subsided by an amount equal to the sum of the convergence in workings A, B, C, etc. With the elastic flexure model, the centre of the undermined roof strata bears with its full weight force γH on the caved area only when a working width of 1λ , as in Fig. 59, has been reached.

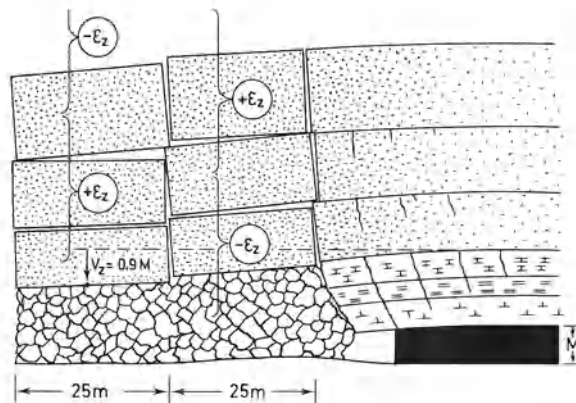


Fig. 115. Over roof beds broken into small fragments, blocks of overlying strata subside in sudden movements as a longitudinal wave (Lautsch), with alternating expansion and contraction zones

In place of a flat trough as in the flexure model, a broad trench with a level floor is what forms in the punch model. The ground over the unmined seam beyond the vertical trough wall does not subside unless there is friction up the rupture plane which develops over the face edge, in which case it will subside to some extent (curve a). The amount of subsidence in this zone beyond and above the face edge thus depends on the amount of loosening that occurs in the rock fabric. In the flexure model it is 50% with a critical area of extraction and under 50% with a subcritical area (Fig. 179); in the punch model with friction (curve a), subsidence is less than 50%, and without friction (curve b) 0%, of the maximum subsidence at the centre of the trough in each case. The more the rock fabric is loosened, the less the zone over the unmined seam and above the face edge subsides (less than 50% v_{zmax}), but the more the subsidence over the panel will be – a phenomenon which accords well with subsidence measured over multi-seam caving panels (Chap. 8).

If the cracks in the overlying strata do not develop in such straight lines as shown in Figs. 113, right, and 115, but are displaced laterally from bed to bed, or from seam to seam, then a model of a clastic rock mass, composed of cubes as in Fig. 114, right, arises. In structure and kinematics this occupies an intermediate position between the flexure model and the punch model: the free-fall movement of the cubic blocks of strata is restrained both by friction at the rupture surfaces and by the form fit of the blocks. The trough line incorporating steps at each horizon begins over the unworked seam. A weak layer of overburden would even out these subsidence steps. Only the lowest cubes over the panel, within the limit-angle cone – the zone of full subsidence mentioned in Sect. 3.31 – can move down freely by the full amount aM . The form taken by the network of joints determines the size of the limit angle. If the panel is wide enough ($2R$) for the inside arms of the limit angle to intersect at the surface of the model, then the top block of strata in the centre of the trough can also go down by the full amount aM (critical area, full subsidence).

If the fissuring and fragmentation of the blocks of strata proceed to a sufficient degree, the model arrived at is a stochastic one consisting of *sand*. And if the roof bed in the model represented in Fig. 116 is lowered progressively in equal stages from 1 to 5 starting from the left-hand side, then the following subsidence profiles appear at the two strata horizons z_1 and z_3 and at the ground level z_5 :

In sand compacted by vibration, which corresponds in practice to a rock mass broken up small by repeated mining, the maximum surface subsidence of 2 mm at the left-hand edge of the model is only half as great as settlement of the roof bed amounting to 4 mm. The sand thus becomes loosened from horizon to horizon upwards, while the roof strata expand over the panel. The subsidence wave over the progressively advancing working front becomes evened out according to the St. Venant principle (near and distant action). Above the face edge the surface has subsided by only 25% of the maximum subsidence. In the rock mass the silo-like subsidence of the roof strata has not extended to the zone ahead of the face (see funnel trough, Chap. 7).

If the experiment is repeated with sand that has already been loosened, the sand mass contracts and becomes denser through movement within itself, as happens when grains of sand change their position from a cubic to a rhombic pattern

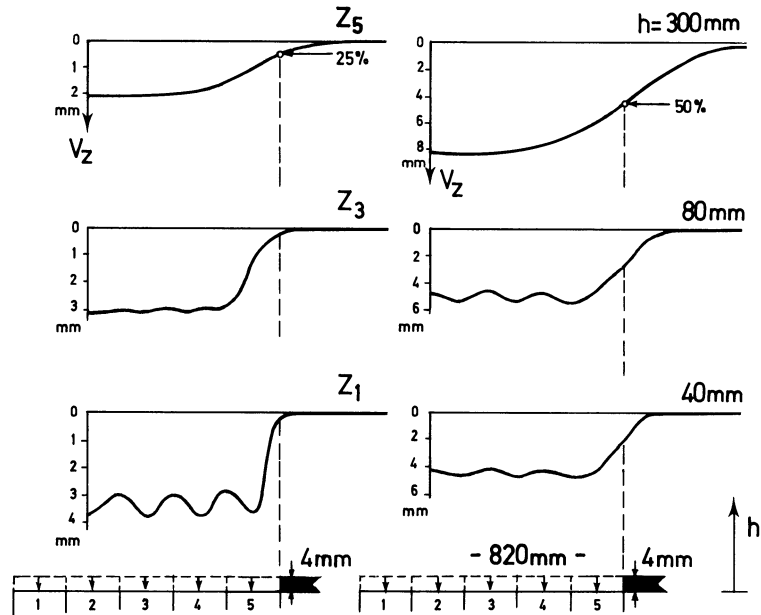


Fig. 116. Subsidence of horizons z_1 to z_5 over a mine excavation enlarged progressively from 1 to 5, in a model made with compacted sand (*left*) and with loose sand (*right*). (After Knothe, 1970)

(Fig. 279). Subsidence of the surface amounting to 8 mm is consequently greater than settlement of the roof bed amounting to 4 mm. The trough becomes deeper and wider towards the surface and includes the zone ahead of the face. In the lower horizons a supporting dome forms over the individual sections 1 to 5 of the mine working.

Both experiments make it clear in relation to the cube model why, in the loosening-up phase of a fractured rock mass, subsidence decreases as it goes higher and is comparatively slight over the edge of a working (the “silo effect”), and why, on repeated working through a rock body, greater subsidence is found in practice than had been predicted (compaction through the rearrangement of particles). As fracturing and loosening-up of the rock fabric progresses, the freedom of movement of the blocks of strata increases. They become released from the form fit and strata bonding and can slide downwards, causing the horizontal fissures between the beds to close up again (compaction), as can be seen from the blocks in the inner limit-angle cone of the cube model in Fig. 114, right. Vertical expansion then decreases, and subsidence increases.

In strata that have been fractured and loosened up by intensive mining, the roof of a working will not sink uniformly as a single group of beds. The loosened blocks of strata will move more in sudden jerks, with the result that, in a vertical column of rock, sections that have expanded vertically will be followed by others that have contracted vertically (Fig. 115). These phases of vertical deformation $+\epsilon_z$, $-\epsilon_z$ are transmitted through the overlying strata as a longitudinal wave travelling from the bottom upwards (Lautsch, 1974). In a section which has expanded vertically, vertical loosening or dissociation increases, while in one that

has contracted vertically, the blocks of strata have already sufficiently broken out of their strata bonding as to rest on the block at the bottom. Horizontal cracks in the vertically contracted section close up again, and under the weight force of the overlying blocks of strata the perpendicular load relief of the rock, which had taken the form of vertical expansion during the loosening phase, recedes again (vertical contraction).

Similar phenomena appear in less strongly stressed strata which have been fractured but not yet loosened up by mining. If fractured and therefore easily bent beds alternate with unbroken, stiff beds in such strata, then the described sequence of expansion and contraction in roof strata is seen, as in the cube model, with a stiff bed temporarily bridging a subsided weak bed, and resting its whole weight on the lower bed only as the span widens.

Summing up on the subject of the fracturing and loosening-up of strata in relation of the field of stress and movement, it can be stated that a rapidly advancing caving operation concentrated on the seams of a narrow block of strata will lead to a clastic strata structure and induce abnormal pressure stresses and convergence both in the unmined portions of the deposit and over the face edges (Table 9).

Table 9. The impact of modern coal-mining practice on the fabric of strata

Mining causes	Rock-mechanic effects: Alteration in fabric of strata	Effects as seen at the surface: Alteration in trough shape
1) Concentrating the extraction of several seams within a tectonically delimited block of strata	1) Fissuring and breaking up of overlying strata (increased elasticity in lower roof beds, decreased bending strength in strata)	1) Acute-angled trough profiles (pre-defined movement paths in the fissured and weakened centre of the roof-rock in a mining block, leading to twice as much tilt and compression)
2) Caving instead of stowing (twice as much convergence in the working area); preferential extraction of thick seams	2) Ribside effect at the block margin (joints and cracks penetrating some distance above ribsides serve as movement paths)	2) Asymmetrical trough profiles (trough bottom displaced towards focal point of block extraction in cross-cut profile, and towards stationary opening-out face in strike profile)
3) High rate of face advance (5 m/d instead of 0.75 m/d) and continuous extraction from Monday to Friday, with face then stationary for 2 days	3) Perimeter effect at block margin (mobilization of latent subsidence potential at the perimeter of old panels on undermining or overmining)	3) High rate of subsidence and deformation after passage of face – e.g., 20 mm subsidence and 0.1 m/mm deformation per day
4) Rapid sequence of workings	4) Intensified conversion of potential energy into kinetic energy through rapid undermining of a block of strata	4) Special or secondary troughs as result of advance stressing of strata or mobilizing the subsidence potential of old workings

6.6 The Swelling of Clay-bearing Beds

Strata that have been worked through in mining become loosened up – “de-consolidated” – by fracturing and expand elastically when once relaxed in this way. They can also increase in volume by *swelling*, if the beds contain clay and become exposed to mine water or to humidity in the mine ventilation. This expansion in the volume of beds uncovered in mining, which is quite independent of stresses, can constrict the mine excavation so severely that strata movements induced by mining and tending towards the remaining excavation are markedly reduced.

It has long been known that the lime-mica clay-mineral montmorillonite (Ca illite) has the capacity of absorbing molecular water between its platelets and of swelling as a result. Absorption of a layer of water is facilitated in this mineral by the fact that the weakly negative charge of the silicate sheets causes them to repel each other to some extent and thus open the way for the entry of cationic water bearing a weakly positive charge. Swelling montmorillonites are, however, rarely found in the Ruhr Carboniferous. On the basis of comprehensive investigations in associated strata of the Ruhr Carboniferous (Klosse, 1967), however, a connection was established between swelling capacity, content of commonly occurring muscovite-illite clay minerals in samples, and the ion potential of an introduced salt solution. If the K^+ ions in this potassium illite are leached out and replaced by smaller Ca^{2+} or Mg^{2+} ions, the mineral can be converted into an “expandable illite” related to montmorillonite, which will accept water within the crystal lattice. Conversely, a calcium illite treated with a potassium salt solution reverts to a normal, non-swelling illite. The swelling behaviour of associated strata depends therefore both on the proportion of “expandable illite” present and on the composition of the mine water, particularly its Ca and Mg content. Potassium salt solutions with a low ion potential reduce swelling in argillaceous shales.

On the introduction of water, the shales encountered swell at right angles to the stratification by 1%–3%, and sometimes even by as much as 10% or even 20%, of their seam thickness. That implies an increase in thickness of 5 to 15 cm (sometimes more) in a 5 m thick bed containing illite. By comparison, swelling parallel to the stratification is only half as great. In calculating roof settlement therefore, the swelling of clay-bearing roof and floor strata must be taken into account by reducing the thickness of seam worked M by the amount of swelling perpendicular to the stratification – e.g. by 5%, 10%, or as appropriate. The subsidence factor a should similarly be reduced if swelling or diminished compressive strength is to be expected in wet stowing material. As is known, rock loses strength on taking up water.

Finally, mention should be made of strata movements associated with the submerging of halted underground workings. If air is forced out of the workings by water, the column of water exerts a pressure on the floor of the open panels and the mine roadway floors, the magnitude of which ($1H$, in N/cm^2) is determined by the height H (in m) of the water column. This additional load of, for example, $800 N/cm^2 = 8 MPa$ at a depth of 800 m produces, at the lowest mining level, a lowering of the floor strata by several centimetres in the flooded

area. Since the water pressure is, of course, equal in all directions, it will also act on the roof surfaces of the workings and mine roadways, with the result that the mine excavation is very slightly enlarged, uncompacted stowing material is relieved, and roof beds are slightly lifted. Strata lying between two workings will be compressed by the differential pressure $1 \Delta H$ (in N/cm^2). In the upper horizons, a heave measurable in centimetres occurs on the access of water if there is interstratification of clay beds and loose sand, because grains of sand surrounded by water experience some buoyancy, which relieves an underlying clay bed in the supporting grain structure, thereby enabling it to swell.

Part II The Study of Ground Movement

7. The Components of Ground Movement

7.1 Historical Development

Since the middle of last century, when the coal industry went over from drift mining to deep-level mining, attention has been given in Europe to the ground movements which occur on the extraction of coal. As early as 1860, the railway companies in the Ruhr were taking ground-level measurements, in order to bring evidence of the lowering of their tracks in the event of disputes about mining damage. On the basis of these and similar levellings, there appeared from 1870 onwards, in Germany and in other European countries, a number of scientific publications on ground movements in mining and on mathematical formulae for the prediction of ground subsidence.

In the literature, the first formula to appear for subsidence over the centre of a caved panel was:

$$v_z = M \cos \alpha \quad (111)$$

where M signified seam thickness and α the angle of dip of the strata (Dumont, 1871). On this basis, subsidence in flat-lying measures ($\alpha \approx 0$) would be equal to seam thickness. The formula

$$v_z = aMz \quad (112)$$

used around 1895 for determining proportional mining damage, caused by neighbouring mines, already took account, through the factors a and z , of the method of extraction (caving, stowing) and of the age of the working. Later, the quantity of coal extracted was related to the volume of the surface depression, and the depth of the workings was introduced, as well as a coefficient for the increase in volume of strata loosened up by subsidence (Goldreich, 1913).

Research into mining subsidence received a strong impetus in 1899 with the founding in the Ruhr of the “Emschergerossenschaft”, an association devoted to draining the mining district round the River Emscher, a tributary of the Rhine. To ensure the safe channelling of waterways, regular observations of any lowering of the ground surface and calculations of foreseen subsidence were instituted. One result of these observations was that the size of the extraction area, ignored in Eq. (112), was evaluated in relation to a reduced thickness of seam worked M_{red} and finally taken into account through the relation between extraction area and area of influence. A method that could be used in today's terms for pre-calculating subsidence at the surface was not, however, enunciated until it had been discovered that the amount of subsidence depends not only on the size of the

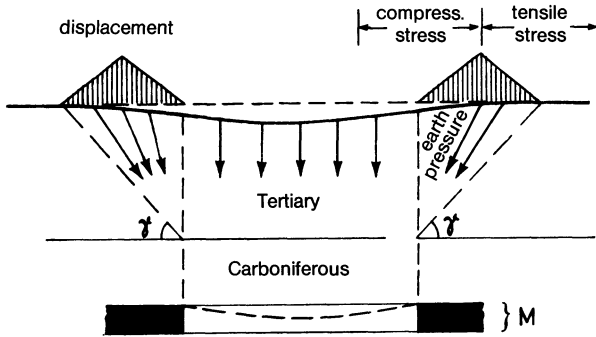


Fig. 117. Movement of strata overlying an extraction panel. (After Goldreich, 1913)

extraction area but also on the relative position of surface points to the workings (Keinhorst, 1925; integration-grid method). The horizontal displacement of surface points first came to notice around 1900, after changes in the lie of buildings, and tensile and compressive stresses in tramways (streetcar tracks), had been detected.

The ground-movement process over a near-surface working had at first been seen as consisting in the roof strata sliding down into the mine excavation as a single block, between rupture planes leading up through the rock mass from the edge of the workings to surface cracks visible at the subsidence trough margins (*angle-of-break theory*; Gonot, 1839). With increasing working depth it was then recognized that ground movements extend beyond these surface cracks (*angle-of-limit theory*) and that the rock body over a large working panel becomes loosened up, or “de-consolidated” (increase in volume). To explain the horizontal displacement of points in the rock mass, these two theories were combined (Fig. 117). Over the working itself all points were believed to drop vertically, while in the neighbouring trough-margin area points were thought to move in the direction of the working face. Compressive stresses, as with active earth pressure, thus arose over the working face, while tensile stresses arose at the trough margin.

A collapse of the trough core down steep rupture surfaces only occurs, however, over near-surface workings (see Fig. 152). In general, no continuous planes of separation and movement develop in roof strata of firm rock exceeding 200 m in thickness. In the next years therefore the view developed that the ground-movement process could be compared to the bending of a beam or slab (Eckardt, 1913) or to the forming of a geosyncline (K. Lehmann, 1919). As is known, when a rock bed sags like a beam or slab, the upper side is compressed at the centre and extended at the edges (Fig. 118). Similar phenomena are found in a geosyncline. The soft layers of earth moving downwards and sideways towards the syncline axis are also compressed at the centre of the trough and extended at its margins (*trough theory*). The qualitative agreement between the propositions of this model and practical experience of ground movement in mining was confirmed by the results of observations systematically assembled in the various mining districts. The findings on the critical area of extraction, the trough profile, and linear change, all derived from these observations, close the first phase in the development of mining subsidence engineering (Fig. 119).

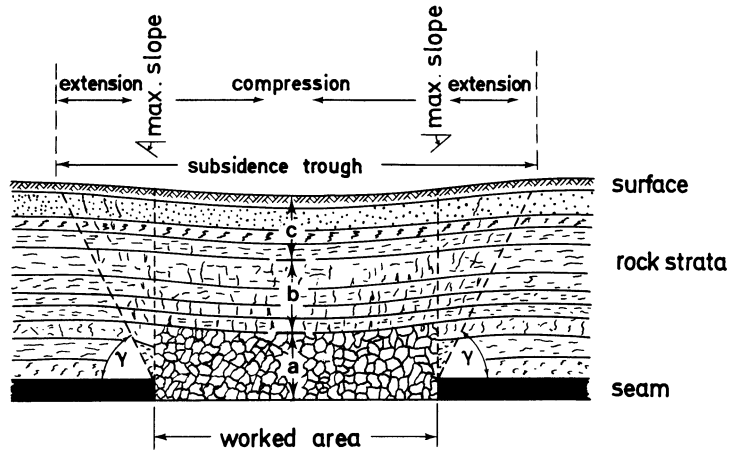


Fig. 118. Deformation of strata above an extracted area. *a* Zone of shattered roof beds over mine excavation; *b* fissured intermediate zone; *c* surface zone; γ limit angle from edge of working to trough margin

The state of knowledge in this field was advanced in the 1930s and 1940s and extended to displacement calculations, steep-lying conditions, and the dynamics of ground movement. After the 1939–45 war, new impetus came from the stochastic process (Litwiniszyn, 1957), the programmed computing of profile curves (Niederhofer, 1960), and the application of the finite-elements method to the study of ground movement in mining subsidence.

Most recently, attempts have been made to reconcile calculation methods with actual, measured subsidence and displacement by means of additional functions and locally valid parameters, with due regard being paid to the time factor – developments which arose from the recognition that, when strata have been repeatedly worked through, the shape of the subsidence trough varies from the normal profile (Lautsch, 1974).

Information derived from coal-mining has been successfully applied to the draining of lignite and to iron-ore and salt-mining. In petroleum and natural-gas fields also, trough-shaped depressions can form at the surface as a result of the

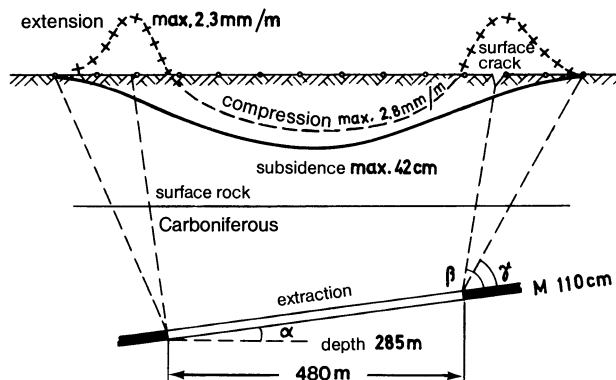


Fig. 119. Section through a subsidence trough. (After K. Lehmann, 1919)

drop in pressure in a yielding reservoir rock (see Fig. 26). The shape of these depressions can also now be pre-calculated (Geertsma, 1973).

7.2 Ground Movement with Flat-lying Measures

7.2.1 Shape of the Trough

Movement at the surface over a working at a depth of 730 m is depicted in Fig. 120. The surface over a panel in the Karl seam has subsided by over 60 cm in the shape of a trough, and the marked observation points (represented by arrows) have been displaced horizontally by up to 30 cm towards the centre of the trough. The lines of equal subsidence enclose the extraction area more or less concentrically, and the point of maximum subsidence lies roughly over the centre of the panel. Movements of points as much as 200 m outside the surface projection of the extraction area were recorded. The bulging-out of subsidence contours in the upper left-hand corner of Fig. 120 indicate delayed subsidence in an old working in the Blücher seam.

Surface points lying within the area of influence of a mine working are thus lowered or displaced horizontally to a greater or lesser extent depending on their position in relation to the extraction area. For example, the point P in Fig. 121 has moved to the position P', following a curved path, during the period of mining influence. The straight line joining the point of departure P with the final position P' gives the displacement vector $\vec{V}$ on a three-dimensional basis x, y, z ,

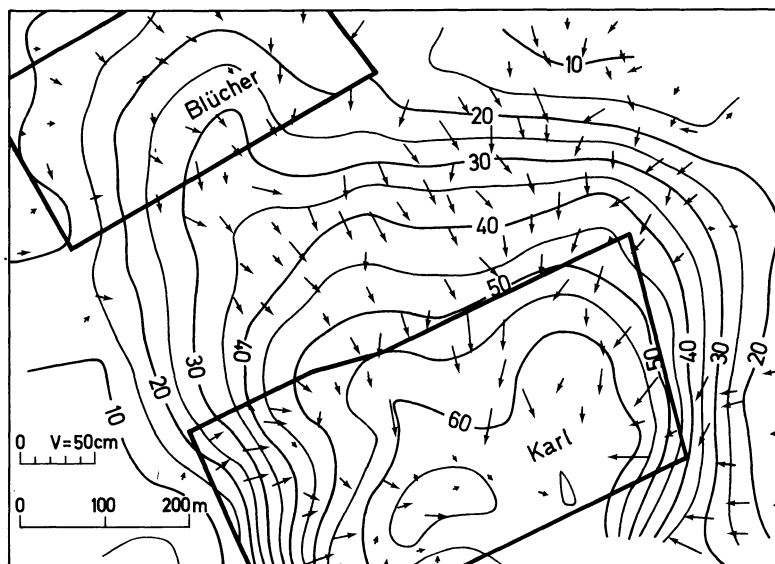
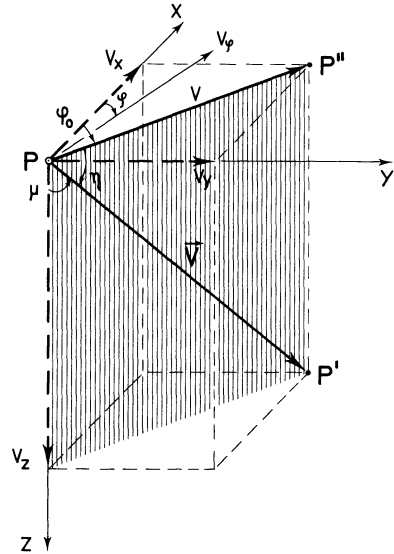


Fig. 120. Diagram from aerial photos showing subsidence and displacement from June 1976 to May 1977. (After Pfläging, 1978). Flight altitude 375 m, scale 1 : 2500. Working in Karl seam ($M = 150$ cm, caving, depth 730 m)

Fig. 121. Change in position of point P to P' (final situation) as a result of mining; and components of the movement



at the angle of inclination η and the angle of direction ϕ_0 . This spatial vector can be projected on the horizontal plane xy as the displacement magnitude v and analysed into the components v_z (subsidence), v_x and v_y .

Only the vertical component of point movement, namely subsidence v_z , can be measured directly in a simple manner by means of levelling. Horizontal displacement v must be determined trigonometrically with a theodolite or an electronic tachymeter, using angles and distances measured from a known point. For this purpose a traverse is made from a fixed point outside the trough area to the observation point. In the right-angled system of coordinates, the measured change in position of the point can be plotted from reading to reading and supplemented by the relevant subsidence figure (Fig. 122). With a row of measuring points going in a straight line through the centre of the trough to the trough margin ($v = 0$), displacement v can be derived indirectly from the remeasured dis-

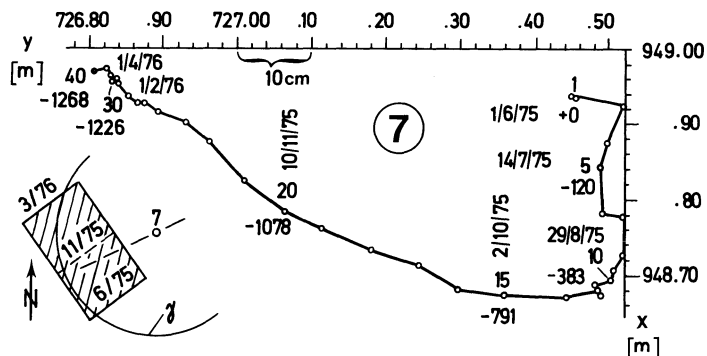


Fig. 122. Horizontal movement (change of position) of ground point 7 between June 1975 and April 1976, shown in a system of coordinates. (After Steudel, 1976). Nos. 1 to 40: observation measurements. -791 subsidence of point in mm

tances s_i between points, and thus the linear change in those distances Δs_i , because the direction φ_0 of the displacement v agrees in this case with the direction of the line of measurement. In lines of measurement which do not run to the centre of the trough, only the components v_x and v_y of horizontal point movement in the direction of the line of measurement can be derived from linear changes obtained without measuring angles.

The example of a line of measurements running parallel to an approaching face will serve to make clear the *movements* and *deformations* which take place at the surface in the y -axis (Fig. 123). In three face workings side by side, the extraction front advances to the measurement line and passes under it. At positions I, II, and III of the front, the differences in height of points 46 to 84 are measured with a levelling instrument, and the changes Δs in the distance intervals between points with a steel tape. The measuring points are permanently marked with wooden pegs, iron rods, or bricks at intervals of $s = 30$ m, 50 m or 100 m. From extraction position I to extraction position II and the final position III, the diagram shows increasing values in subsidence v_z , in displacement v_y derived from measured changes in distance intervals, and in interval changes $\Delta s/s$ expressed per length of 1 m, — i. e., tensile strain $+\epsilon$ and compressive strain $-\epsilon$.

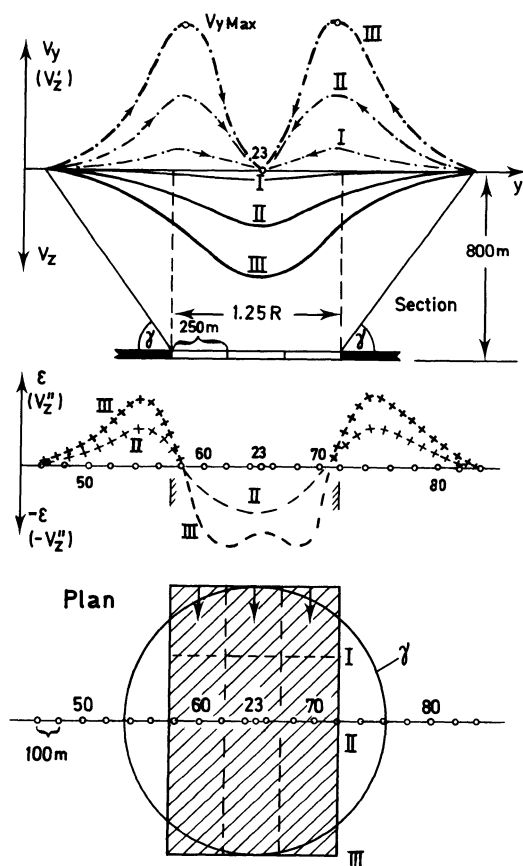
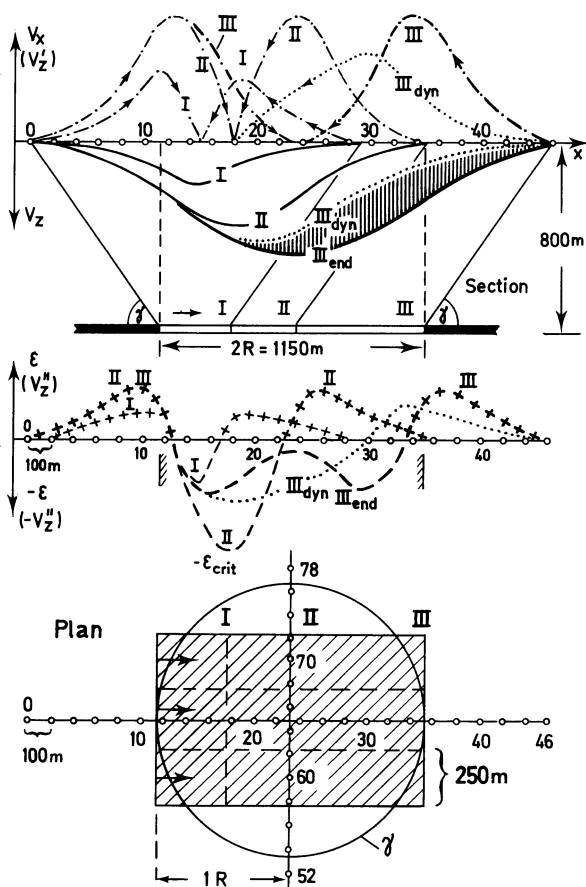


Fig. 123. Ground movement parallel to the extraction front (end situation), before (I), at (II), and after (III) passage of three side-by-side faces under a line of measuring points numbered 46 to 84

Fig. 124. Ground movement (end situation) at right angles to the extraction front, over subcritical areas I and II and over working III of length $2R$. Dotted line: dynamic trough shape on reaching extraction position III (without delayed subsidence, hatched)



The subsidence trough becomes deeper along the line of measurement as the extraction area increases in size. Over the ribside, subsidence is approximately half as great as the maximum $v_{z\max}$ over the centre of the panel, near Point 23. At each side of the trough the displacement v_y of the points is in opposing directions and towards the centre of the panel. Within the curve the direction is marked with arrows (see Fig. 131), to avoid distinguishing between $+v_y$ and $-v_y$. Maximum displacement $v_{y\max}$, which increases as extraction proceeds, occurs over and just inside the edge of the working.

Tensile strain $+\epsilon$ occurs over the unworked seam at both sides, compressive strain $-\epsilon$ only over the panel. The transition from tensile to compressive strain occurs over and just inside the ribside, at the point where displacement v_y is at its maximum. At extraction position III, compressive strain over the panel centre recedes somewhat. It is noteworthy that at no point along the line of measurement is there a change from tensile to compressive strain or a change in the direction of displacement v_y . The zones of tensile and compressive strain are stationary.

The ground movements shown in Fig. 124 for the same working but in the x -axis appear to be more complicated than those in Fig. 123. The line of measure-

ment of points 0 to 46 lies here in the direction of the face advance, which proceeds from left to right along the strike of the seam. The extreme values of ground movement therefore travel with the working face on the right-hand side of the subsidence trough. The trough grows longer in the direction of the advancing face at positions I, II and III, and grows deeper by the stationary left-hand rib. The dynamic trough outline measured at the time when the face is at position III is shown as a dotted line. When the face ceases to advance the final trough shape III_{end} develops within 6 months to 1 year, depending on the nature of the rock mass. Delayed subsidence is shown in vertical hatching. It should be noted that the amount of movement over the stationary left-hand ribside, and over the unworked seam adjoining it, increases only until the area of extraction reaches half the critical size. A face advancing to the right from position II has no further effect on the area over the unworked seam on the left. By the time of position II, displacement has consequently also reached its maximum value $v_{x \text{ max}}$ over the left-hand ribside between points 12 and 14. The same applies to tensile strain in the region of point 10. Above the stationary face-position III the dynamic displacement curve (dotted line) is transformed into the final curve III, with a maximum value near point 34 which corresponds to the displacement maximum over the left-hand ribside.

The zone of tensile strain travels to the right with the working face, while compressive strain increases over the panel. The greatest compressive strain has developed near point 18 at face position II – i.e., at a panel length of 1 R. This so-called *critical compressive maximum* (Rellensmann, 1940) is twice as great as the peak of tensile strain, which is also at its greatest at a panel length of 1 R and, depending on its amount, remains almost unaltered as the face advances. For the coalfields on the left bank of the Rhine in Germany, a ratio of tensile to compressive strain has been established which depends on the size of the extraction area. Taking 100% as a full critical area of extraction, the ratio is 1:3 with a subcritical area of 40%, 1:2 with one of 60%, and 1:1.5 with one of 80% (Reichelt, 1972). The maximum value for compressive strain becomes less again

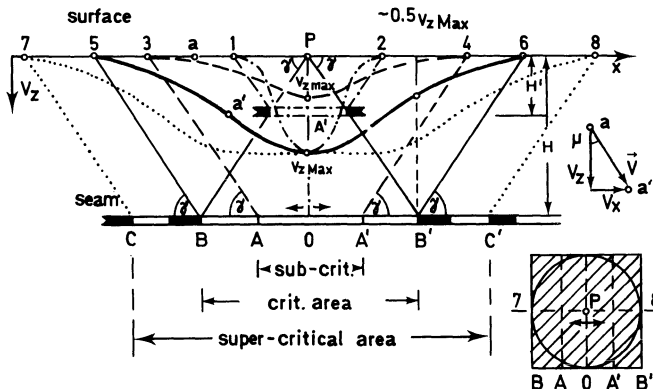


Fig. 125. Comparison of various trough profiles for a subcritical area A – A' (dashed line), critical areas B – B' (continuous line), and supercritical area C – C' (dotted line). Maximum possible subsidence of central surface point P also produced by extraction of a critical area at depth H' (dotted and dashed line)

if the panel length is extended beyond $1R$; and at a panel length of $2R$ it separates into two compressive maxima, which correspond in size to the tensile maxima. Over the centre of an extraction area which is more than $2R$ long and wide (a "supercritical" area), compressive strain in fact drops to zero. Along a line of measurement which crosses the trough from edge to edge in a straight line, whether through the centre or not, the sum of tensile strains must be equal to the sum of compressive strains – ignoring, that is to say, the fact that the curved trough-section line is slightly longer than the line of measurement (the chord) before subsidence.

To illustrate the various *trough shapes*, which themselves depend on the size of the extraction area and the depth of the workings, it is useful to start from the final position of ground movement over a horizontal panel (Fig. 125). In the sectional diagram, let extraction in both directions starting from 0 extend in the first place only to A and A'. At this stage in extraction a sinusoidal trough (dashed line) will form at the surface along the axis 7–P–8, once movements have settled down, and point P over the centre of the extraction area will have subsided the most, by $v_{z\max}$. The shape of this trough curve varies from coal district to coal district. With subcritical extraction, subsidence of 30%–60% of maximum subsidence is found over the face edges A and A'.

These variations in subsidence depend on variations in the composition of the rock mass (hard and soft beds) and on its increasing fragmentation if repeatedly worked through. The angle from the horizontal drawn from the face edge to the trough margin is designated the "limit angle" γ^1 . In the Ruhr this angle varies between 55 and 72 gr, but in soft strata it can be as little as 40 gr. The trough margin is regarded as the point where a clear subsidence of 1 or 2 cm can be found by levelling, provided there is no question of ground settlement through non-mining causes (e.g., de-watering or shrinkage). With increasing depth the arm of the limit angle becomes steeper, because the proportion of firm rock in the overlying strata increases. The breakup of those strata also increases on repeated working. The limit angle flattens out only in the uppermost zone of strata. The size of this angle thus depends on the strength of the rock strata, the degree of fragmentation, and the depth of working. The dip of the beds, the thickness of seam worked, the forefield convergence and geological faults also affect it (see Figs. 35, 48, 65 and 95).

If now the area of extraction is increased to B–B' in such a way that lines sloping inwards at the limit angle drawn from the new face edges intersect at the point P on the surface, then this point will undergo the greatest possible subsidence according to Eq. (16) – i.e., so-called "full or critical subsidence" $v_{z\max} = aM$. In such a case, extraction extending from B to B' at depth H is referred to as being a "critical" area of extraction in relation to the point P. The size of this critical area for any given depth of working in flat-lying measures is thus determined by a cone having its apex at the point P and its envelope inclined from the horizontal surface at the limit angle γ . At the smaller depth of working H' therefore, extraction of the small critical area A' is sufficient to lower the

¹ The "angle of draw or influence" from the vertical is $\zeta = 100 - \gamma$. In the Saar-district the size of critical area is determined by an angle $\gamma' > \gamma$ in point P – e.g., $\overline{OB} < \overline{a5}$ in Fig. 125 ($\gamma' = 69^\circ$, $\gamma = 60^\circ$).

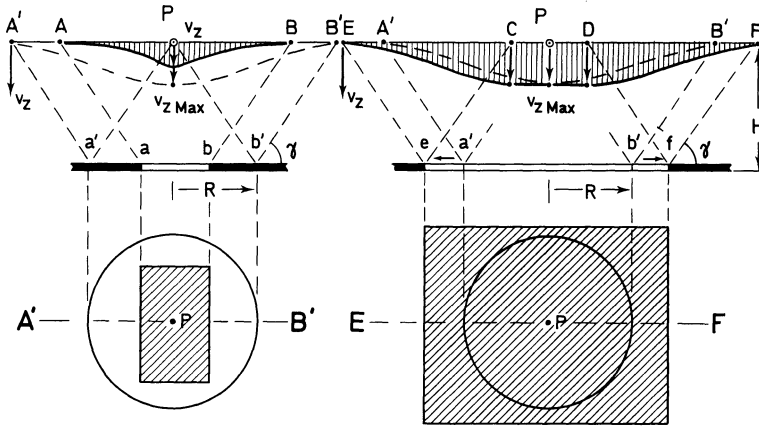


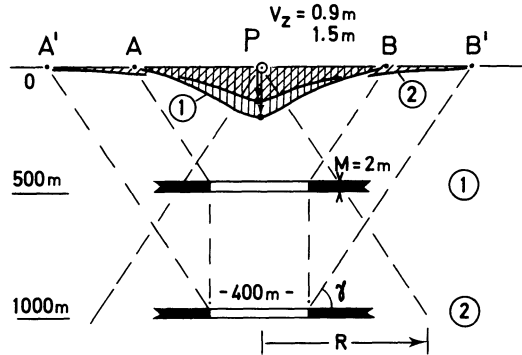
Fig. 126. Drawing in plan of a subcritical area $a-b$, with appropriate trough profile (*left*), and of a supercritical area $e-f$, with trough profile (*right*), each compared with a critical-area trough (*dashed line*, critical area $a'-b'$)

central surface point P by the full amount of subsidence $v_{z\text{Max}}$. Where the depth of working is small, the trough of full subsidence from 1 to 2 (dash-dotted line in Fig. 125) is comparatively steep-sided, whereas over deep workings full subsidence produces a broader and therefore flatter trough from 5 to 6 (continuous line). Depending on the treatment of the goaf, full subsidence amounts to 20% of the thickness of seam worked M with hydraulic stowing, 50% with pneumatic stowing, and 90% with caving.

A working which extends beyond the critical area to the “supercritical” area $C-C'$ produces a trough with a level floor extending from point 1 to point 2 (dotted line) at full subsidence $v_{z\text{Max}}$. Besides the terms “critical” and “supercritical” there is the “subcritical” area, the partial extraction $A-A'$ which does not produce a fully developed subsidence trough. In plan, the “critical” area is seen as a circle, the radius of which $R = H \cot \gamma$ depends on the depth of working H and the given limit angle γ (Fig. 126).

With *increasing depth*, the trough produced by an extraction of constant area will become wider and maximum subsidence will become less (Fig. 127). With this flattening of the trough at increasing depths of working goes a reduction in tilt, curvature, displacement, and linear change (tensile and compressive strain). On this basis, the degree of mining damage should decrease as the mining industry achieves greater and greater depths of working. For this reason the mining subsidence guidelines in some countries provide safety coefficients which, when multiplied by the thickness of seam mined, indicate the critical depth up to which a deeper working causes deformations no greater than those allowed for the relevant building classes (see Table 30). In the Donetz district, assuming $M = 1.5$ m, this depth amounts to $1.5 \times 400 = 600$ m for building class I, and to $1.5 \times 100 = 150$ m for building class V. A deeper working of unchanged area, however, affects a larger area at the surface, and in a densely populated mining district this can imply an increase in mining damage costs. Furthermore, where mining proceeds from seam to seam to greater depth, the rock mass becomes in-

Fig. 127. Subcritical areas of equal size at depths of 500 m (1) and 1000 m (2), with appropriate trough profiles



creasingly fissured and loosened, and this produces trough margins with steep tilts and high compressive strains.

The trough outlines from coal-mining selected here apply equally to ore- and salt-mining, if the area of deposit worked is large (see Fig. 253).

7.2.2 Calculation of Trough Components

An analysis of the *movement of points* in the trough area shows that, when the ground has come to rest again, points on the surface – with the exception of point P (the trough centre) – do not lie vertically under their starting position but have been displaced diagonally in the direction of the focal point of the extraction (for example, point a in Fig. 125). The three-dimensional displacement vector is thus at a greater or lesser diagonal to the working, depending on position.

a) Vertical components. If horizontal displacement v is ignored and the amount of subsidence v_z experienced by measurement points is marked off vertically under their original positions on an exaggerated scale (Fig. 125, point a), the result is the profile curve of the trough, or the “*subsidence curve*”, from which the line of tilt or slant v'_z can be constructed as a first derivative, and the line of curvature v''_z as a second derivative (Fig. 128). The slant position, or *tilt* (also called “*slope*”), related to an interval of 1 m between points, is obtained geometrically from the difference in subsidence between two points 1 and 2, as well as from their horizontal distance apart s (Fig. 129), according to

$$v'_z = \frac{v_{z1} - v_{z2}}{s} \quad (113)$$

where the interval l between points before subsidence can be taken in place of s . All along the trough the greatest degree of tilt, or subsidence slope, points in the direction of the centre of the working. The point of maximum tilt of the ground lies above a point just within the edge of the goaf, at the transition point W of the subsidence curve (Fig. 128).

The radius of *curvature* ρ_z of the subsidence profile can be obtained as an approximation, assuming its curves to be segments of circles, from the sub-

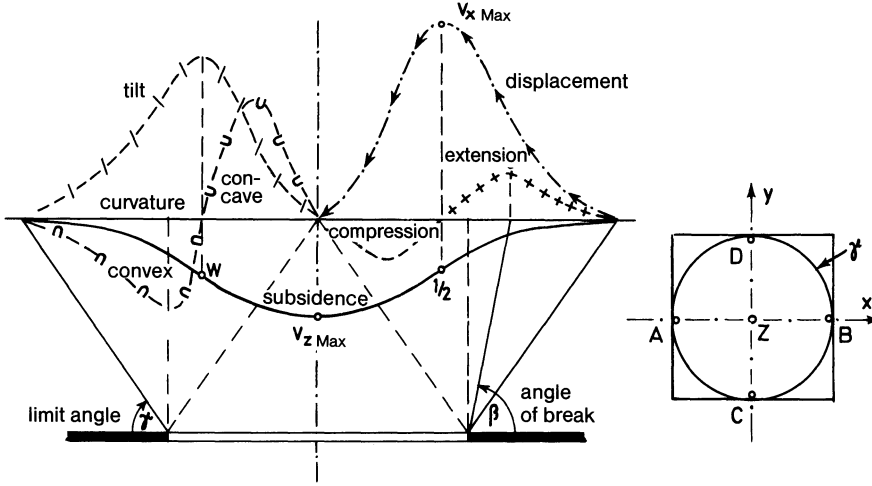


Fig. 128. The components of ground movement over a critical area of extraction. *Left half* of profile: vertical components. *Right half*: horizontal components. *Inset*: plan view of critical area

sidence of three measuring points, using the height of crown f and the chord $l' \approx l$, according to

$$\rho_z \approx \frac{l^2}{8f} \quad (114)$$

where the height of crown, if the distance between points is approximately equal, is

$$f \approx v_{z3} - \frac{v_{z1} + v_{z2}}{2}. \quad (115)$$

The curvature v_z'' , expressed as $1/m$, is the reciprocal of the radius of curvature ρ_z

$$v_z'' = \frac{1}{\rho_z}. \quad (116)$$

If the consecutively numbered points in a line of measurement 1–2–3... are at equal intervals $s = s_{(1-2)} = s_{(2-3)}$, then the radius of curvature can be obtained, given $l = 2s$, by introducing Eq. (115) into Eq. (114):

$$\begin{aligned} \rho_z &\approx \frac{s^2}{2v_{z2} - v_{z1} - v_{z3}} \text{ (concave)} \\ &\approx \frac{s^2}{v_{z1} - 2v_{z2} + v_{z3}} \text{ (convex)}. \end{aligned} \quad (117)$$

Curvature is convex in the area of the trough margin and concave in its inner zone, over the extraction area. The line of curvature has four peaks (see Fig. 128).

Table 10. The components of mining-induced ground movement within the influence trough

Vertical components			Horizontal components		
Designation	Symbol	Unit	Designation	Symbol	Unit
Subsidence	v_z	mm	Displacement	v_x, v_y, v	mm
Tilt	v'_z	mm/m	Linear change	$\pm \Delta s$	mm
Curvature	v''_z	1/m	Extension	$+\varepsilon$	mm/m
Radius of curvature	ρ_z	m	Compression	$-\varepsilon$	mm/m

Other quantities: Subsidence factor a Thickness worked M
Influence factor e Time factor z

Tilt and curvature are thus the trough components that are derived from vertical movement (Table 10). Knowledge of the amount of tilt is important for determining the decrease in value of a building in a mining area, and for altering the gradient of railway tracks. Curvature causes arching in extended structures, which can lead to failure.

b) Horizontal components. The second group of trough components – namely, displacement v and changes in point spacing (linear change), $\pm \Delta s = s - l$ or Δv_x in Fig. 129 – covers the effects of horizontal ground movement. If the ascertained displacements v , v_x or v_y (Fig. 121) are erected as perpendiculars midway between pairs of points, a symmetrical *curve of displacement* will be obtained, which reaches a maximum over a point just within the goaf perimeter, approximately where subsidence is at half its maximum, and returns to zero at the centre of the extracted area (Fig. 128). By analogy with the expression “full subsidence”, and in contradistinction to the maximum amount of displacement in any given profile, the greatest possible displacement may be referred to as “full displacement” $v_{\text{Max}} \approx 0.4 v_{z\text{Max}}$, even though this amount of displacement is reached in an extraction of half the critical size – e.g., over the centre of a working front of length $2R = \overline{CD}$ at the points A and Z (inset to Fig. 128). Over the square, critical-extraction area taken as a whole, full displacement is also found on the other diametral transverse section y at the points C and D, i.e. at four places. Movement is towards the focal point of extraction, and so, for all points on the diametral transverse-section and axis lines x and y in the inset to Fig. 128, the sum of displacement $\pm v = \pm v_x$ or $\pm v_y$ amounts to zero. The displacement curve resembles the curve of tilt.

The difference Δv_x in the displacement of two points (Fig. 129) taken in the direction of the line of measurement gives the change in spacing, or *linear change*, $\pm \Delta s = v_{x2} - v_{x1} = s - t$, expressed in mm ($-\Delta s$: shortening; $+\Delta s$: lengthening); or, related to a length of 1 m, the horizontal linear change $\pm \varepsilon$ in mm/m:

$$\varepsilon = \pm \frac{\Delta s}{s} = \pm \frac{v_{x2} - v_{x1}}{s}. \quad (118)$$

As *compressive strain* this is negative above the extracted area, and as *tensile strain* in the direction of extraction it is positive in the trough-margin zone.

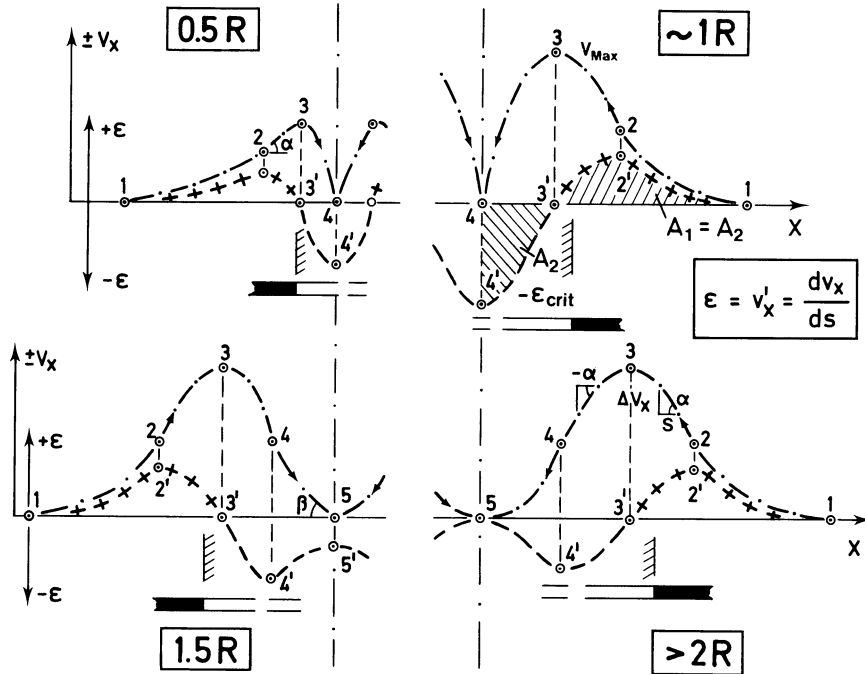


Fig. 130. Deriving linear change ϵ from the curve of displacement v_x for the working up to super-critical-area size (long-term position)

This negative angle increases, if the working area is less than $1R = H \cot \gamma$ in size, up to the central point 4, where compression is at a maximum. Over an extraction area up to $1R$ in size the displacement curve is consequently convex in shape, and the two curves of inwards displacement which meet at point 4 form an acute angle. Point 4 is a transition point in the displacement curve, which curve should be imagined as extending downwards in respect of the $-v_x$ values.

Over an extraction area over $1R$ in size (Fig. 130, bottom), the displacement curve between the edge and the centre of the working has a further transition point (point 4), with the result that the peak value of compression is reached before the centre 5. This central point 5 (a transition point) over an extraction area of $1.5R$ intersects the displacement curve at an angle of β , the size of which determines compression at the trough centre 5. If the working has exceeded the critical area $2R$ in size, the displacement curve passes through point 5 horizontally, and with $\beta = 0$ the amount of compression at the trough centre also recedes to zero. The section 1 to 3 of the respective curves is the same for workings of size $1R$, $1.5R$, and $2R$. The whole displacement curve from trough margin to trough margin thus has three transition points over an extraction area up to $1R$ in size, and five transition points over one larger than $1R$; the curve of linear change accordingly shows 3 or 5 peak values. A similar picture is presented by the curve of tilt (corresponding to displacement) and of curvature derived from it (corresponding to linear change).

In Fig. 118 the components of a subsidence trough are shown as an illustrative strata section. From the shape of the trough it is clear that the curvature,

which is not indicated, is convex in the zone of tension and concave in the zone of compression.

The order of magnitude of all components of ground movement, such as might be expected in their final state over a 500 m deep panel ($M = 2$ m, caving), can be found in Fig. 131. There is a noticeable similarity between the curves for tilt and displacement, and between those for curvature and linear change (tensile

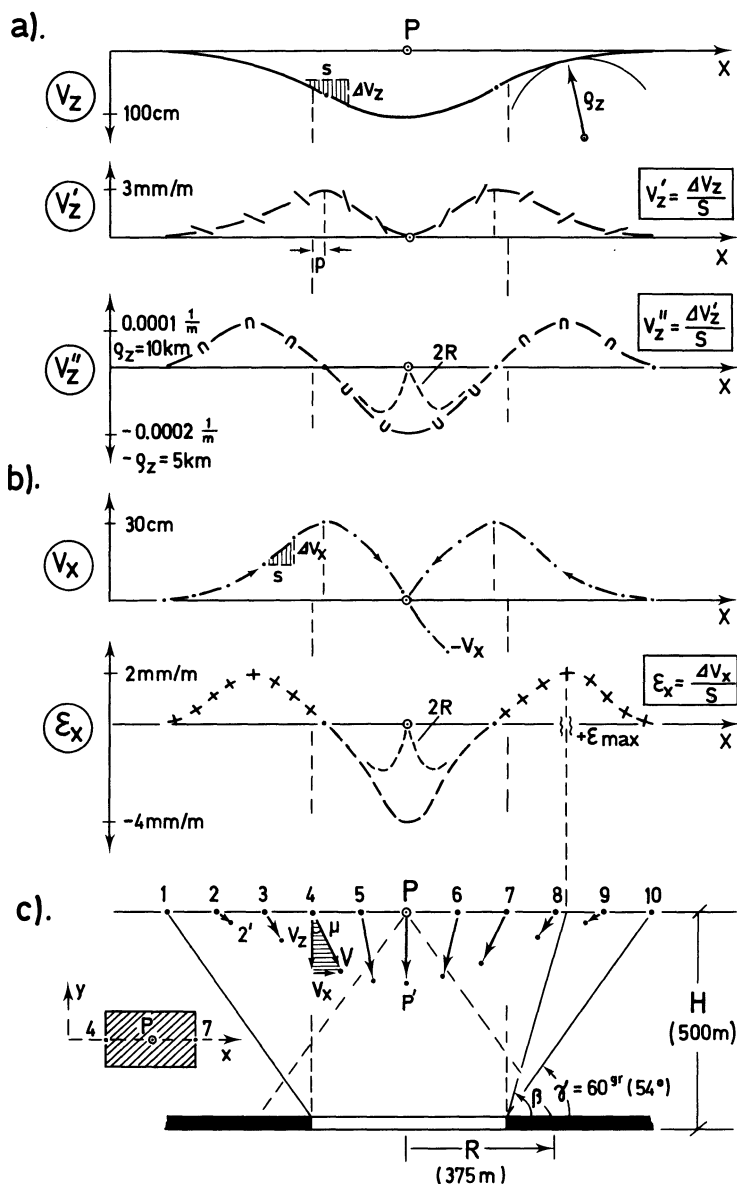


Fig. 131. Comprehensive diagram of all components of movement, shown as profiles. **a** Vertical components – subsidence, tilt, curvature; **b** horizontal components – displacement, linear change; **c** cross-sectional profile of line of measuring points 1 to 10 and of extraction (subcritical area)

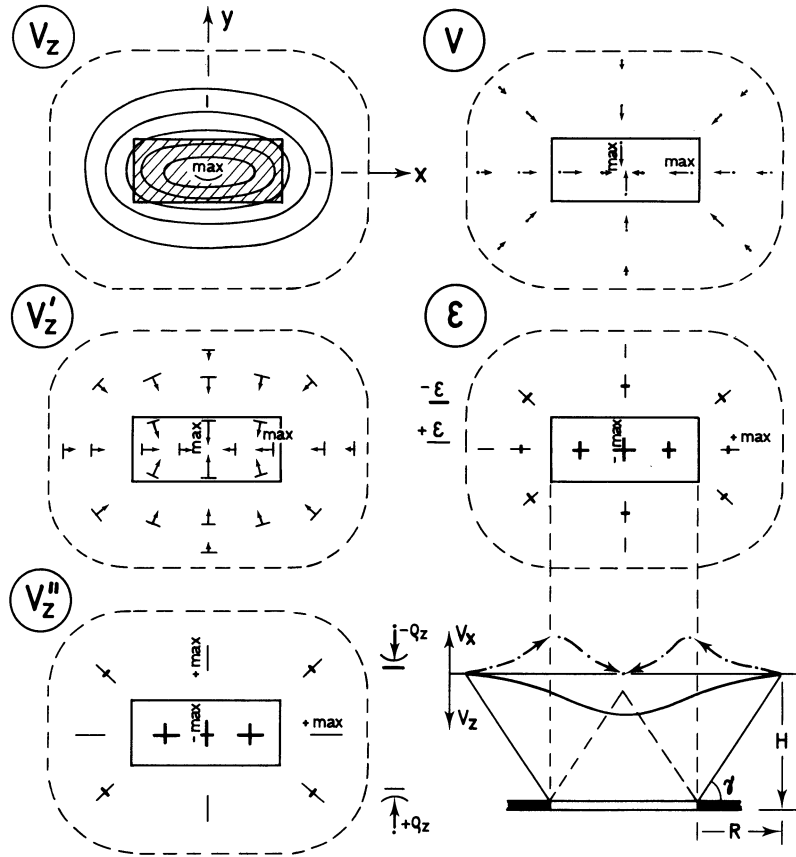


Fig. 132. Diagrammatic representation of all components of ground movement, seen in plan over a rectangular (subcritical) extraction area. *Left:* subsidence, tilt, curvature. *Right:* displacement, linear change, and profile of extraction

and compressive strain). The regression of curvature and of compressive strain over the centre of the panel as a critical extraction area ($2R$) is approached is indicated by the dashed lines. The formulae inset in Fig. 131 give expression to the interrelationship of the various components of a trough – i.e., the deriving of tilt from subsidence, curvature from tilt, and linear changes from displacement.

A convenient summary of the trough components seen in plan and of extreme values for purpose of planning buildings is given in Fig. 132. It can be seen that a structure above and just within the perimeter of extraction will be displaced and tilted to the maximum, that one over the centre of the panel will be subject to the maximum compressive stress and concave curvature, and that one over the unextracted zone immediately surrounding the panel will experience the maximum tensile stress and convex curvature.

c) Directional components. The ground movements which so far have been examined in relation to lines of measurement will now be looked at in terms of an elementary surface and of an orthogonal data grid. In the trough surface element

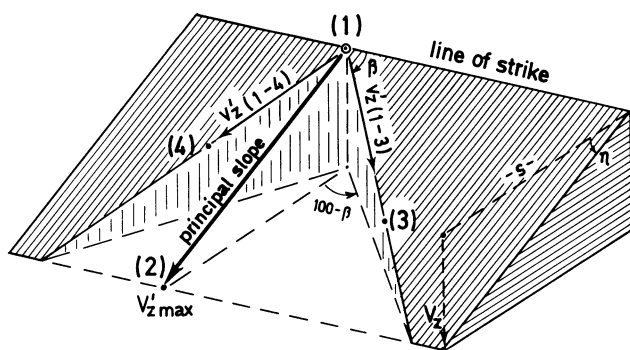


Fig. 133. Tilt of a surface element from point 1 to points 2 (principal tilt), 3 and 4

$$v'_{z \max} = \tan \eta = v_z \div s$$

$$v'_z(1-3) = v'_{z \max} \sin \beta$$

Fig. 134. Diagrammatic determination of principal tilt $v'_{z \max}$ from two known tilt movements 1-3 and 1-4

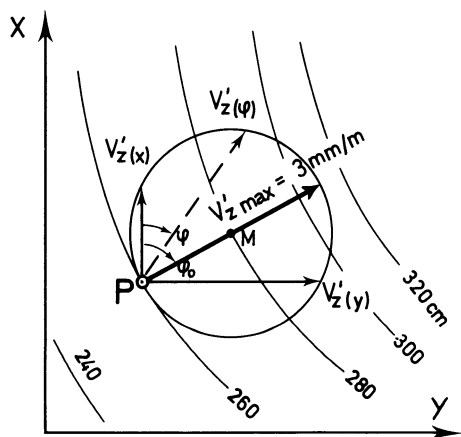
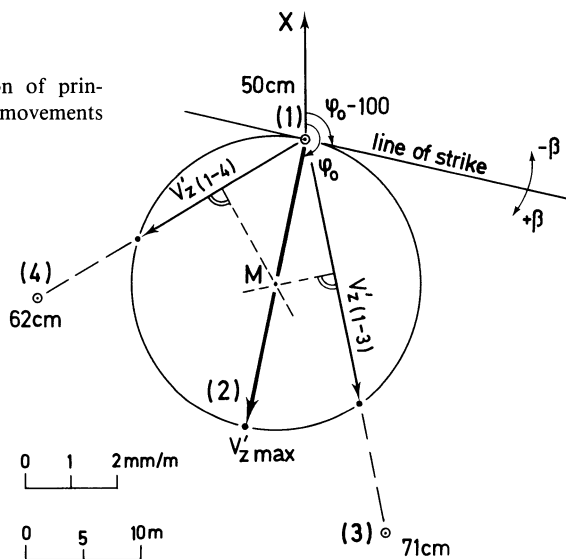
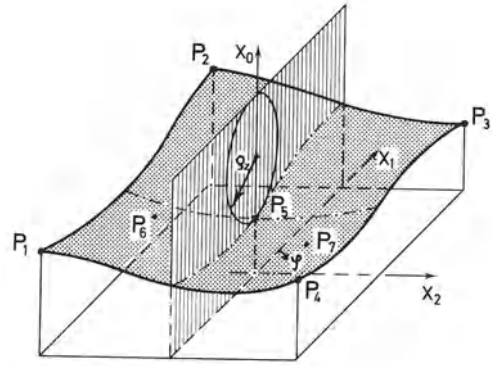


Fig. 135. Calculation of tilt positions, given lines of equal subsidence

Fig. 136. Determination of radii of curvature in a warped surface element of a subsidence trough



represented in Fig. 133, which is seen as being flat, with curvature ignored, the slant or tilt amount v'_z alters from point 1 at an angle of direction β . For $\beta = 0$, *tilt* along the "line of strike" is nil. In a vertical direction, on the other hand, towards point 2 ($\beta = 100$ gr), tilt is at a maximum. From this principal or transverse tilt position $v'_{z\max}$, tilt values in any desired direction can be derived with the function $\sin \beta$. Graphically, given two known direction values 1–3 and 1–4 (Fig. 134), principal tilt can be determined by means of a circle.

In a scalar field of subsidence defined by contour lines, the vector of principal tilt $v'_{z\max}$ intersects the lines of subsidence from P at right angles (Fig. 135). Directional tilt is given by

$$v'_{z(\varphi)} = v'_{z\max} \cos (\varphi_0 - \varphi) . \quad (119)$$

If the tilt values $v'_{z(x)}$ and $v'_{z(y)}$ for the point P are calculated in the direction of the x and y axes, as is foreseen for electronic data-processing programmes, then the principal tilt is obtained by

$$v'_{z\max} = \sqrt{v'^2_{z(x)} + v'^2_{z(y)}} \quad (120)$$

and its direction φ_0 by

$$\tan \varphi_0 = \frac{v'_{z(y)}}{v'_{z(x)}} . \quad (121)$$

In Fig. 136 the curved trough surface is defined by the position and height (subsidence) of the points P_1 to P_7 . If the *curvature* of this surface in various directions from point P_5 is investigated by turning the vertical plane around point P_5 and determining the curvature v''_z of the line of intersection between that vertical plane and the surface by means of a circle ρ_z which follows the outline closely, then it will be found that the curvature with the largest and smallest radii are the principal transverse curvatures v''_{z1} and v''_{z2} . The lines of principal curvature generally intersect at right angles. If there are at least six points $P_i(x, y, z)$ of the surface to be investigated, of which the height is known through levelling or subsidence calculation, in any given arrangement, then the curved

surface can be substituted as an approximation by a paraboloid. From such a surface equation of the 2nd order

$$\begin{aligned} X_0 &= f(x_1, x_2) \\ X_0 &= x_1^2 b_1 + x_2^2 b_2 + x_1 b_3 + x_2 b_4 + x_1 x_2 b_5 + a \end{aligned} \quad (122)$$

the coefficients b_i of which are found by a multiple regression (Chilian, 1979), the radii of curvature of the surface element at the point P_5 , in the direction of the selected axial systems x_1 and x_2 , can be calculated as follows. Since in general

$$\rho = \frac{(1 + X_0'^2)^{3/2}}{X_0''} \approx \frac{1}{X_0''} \quad (123)$$

the procedure is to differentiate Eq. (122) on the basis of the parameters x_1 and x_2 :

$$\begin{aligned} \rho_z(x_1) &= \frac{[1 + (2b_1 x_1 + b_3 + b_5 x_2)^2]^{2/3}}{2b_1} \\ \rho_z(x_2) &= \frac{[1 + (2b_2 x_2 + b_4 + b_5 x_1)^2]^{2/3}}{2b_2} \end{aligned} \quad (124)$$

The horizontal angle ϕ in any desired direction is obtained by transforming the coordinates of the points P_i into the system of coordinates as turned through the angle ϕ and introducing the new values into the surface equation (122). The principal radii of curvature ρ_{z1} and ρ_{z2} are found by progressively revolving the system of coordinates. It is only for the foundations of sensitive structures and machinery that surface curvature needs to be calculated as exactly as this, with all high points in the vicinity brought in as well. With structures which are sensitive to lateral buckling, such as retaining walls, basement walls, and rail tracks, curvature along the horizontal or the radius ρ_{xy} , resulting from differential displacement, can be of importance.

In most electronic data-processing programmes a mathematical description of the complete surface outline is dispensed with, and the radius of curvature ρ is determined from the amount of tilt as calculated or measured between three points in a straight line $v'_{z(1-2)}$ and $v'_{z(2-3)}$ and from the average distances between points:

$$\rho_z = \frac{s \times 10^3}{v'_{z(1-2)} - v'_{z(2-3)}} \text{ [m]}. \quad (125)$$

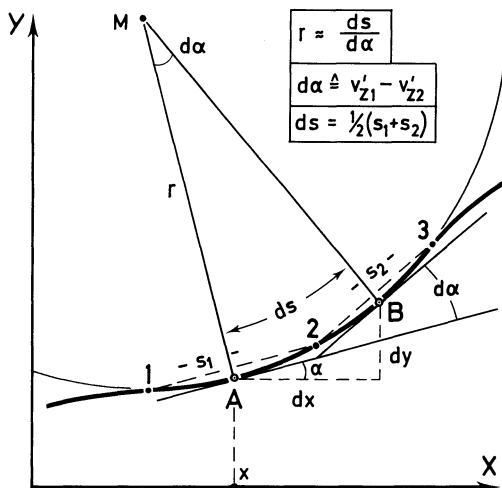
This formula derives from the definition of curvature², whereby the deviation of the curve from a straight line is expressed through the change $d\alpha$ in the angle of inclination α of the tangent as between the two end-points A and B of a short element of arc ds :

$$k(x) = \frac{d\alpha}{ds} = \frac{1}{r} \quad (126)$$

where r is the radius of the arc of curvature which is tangent to the curve at the point P (Fig. 137).

² Curvature is a scalar quantity, independent of direction, at the point P of the curve.

Fig. 137. Definition of curvature in a warped line



The tilt difference in Eq. (125) is calculated in both axes with the aid of points arranged in a cross, the end-points of which P_1 to P_4 are 20 m from the central point P (see Fig. 214). The points cross is gradually revolved until the principal curvature are found.

For the design of structures the ground curvature to be expected after a mining operation is an important piece of information, which nevertheless reflects unavoidable errors contained in three subsidence values. If a uniform standard deviation (mean error) of $\pm m_z$ is assumed for all three points, then the *error* in the radius of curvature is:

$$m_p = \pm \frac{2|\rho|}{|\Delta v'_z|} \frac{m_z}{s}. \quad (127)$$

For example, if the height measurements of the three points $s = 5$ m apart are affected with a height error of ± 1 mm, and if the degrees of tilt amount to -1 mm/m and -3 mm/m, then the radius of curvature and the error in it will be:

$$\rho_z = \frac{5 \times 10^3}{-1 - (-3)} = 2500 \text{ m}$$

$$m_p = \frac{2 \times 2500}{2} \times \frac{1}{5} = \pm 500 \text{ m}.$$

Since the error is inversely proportional to the point interval s , the distance s between points may not be shortened at will. The degree of uncertainty in the calculated radius of curvature is therefore relatively great.

For a large surface, *displacement* v can be represented diagrammatically in various ways. In Fig. 138 the displacement vectors v_i , drawn to scale, have been marked against the grid points 1 to 6, showing the amounts of displacement in cm and the angles of direction from the x-axis. In addition, the field of displacement is defined by a series of lines of equal value and equal direction.

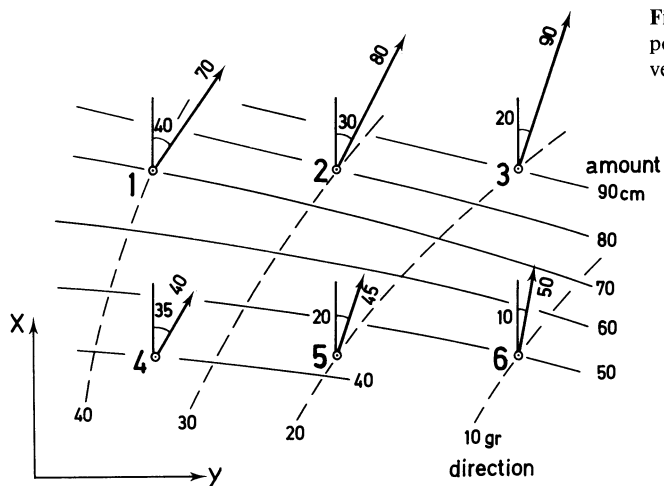


Fig. 138. Displacement of grid points 1 to 6, represented by vectors and lines of equal value

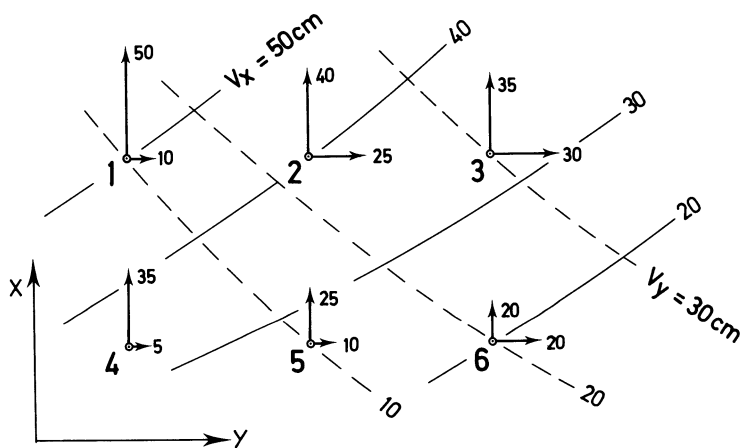


Fig. 139. Orthogonal components of displacement for grid points 1 to 6, and their representation with lines of equal value

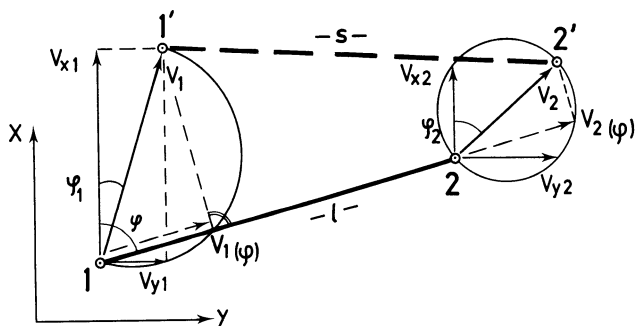


Fig. 140. Linear change in the line 1-2 as a result of displacement of the end points from 1 to 1' and from 2 to 2'

If in the displacement field the orthogonal components v_x and v_y for the grid points 1 to 6, as calculated with an EDP programme, are known, then it becomes possible to construct lines for changes in the value of components which are independent of direction (Fig. 139). The tilt situation in an extensive surface can be illustrated in a similar manner by means of gradients, components, and grid lines of constant value (Grafarend, 1964).

In the trough area therefore, displacement v varies from point to point both in amount and in direction. If now it is required to find the linear change in a structural axis which in its x -axis includes the angle ϕ , and for the end-points 1 and 2 of which the displacement components v_{x1} , v_{y1} , v_{x2} and v_{y2} have been calculated (Fig. 140), then as a first step these components can be used to calculate the amounts and directions of the displacement vectors:

$$v_i = \sqrt{v_x^2 + v_y^2} \quad (128)$$

$$\phi_i = \arctan \frac{v_y}{v_x} . \quad (129)$$

The second step is to calculate, from the displacement vectors v_1 and v_2 , the components in the structural axis:

$$v_{i(\phi)} = v_i \cos (\phi - \phi_i) \quad (130)$$

and then finally the *linear change*:

$$\Delta s = v_{2(\phi)} - v_{1(\phi)} . \quad (131)$$

From the linear change, tensile and compressive strains can be derived as in Eq. (118). Linear change can also be obtained direct from the displacement and direction components of the structural axis:

$$\Delta s = (v_{x2} - v_{x1}) \cos \phi + (v_{y2} - v_{y1}) \sin \phi . \quad (132)$$

The vector fields of displacement and tilt are linear, since at each point of the surface the change of position and the slope can be described by a single vector, the gradient v or $v'_{z\max}$. In a vertical direction of $\phi_0 + 100$ gr to the gradient, the vector field has the value nil. By contrast, the deformation field of linear change and curvature is bilinear, and at right angles to the largest linear change and curvature the deformation is minimal (Fig. 141). Both extremes influence the magnitude of a given direction. Linear change and curvature are therefore tensors and may not be added vectorially. In a polar diagram as in Fig. 141, the directional magnitudes of tension and compression at the point P of a trough surface produce either a waisted figure (where both principal magnitudes ϵ_1 and ϵ_2 are tensile or compressive strains) or a rosette (where both principal magnitudes are of a differing sign). A circular ground element is deformed into an ellipse by the extraction effect, and the measured radial difference between circle and ellipse yields the deformation figure a) or b) of Fig. 141.

If all that is known is the deformation components ϵ_x and ϵ_y and the direction ϕ_0 of maximum principal deformation ϵ_1 [which can be equated approximately to the direction of principal tilt, as in Fig. 120, and in general can be calculated by Eq. (121), along with subsidence and tilt], then the principal magnitudes ϵ_1 and ϵ_2

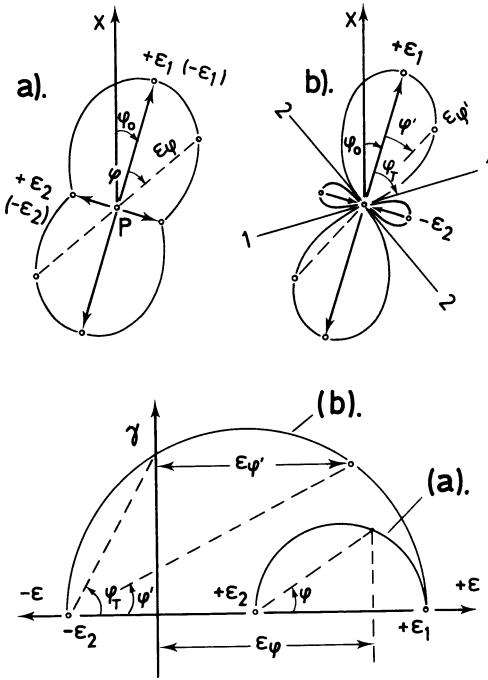


Fig. 141. Polar representation and diagrammatic determination of horizontal deformation in trough surface element

and the directional value ε_φ can be obtained by the following formulae³ (Kratzsch, 1980):

$$\varepsilon_1 = \frac{\varepsilon_x \cos^2 \varphi_0 - \varepsilon_y \sin^2 \varphi_0}{\cos 2 \varphi_0} \quad (133)$$

$$\varepsilon_2 = \frac{\varepsilon_y \cos^2 \varphi_0 - \varepsilon_x \sin^2 \varphi_0}{\cos 2 \varphi_0} \quad (134)$$

$$\varepsilon_\varphi = \varepsilon_1 \cos^2 \varphi + \varepsilon_2 \sin^2 \varphi \quad (135a)$$

or

$$\varepsilon_\varphi = \frac{\varepsilon_1 + \varepsilon_2}{2} + \frac{\varepsilon_1 - \varepsilon_2}{2} \cos 2 \varphi. \quad (135b)$$

With the principal deformation values ε_1 and ε_2 , a semicircle as in Fig. 141, bottom, can be constructed, and from this the relevant directional value ε_φ for any given angle φ can be read off. At the directional angle φ_T , deformation ε_φ is nil if tensile and compressive strains occur in a ground element (case b), and in that direction the tangents lie between the loops of tension and compression. In the deformation semicircle the angle φ_T can be read off at the intersection of the circle with the axis of the angle of shear γ .

The various *deformation figures* of the trough surface in zones I to IV are shown in Fig. 142. Over the extraction area (zone I), only compressive strains

³ $\varphi_0 \neq 50 \text{ gr}$; $\varphi < 100 \text{ gr}$.

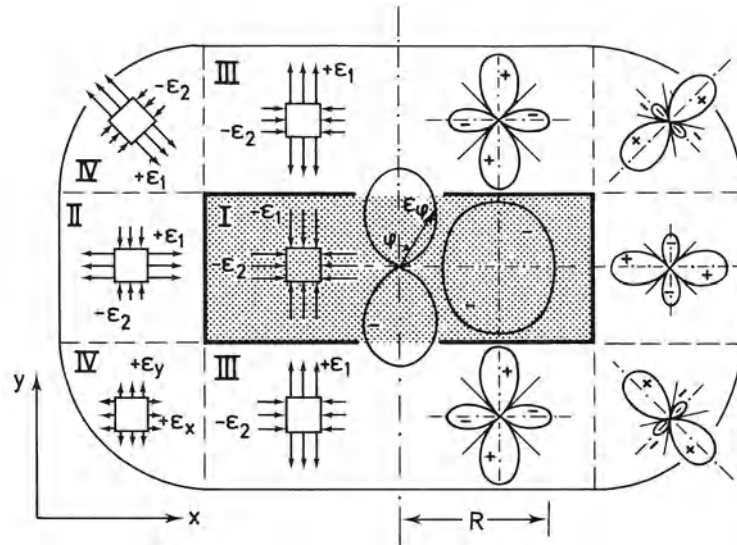


Fig. 142. Horizontal deformation (tensile and compressive strain) in a subsidence trough, seen in plan. *Left side:* principal deformations. *Right side:* polar representation of them. Extraction area dotted

occur in all directions. Over the unextracted seam (zones II to IV), the radial deformations are tensile strains, and the lines parallel to the edge of the extraction indicate compression. In the corner zone IV, both components ϵ_x and ϵ_y are tensile strains. Over the centre of a subcritical extraction area of length $2R$, the lines of compression become a figure of eight. A deformation diagram such as this is of great value for the siting of structures. The figures described by curvature diagrams are similar to these. In their position in the trough, tensile strains $+\epsilon$ coincide with convex curvature and compressive strains $-\epsilon$ with concave curvature.

7.3 Anomalies in the Trough Outline

Subsidence troughs, as generally depicted, reflect ground movement in an idealized form not always encountered in mining. The uniform outline given to troughs presupposes a little-worked rock mass, free from geological faults, with a thick and soft surface layer which acts like a cushion in evening out irregular movements in the bed rock. If these assumptions are not met, the trough will differ from the normal outline as depicted in Fig. 128, especially in amplitude of subsidence, or will exhibit localized anomalies such as surface cracks and steps. Abnormal trough outlines may have a tectonic, a mining-engineering, or a hydrological origin (Table 11).

Tectonic fault planes divide the rock body like joints and act as slip planes if the overlying mass moves under the influence of mining. A distinction can be drawn between action on a fault plane exerted by the footwall block and action

Table 11. Classification of surface cracks and trough anomalies by shape and origin

Shape	Origin
Downwarp	Ground wave over a fault line passing incompletely through overburden in the trough margin area
Hump	Small, saddle-shaped buckle at the trough centre, over a roadside pack, or over the boundary between two panels
Surface crack or cleft	Tension crack on ground strain of 3 – 7 mm/m (through exceeding the cohesive strength of the ground)
Surface step	“Tracing through” of a network of fissures from the underlying strata (jointy marl bedded on float-sand) Break in ground at the outcrop of a fault Down-sliding of a wedge block at the associated surface crack
Benching	The forming of parallel surface cracks on continuous and vigorous mining activity in several seams having margins in a common vertical plane
Surface trench	Subsidence of a wedge block on increasing tension at the surface Collapse of a block of marl into float-sand under tension
Overthrust, arching	Upthrust of ground layer in zone of strong compression at trough centre
Shear crack	Differential horizontal displacement of blocks of overburden towards the focus of mining
Secondary troughs	Localized depressions at the trough margin resulting from delayed convergence in old extraction panels Localized settlement on the escape of ground water or float-sand into parts of the mine (shrinkage of cohesive soils, such as lenses of peat, or washing away of solid or soluble particles)
Cratering, caving-in	Collapse of surface layers over near-surface workings, washed-out karst hollows, or sites of float-sand escape into the mine
Steep trough sides and asymmetrical trough profile	Repeated mining of the strata Intense perpendicular fissuring of the mine roof, accentuated by caving, concentration of mining within blocks of strata, and a high rate of face advance (over 3 m/d) Lasting shear movement of strata at limit planes (slip planes) and conditioned by direction of face advance

exerted by the hanging wall, and again between fault planes traversing only the bed rock and those traversing also the surface layer (Fig. 143). If the fault does not continue into the surface layer, either step-breaks or down-warps (ground waves) will occur at the surface at a point in prolongation of the fault, when the rock mass is mined through repeatedly. Which of these two surface phenomena will occur depends on whether the workings are on the footwall or the hanging-wall side of the fault (diagrams a and d in Fig. 143). At the surface termination of a continuous fault, a surface crack (diagram b) or even a surface step will form on the second undermining, if extraction in the overlying strata proceeds up to a fault plane which is steeper than the limit angle γ (diagram e). The strata movement which shears off at the fault produces an excess of subsidence over the inner

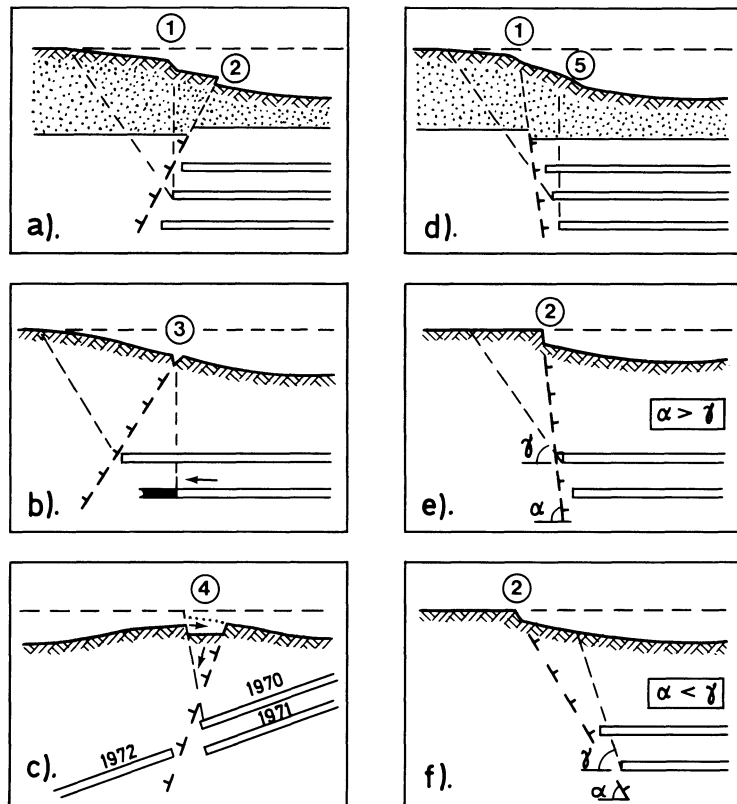


Fig. 143. Anomalies in the trough outline caused by geological faults. 1 Ground wave (down-warping); 2 surface step; 3 surface crack; 4 graben or trench

side of the face edge. If the fault dips at less than the limit angle, strata movements in the hanging wall will extend beyond the limit angle (diagram f), and a smaller step than in diagram e will be formed at the fault outcrop. In inclined measures, a surface step produced by workings in the footwall may be widened into a trench by later working in the hanging wall of the fault (diagram c).

The height of surface step in Fig. 143e and f can be predicted from the amount of precalculated undisturbed subsidence at this point, enlarged by the empirical extent b by which the hanging-wall mass slides down vertically at the fault plane. This sliding b depends on the mining convergence aM , extraction depth H , dip angle α of fault, and thickness d of loose overburden⁴.

The next few figures illustrating observed trough anomalies make clear the *influence of tectonics*. A ground step in prolongation of a geological fault can be up to 1 m and more in height if the overburden is of a marly consistency (Fig. 144, left). If there is no overburden, steps in the ground will appear directly at the outcrop of the fault. A working 200 m deep, carried in the hanging wall along a very steeply dipping (90 gr) fault plane, produces a subsidence step 155 mm deep

⁴ E.g., according to equation, $b = aMH \sin \alpha / 2000 (0.1 d + 1)$ in metres (Kowalczyk, 1982).

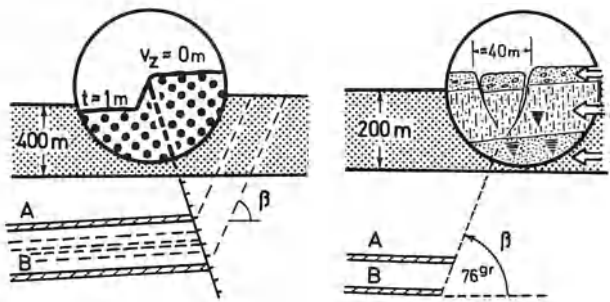


Fig. 144. Tectonically induced trough anomalies. *Left:* surface step at trough margin. *Right:* surface cracks forming wedge block and trench. 1 Gravelly sand; 2 fine sand and clay; 3 water-bearing sand bed. (After Mehrhoff, 1974)

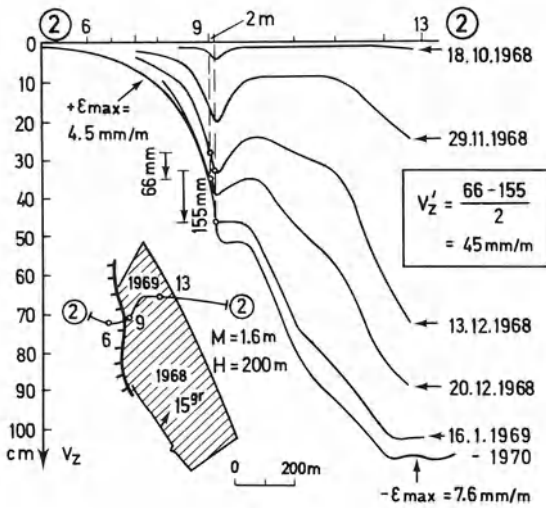


Fig. 145. Subsidence over a 200 m deep panel measured parallel to the face. Anomaly near point 9, where a geological fault outcrops. (After Pfläging, 1974)

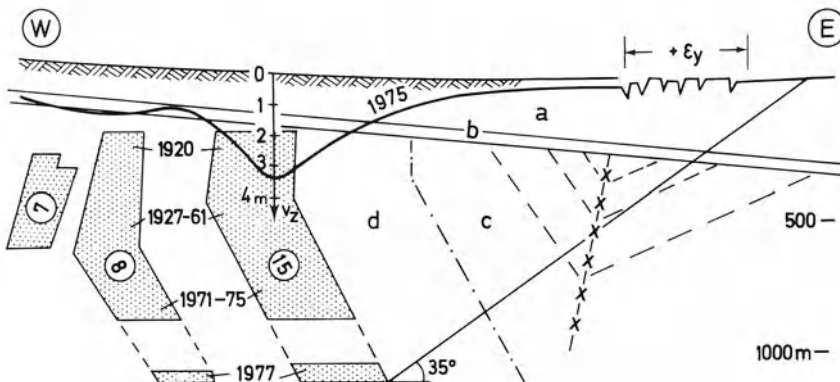
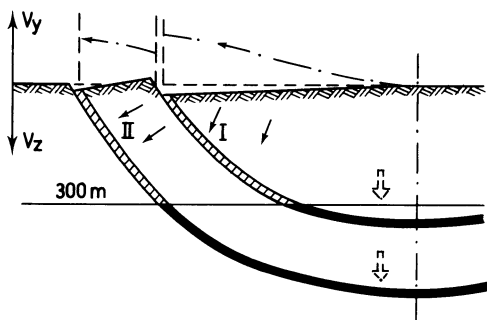


Fig. 146. Tension cracks at the margin of the trough on mining 7, 8 and 15 steep-lying coal-seams. a Bunter Sandstone (Lower Triassic); b Permian; c Stephanian (Upper Carboniferous); d Westphalian (Upper Carboniferous). (After Bordonné, 1978)

Fig. 147. Suggested pattern of strata movement on mining seams *I* and *II* above the 300 m level. *Broken arrows*: Subsidence of trough core on later mining of these seams below 300 m. (After Matouš, 1979)



near point 9 in the measuring line (Fig. 145). The line of weakness in the strata has an earlier and more intense influence on the effects of mining than do undisturbed strata (note the measurement made on 18 October 1968). There is even a secondary trough formed in the area of the outcrop line at the beginning of the mining influence. Even in later workings this zone of weakness will draw part of the subsidence potential to itself, and the surface step will become deeper. In the French coal-mining industry, steps of up to 80 cm in height have been observed at a very distant trough margin above a tectonic downfold (Fig. 146). By means of levelling it was possible to demonstrate that there was a narrow additional trough along the line of the downfold, in which the Bunter Sandstone (a) had sheared off in steps along vertical joints through tensional stresses. The sharp folding of the Carboniferous strata no doubt forms a barrier to the perpendicular, westwards-directed movement of the overlying Carboniferous beds (d) and (c), with the result that subsidence is concentrated at the margin of the trough as though by a geological disturbance. At the outcrop of extracted seams, surface steps can also occur if the steep beds at the margin of the basin subside over the extraction area with a tilting and revolving movement (Fig. 147). Displacement v_y is then directed outwards.

A fissured layer of marl bedded on soft shifting sand, if exposed to tensional and bending stresses at the trough margin, can break off and subside in blocks into the soft underlayer and give rise to benching (Fig. 148). The interval between steps depends on the jointing system in the layer of marl. Even without strong tensional stresses, steps in the ground can arise in places where the strength of the upper strata varies, as may happen at the dividing lines of geological formations – for example, at the transition points of Carboniferous, Cretaceous, and Tertiary strata if the Quaternary cover is thin or liable to gaps. In salt-mining it has been established that subsidence profiles show distinct bulges in the neighbourhood of basalt zones. Seen from the mine working, subsidence on the near side of basalt dikes is somewhat greater, and on the far side somewhat smaller, than would be expected in normal, regular development (Esslinger, 1975). Above a residual pillar, convex curvature and tensional stress can be so great that several parallel surface steps are formed. Above the loosened layer of marl the soil may then be washed away through deep cracks by the percolating rain water, causing the undercut blocks of marl to collapse and form trenches. In near-surface workings, if the roof rock is jointy and there is no overburden, individual blocks of rock may move in relation to each other along joint planes while subsiding,

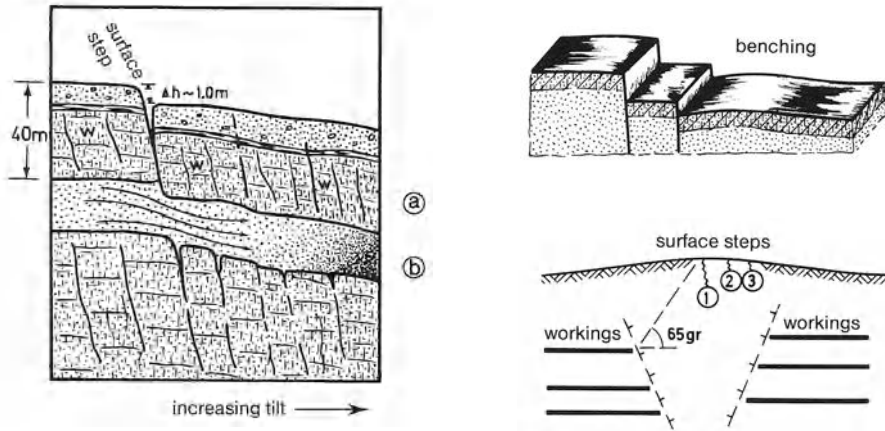


Fig. 148. Left: the forming of surface steps at the trough margin in jointy marl (a), overlying a bed of float-sand (b). Right: the development of benching at the interval between joints on further undermining and consequent increase in ground curvature. (After Mehrhoff, 1974)

with the result that the pattern of joints is “traced through” to the surface layer of soil (Fig. 149). At the interval of the principal joints – say 2 m – there will arise at the surface a network of fissures and small steps, which are reproduced in the foundations of houses. Paths of strata movement that are fixed in advance in this way are another cause of unsymmetrical trough profiles.

Figure 150 provides a summary of anomalies caused either by aspects of *mining* or by *hydrological phenomena*. A former panel boundary in seam A, a mine roadway with timber-prop packs on both sides in seam B, or a gap in mining – all are recognizable in the trough section as a hump or swelling (diagram a). Where the ribsides in several seams lie vertically one above the other, the tensional influences of the individual workings become cumulative at the angle of break, and the ground will crack under a tension of 5–7 mm/m (diagram b). Usually a parallel counter-crack opens up at a distance of 30–50 m in the direction of the trough. If the subsoil is of fine sand and soft, and if the tensional stresses increase, the wedge of ground between the two lines of cracks

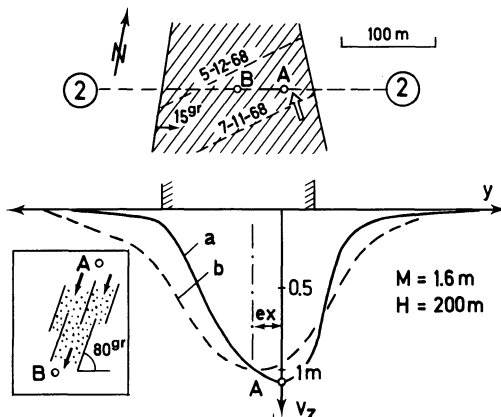


Fig. 149. Eccentrically displaced trough profile (a), as compared with the calculated profile (b), for the line 2–2. Because of strata movement in the direction of steep joints (inset) the focus of subsidence shifts from B to A. (After Pfläging, 1974, modified)

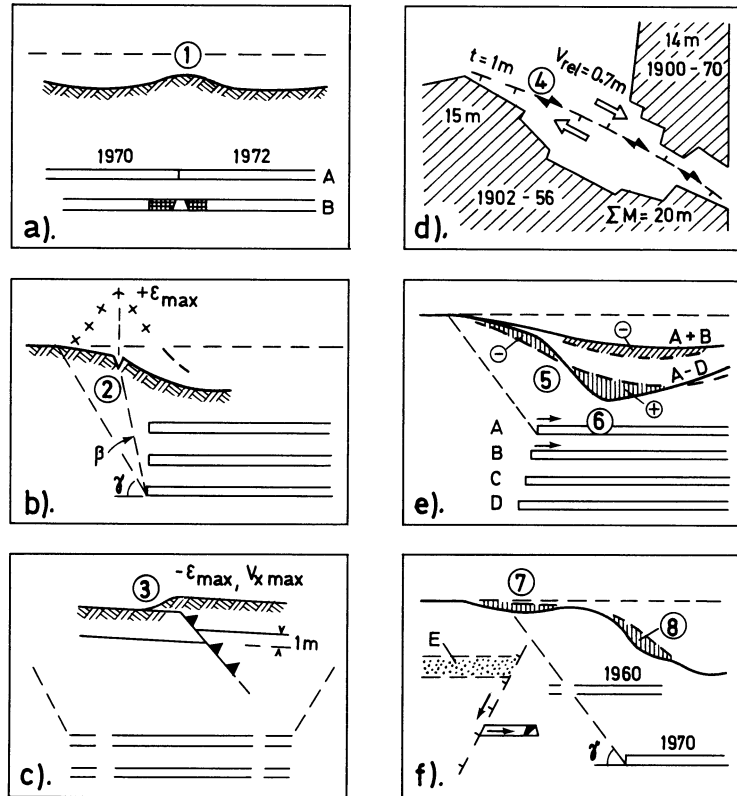


Fig. 150. Trough anomalies of mining origin and hydrological origin. 1 Surface hump; 2 surface crack; 3 vertical thrust-break; 4 horizontal shear-break; 5 steep trough side; 6 eccentrically displaced trough bottom; 7 secondary trough; 8 delayed subsidence in upper panel

may subside and form a trench (Fig. 144, right). Conversely, in the area of compression, a thrust crack and upthrow of the ground layer can occur (Fig. 150, diagram c). A horizontal shear crack between two panel areas can develop if an unextracted block of strata (a residual pillar) is displaced in relation to the adjoining focal points of extraction (diagram d). Above extraction (with caving) which is concentrated in blocks in seams A to D, an unsymmetrical trough with steeply sloping sides can develop (diagram e). The eccentric location of the trough centre is determined by the direction of the face advance or by joints in the rock, which channel the movement. By comparison with the normal trough outline (hatched), subsidence on extraction (by caving) of the first two seams A and B is smaller (a subsidence deficit), on extraction of the lower seams C and D greater at the centre (an excess of subsidence), and at the trough margin smaller. Secondary troughs can form if the water in a bed of float sand is enabled to flow away into an underlying roadway via a fault fissure penetrated by workings (diagram f), or if the strata movement over an old working (1960) is reactivated by the abutment pressure of a new working (1970) to the point where the margin zone is completely compacted (see Fig. 107). In one case, 100,000 m³ of sand was deposited

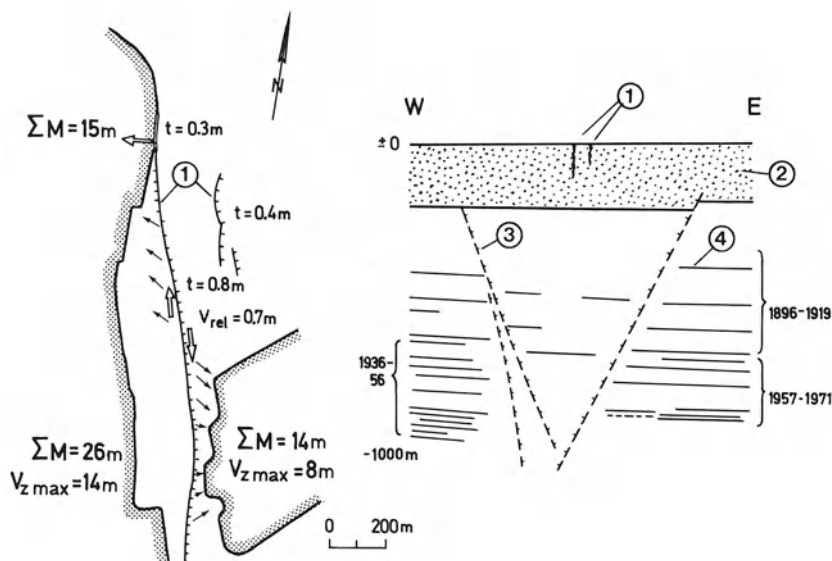


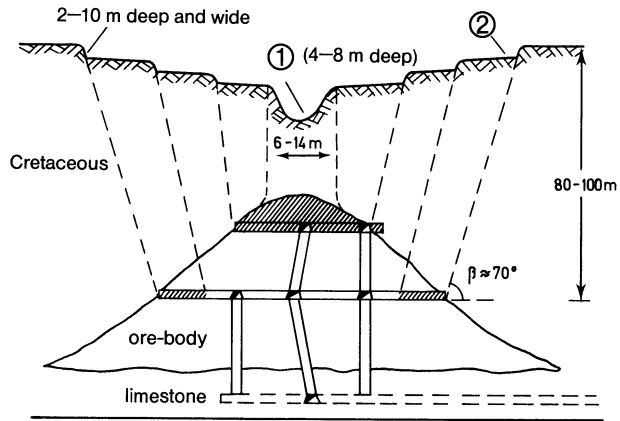
Fig. 151. Horizontal shear movement of up to 70 cm at a surface step (plan view, *left*), over the centre of an unmined, fault-delineated wedge block (section, *right*). 1 Surface step, up to $t = 80$ cm in height; 2 overburden; 3 geological faults; 4 mined coal-seams. (After Mehrhoff, 1974)

in mine workings within a space of 16 h. At the surface there formed very rapidly a trough up to 400 m wide and 6 m deep, with a compressive strain of up to 10% and an upswelling of 1 m in the asphalt of a motorway (Sommer, 1977).

Figure 151 provides an example of a *horizontal shear movement* at a surface step. On either side of a wedge-shaped trench block of strata formed by contra-dipping faults, up to 14 seams were mined in the years between 1896 and 1971. Mining went deeper and deeper, and each new working undermined the ribside of the previous panel. The surface crack which formed over the centre of the unmined wedge block probably did so after the first or second extraction, under the influence of the tension from each side. As each new and deeper working advanced farther towards the line of the surface crack, the crack developed first into an 80 cm high step and then into a series of steps. In addition, the blocks of strata on either side of the surface crack were displaced horizontally by up to 70 cm towards the focus of mining at the time — the eastern block towards the south, and the western block towards the north.

In *ore-mining*, metre-high surface steps have been observed (Fig. 152), which spread outwards as successively deeper slices of the mine extend in area. At the beginning of mining, within a few weeks of the removal of ore, the strata above the pyramid-shaped deposit become loosened in the form of a chimney reaching to the surface, where a narrow, 8 m wide trough forms, followed later by cracks and steps in the ground in an outward direction. On examination of a 1 m high overthrust, or ground wave, deformation of the sandy surface layer to a depth of 10 m was observed over a bed of clay (Fig. 153). Horizontal displacement in the area of the overthrust amounted to 190 cm. In the 9.6 m deep trough, the overthrust was located over the panel approximately at the point of maximum dis-

Fig. 152. Stepped trough profile over a mined ore deposit. (After Eckmann, 1971). The angle of break delimits the trough width. Compression at trough centre up to 477 mm/m. 1 Funnel crater at beginning of mining in centre of deposit; 2 tension cracks



placement $v_{x\max}$, in the transition zone between tension and compression. Along the ground wave the surface was sheared off horizontally by up to 40 cm.

It has been known for a long time that the *asymmetry* of the interim, dynamic trough profile persists on a diminished scale after the suspension of working (Fig. 124): over the rib where the face was opened out the trough is then deeper, and over the halted working face it is flatter, than in the textbook example of a symmetrical trough. Often the limit angles over the working front are smaller, and the trough-margin zones wider, than on the side of the stationary face. A rock mass that has been worked through in several seams, and consequently fissured and broken up, subsides in a funnel shape between steeper limit angles – for example, of 67 gr (Fig. 102); the result is that at the trough centre there is an excess of subsidence as compared with the usual flat, platelike shape, and at the trough margin a deficit. Steep trough margins not only exhibit severe tilts of sometimes over 10 mm/m but also produce narrow trough floors at the centre, and consequently high compression and curvature. The area of maximum tension moves to the ribside, and the angle of break increases to 100 gr. One result of the fissuring of the rock mass is that the compressive maximum persists even when there is a critical or supercritical area at the trough centre.

The analysis of ground movement, the referring of measured subsidence and linear changes to individual workings, and the determination of limit and break angles are in many districts rendered more difficult by the fact that several workings are affecting the upper surface simultaneously.

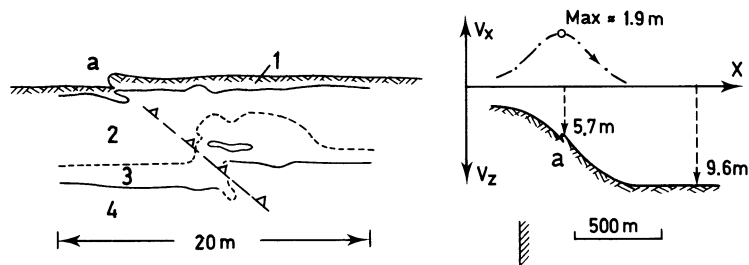


Fig. 153. Overthrust of sandy soil by 2.1 m (height of step 1 m) in zone of maximum displacement. (After Lippmann, 1979). 1 Surface soil; 2 fine sand; 3 coarse sand; 4 clay and marl

The *uplift* by several centimetres in the surface at the margin of subsidence troughs, often mentioned in the literature, can be explained, where it really occurs, by the oscillating bending profile of a beam resting on an elastic underlay (Fig. 58), or by a tilting movement towards the trough induced in lateral blocks of strata (Quiring, 1938). The uplift in ground at the margin of opencast workings, which has definitely been documented, results from relaxation movements in floor strata relieved of the load of overburden. Uplift and subsidence in areas away from the influence of mining are a sign of tectonic movements in large blocks of strata or of alterations in ground-water conditions. Interpreting such non-mining ground movements and identifying the boundary between them and mining-induced subsidence movements can be of great importance in lawsuits for damages.

7.4 Ground Movement in Steep-lying Measures

In inclined and steep-lying measures, as illustrated in Fig. 48, the angles of limit and of break along the strike (γ, β) both at the lower ribside (γ_H, β_H) and upper ribside (γ_L, β_L) vary in size (Fig. 154). From observation it is known that the *limit angle* γ_H at the lower face, the horizontal leg of which at B points into the roof of the working, becomes flatter with increasing dip, whereas the limit angle γ_L at the upper face becomes steeper. For example, with a dip of $\alpha = 40$ or 70 gr and a limit angle along the strike of $\gamma = 60$ gr, the limit angle in the dip plane can amount to $\gamma_H = 45$ or 40 gr and $\gamma_L = 65$ or 80 gr. In very steep measures this change in angle reaches a culmination point (H. Rom, 1964) and then begins to regress again, with the result that the limit angles for horizontal and vertical stratification are approximately equal. The angle of limit and break fluctuate in size according to the composition and strength of the overlying strata, and they

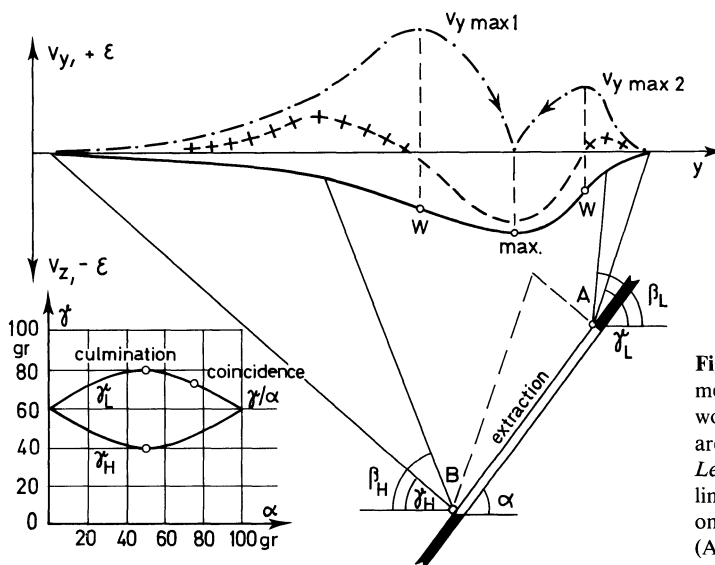


Fig. 154. Ground movement over a steep-lying working (subcritical area), seen in section. Left: systematics of the limit angle, as dependent on the dip of the seam. (After Rom, 1964)

depend in addition on the thickness of the overburden resting discordantly on the steep Carboniferous beds. The limit angles γ , γ_H and γ_L dropped from the surface point P give the cone for the critical extraction area, which is practically elliptical in outline at the seam and at the projection of the working as seen in plan.

With steep stratification the *trough* is always unsymmetrical. Its lowest point becomes displaced towards the upper third of the panel with increasing dip in the seam, and over the lower ribside B the trough becomes very flat and extends far out over the unworked seam. Over the upper ribside A the subsidence influence can also reach far outwards if the limit angle γ_L is smaller than the seam dip α , as is often the case when there is marked heave in the footwall beds. When the stratification is steep and the depth is considerable, a critical area – seldom attained – cannot therefore be determined by dropping the limit angle from the surface point P. The angle γ'_L must be erected at the lower ribside instead (Fig. 155).

The curve of *displacement* shows a large displacement $v_{y\max 1}$ over the lower ribside and a correspondingly small displacement $v_{y\max 2}$ over the upper ribside. The curve of tension and compression derived from this has a tension peak over the lower unmined seam (angle of break β_H) and a strong compression peak over the upper half of a subcritical area (Fig. 154). At the right-hand side of the curve of linear change, the tension peak near the upper ribside is small, and with increasingly steep stratification this part of the curve becomes narrower. The result is that over a very steep working there is practically only one tension peak and only one compression peak. Two unequal zones of compression develop over a critical extraction area, but comprehensive measurement data on these phenomena are lacking.

With a working that is completely vertical, two subsidence troughs form at the surface, if not only the hanging-wall but also the footwall beds become thrust equally into the mine excavation, and if the unextracted part of the deposit does not subside or break through into the workings. Steep strata which run continuously up to the surface often subside in steps at their outcropping edges. A steep seam can moreover form a shear joint over the workings, causing blocks of strata to slide down the joint and give rise to a trench or echelonned crack at the surface.

If a loose overburden layer lies discordantly on steep-lying bedrock, the subsidence and displacement at the surface of the bedrock, which itself displays a

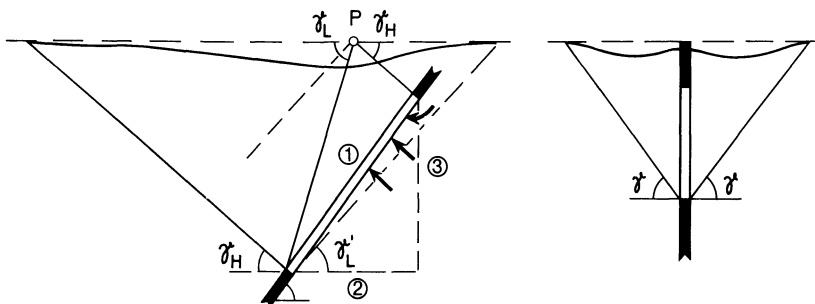


Fig. 155. *Left:* the critical extraction area in steep-lying measures. *Right:* two troughs over extraction of a vertical seam. 1 Critical area at the seam; 2 critical area as projected in plan; 3 heave in relaxed footwall

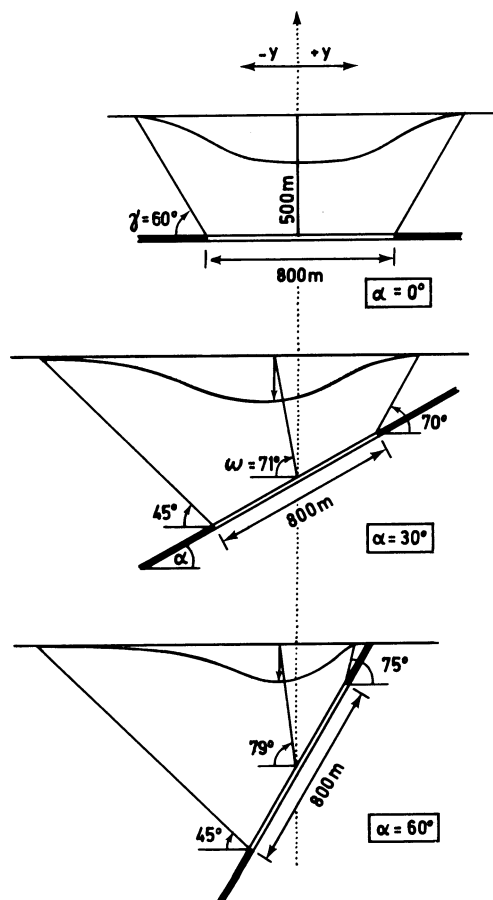
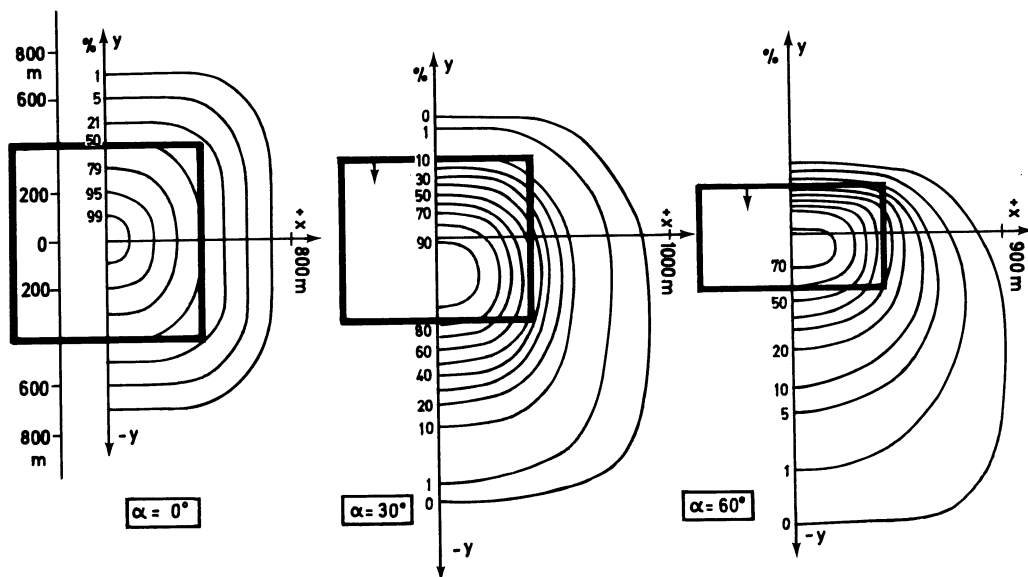


Fig. 156. The trough profile, seen in section, as dependent on dip in the workings and with constant face length. (After Stojanović and Patarić, 1979)

Fig. 157. Subsidence contour lines, seen in plan, for a horizontal, an inclined, and a steep-lying panel of constant size (800 m $\times$ 800 m). (After Stojanović and Patarić, 1979)



movement predominantly in the direction of the mine excavation, may be seen as the starting magnitude for the sagging of the surface layer. This layer deforms over the unsymmetrical trough of the bedrock like a weak slab (see Fig. 227).

Figure 156 shows changes in the trough profile in a direction cross-cutting the dipping beds. A panel 800 m long and wide is tilted out of the horizontal by $\alpha = 30^\circ$ and 60° . In the process the limit angle γ_H at the left-hand, lower ribside decreases from 60° to 45° , while the limit angle γ_L at the right-hand, upper ribside increases from 60° to 70° and 75° – values that are either observed or assumed. The *trough angle* ω , which is measured from the horizontal, joins the centre of the working with the centre of the trough. With horizontal stratification and a symmetrical trough profile, this angle will be 90° , and with inclined stratification perhaps 71° and 79° . Besides the limit angle, this trough angle is an important defining factor for the shape of the trough. Seen in plan, the trough shifts in the direction of the dip (the y-axis) as the stratification becomes steeper, as shown by the contour lines of equal subsidence in Fig. 157. In the direction of the strike (the x-axis) the width of the trough alters little with increasing dip in the beds.

Figure 158 further illustrates the interrelation of the various angles which determine the size of the critical extraction area and the shape of the trough. The limit angles γ_L and γ_H dropped from the surface point P define the length of the critical area A – B in an inclined seam in the direction of dip. The nadir angle μ , which is measured from the vertical, leads from the surface point P to the focal point of extraction Z – the area of extraction which exerts the strongest influence on P. The zenith angle ν , like the trough angle ω , is erected at the centre C of the panel and leads to the trough centre – i. e., to its deepest point, which in this case coincides with P. Comparison with the left-hand inset shows that the zenith angle ν , drawn to the trough centre P_2 , lies in the direction of the dip in a subcritical extraction area. The trough angle ω is then less than 90° . The position of the trough centre, however, depends not only on the length of the panel down the dip but also on its width along the strike, as the dashed trough line in the left-hand inset indicates (trough centre P_1). From all this it will be seen how much more difficult it is to determine the trough profile in steep-lying than in flat-lying measures.

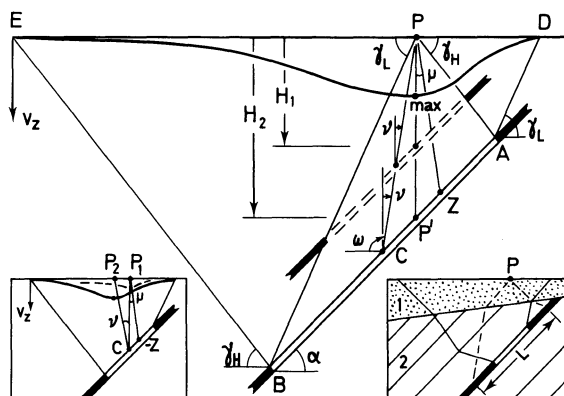


Fig. 158. Factors defining the trough profile at critical area and subcritical area in steep stratification

The right-hand inset shows the construction of the critical area for the surface point P, and of the trough length for the subcritical area illustrated, if the leg of the limit angle changes direction at the transition from firm to weak rock. The wedge-shaped overburden layer (1) lies discordantly on the steep Carboniferous bedrock strata (2). It is questionable whether the right-hand trough margin D and the lower margin B of the critical area should be defined – as they are in the main diagram – by straight lines representing the same limit angles γ_L , since from A to D only a short stretch of the rock mass is traversed, whereas from P to B is a long stretch, passing through different strata to some extent. Similar considerations apply to the lines B–E and P–A. Limit angles will therefore depend on the depth of the workings and will be of a different value for workings at depth H_1 than for workings at depth H_2 .

As already mentioned, the special trough shapes accompanying steep stratification have been little researched, and the critical factors affecting them are still largely unknown. In the literature, for example, the position of the trough centre is sometimes given as being over the upper half of the working, and sometimes over the lower half. Both cases ($\omega \geq 100$ gr – see Figs. 156 and 164) are evidently possible in practice, depending on the dip, depth, and size of the working, as well as on the strata structure – e.g., whether with or without overburden, whether worked through for the first time or already many times previously, etc. It can also happen that in the dip profile the displacement maximum over the upper edge of the working is greater than that over the lower edge – i.e., contrary to Fig. 154 – and also that both maximum values lie outside the horizontal area of the working altogether.

A calculation in this connection by the finite-elements method has revealed the following tendency: a trough angle ω of less than 100 gr decreases somewhat if the dip increases from 45 gr to 75 gr (average depth $H = 550$ m, length of working down the dip $l_d = 200$ m). The greater the proportion of loose overburden in the overlying strata as a whole, and consequently the less stiff the overlying strata, the steeper the trough angle and therefore the deeper the trough. Only when there is a complete lack of soft overburden is the displacement maximum $v_{y\max1}$ greatest over the lower edge of the working (see Fig. 225). If, on the other hand, the Young's modulus E of the strata decreases rapidly upwards, the displacement maximum $v_{y\max2}$ over the upper edge of the working becomes greater, and $v_{y\max1}$ smaller. In practice, $v_{y\max1}$ can increase again if there is relative displacement and friction at the interface between hard and soft strata (see Fig. 227).

8. The Pre-calculation of Ground Subsidence

8.1 Varieties of Calculation Procedure

At the beginning of this century, the mine surveyor was faced with the task of estimating, in increasingly thickly populated mining districts, the mining damage likely to be done to buildings, transportation installations, and agricultural land lying over new mines. He had little more to go on than the mine plans, and had to institute procedures for observing ground movement processes and to develop prediction methods suited to particular mines. In doing so he could confine himself, as was shown in the previous section, to the vertical component of movement – subsidence v_z – and the horizontal component of changes in position – displacement v_x and v_y . As is known, from these components as worked out for two or three points near the surface object under consideration, it is possible to derive all the other factors of movement, such as tilt and curvature on the one hand and extension and compression on the other. The difficulty in covering the movement process in the whole rock body, from mine to surface, in a closed calculation persists even today. Consequently it was necessary to do without a mechanical model and find instead abstract or analytical calculation procedures which were simple to apply to deformation at the surface. The only mechanical characteristics of the rock body available for the calculation were details of the mine geometry such as area, thickness of seam, and depth, which served as quantitative parameters for strata movement, and the results of observations such as limit angle and angle of break. At first only the static end-state of movement could be considered. Later, attempts were made to derive intermediate trough outlines from final shapes by means of time factors.

Pre-calculation procedures for flat-lying to steep-lying measures can be divided either according to their manner of execution, into graphical, numerical, analogue, and programme-controlled methods, or according to the form taken by their results, into point-reckoning, maximal-value, and profile-curve methods, or according to their fundamental principles, into empirical, functional, and theoretical-model methods.

The question as to the accuracy of the various methods, and as to which is the best method to employ, can only be answered to the effect that each method yields good results if its parameters are correctly selected. Each method of calculation must be adapted to the particular strata and extraction conditions under consideration, and what is important is not so much the number of parameters as the possibility of determining the quantities easily from levelling values or rock samples, and of assigning them to their causes. It is therefore not surprising that

in many mining districts they prefer to keep to some method developed there decades before and gradually improved with time, rather than introduce some other method which in theoretical terms is better but the parameters of which would first have to be defined.

With electronic distance-meters, air photos, and computers, there is no doubt that changes in the position of points in the ground can be measured and worked out very rapidly today, but an engineer will be very reluctant to abandon an EDP programme set up at great cost in favour of some other method of calculation. When only a few calculations are needed each year, and when new deposits are being opened up, a graphical method will be the most suitable. A universal method to render all local methods superfluous can only come from a theoretical model, but the problem would remain of defining the parameters in each district afresh, in view of the great variety of rock structures found. The following survey of the calculation procedures best known internationally is intended to contribute to encouraging the engineer to compare notes and improve or enlarge his local method, or introduce a different one, and to facilitate the choice of a suitable method in new mining districts.

8.2 Empirical Methods

The earliest methods for calculating ground subsidence started from the observation that the amount of subsidence depended on seam thickness and dip of beds, and also on quantity of stowing material and age of workings [Eqs. (111) and (112)]. Shortly before and after the First World War, formulae were developed by which the influence of the area of extraction on the amount of subsidence could be allowed for by introducing the volume of the mine excavation as reduced by the loosening-up of the strata (Goldreich), or with the aid of the ratio between extraction area and area of influence. The area of influence of a working was constituted at seam depth by the envelope of the angle-of-break cone, the apex of which lay at the related surface point. When working the outcropping end of a seam it was sufficient to know the maximum subsidence to be expected in the area of caving and funnelling over the workings, but over deeper workings it was already necessary on occasion to determine trough margins and subsidence steps extending far out over the unworked seam, if mining was to take place under railway installations and major roads. In this way the transition from the maximum-value method to the numerical-graphical method, and to the sectional diagram of the area of influence, came about.

Empirical methods offer certain advantages over functional methods and methods involving theoretical models. Whereas the latter rest on model propositions and mathematical assumptions which fit local circumstances to a greater or lesser degree, the former are derived from actual measured values. The reference values found for a particular district can easily be adapted to strata conditions altering with depth, if calculated findings differ markedly from observed values. The easily scanned calculation processes lend themselves to tabulation, and the interdependence of critical factors can be presented in nomograms, if a graphical

solution with strata sections and trough profiles is not preferred. The functions of many theoretical methods, on the other hand, can be varied only in the size of limit angle and consequently are not applicable to all mining districts, so long as the influence of particular strata properties on ground movement is not accurately known.

8.2.1 The Angle-of-Intersection Method

One of the oldest empirical methods, which even today is still used in the Saar, is to find the position of the trough centre and of other significant points of ground movement by means of a graph. A number of points in the subsidence profile are located on strike and dip sectional diagrams by using angles of intersection obtained from observations (Fig. 159). In this a distinction should be drawn between the angles of intersection at the upper and lower face of a working in inclined measures, and also between a solid coal face and an old goaf area as the edge of the working. Figure 160 shows a trough profile constructed by this intersection-angle method, both down the dip ($\alpha = 20^\circ$) and along the strike ($\alpha = 0^\circ$). One noteworthy aspect of these profiles is that the angle of break pointing to the peak of tension is near enough perpendicular to the seam. Between the line of break *b* and the beginning of the transition zone *c* the trough slopes comparatively steeply, as there is generally no plastic, yielding overburden in the Saar, with the result that tilts of several percentage points (>10 mm/m) are not uncommon there. At the point where the thin-bedded Carboniferous is covered by the friable Bunter Sandstone (Lower Triassic), large cracks often form at the break line over the lower face. The narrow transition zone *c*–*d*, reaching to the floor of the trough of “full” (i.e., maximum possible, or “critical”) subsidence, is identical with the zone of compression; but the latter zone, which depends on the shape and size of the extraction area, may also extend beyond the point *d* and, in a subcritical extraction, coincide with the trough centre. The peak of

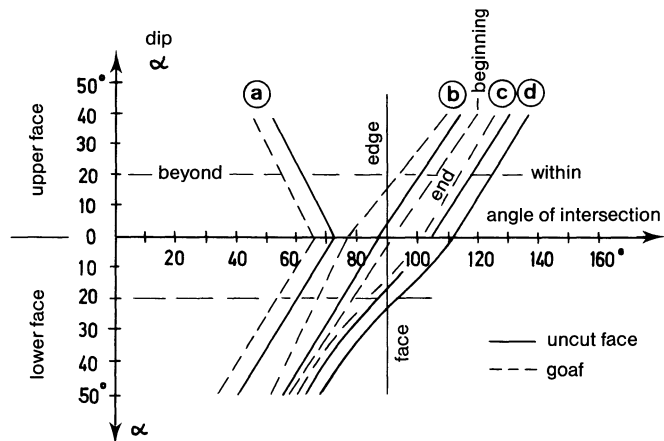


Fig. 159. Diagram for angles of intersection. (After Hellwig, 1950). *a* Limit angle; *b* angle of break, peak of tensile strain; *c* beginning, and *d* end of transition zone

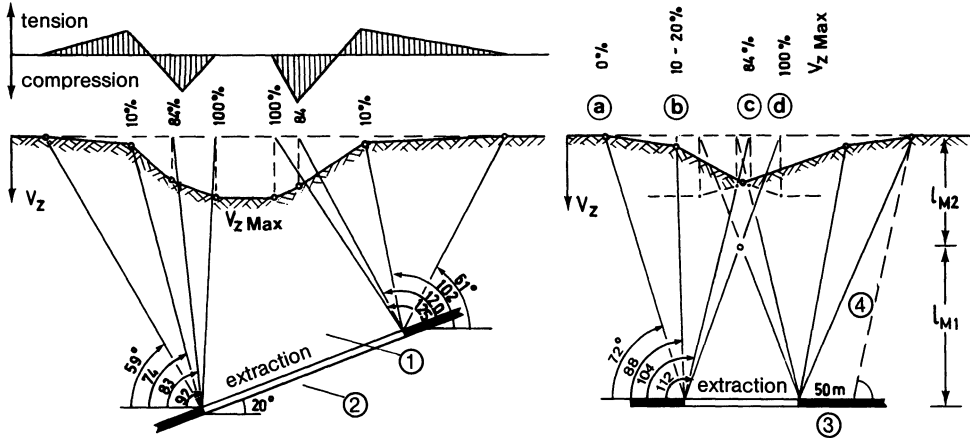


Fig. 160. The angles-of-intersection method. *Left:* subsidence profile down the dip, and graph of linear change. *Right:* subsidence profile along the strike over a subcritical area. (After Loos, 1960). 1 5% loosening-up (volume expansion) of roof strata to a height of between 40 and 100 m; 2 12% heave in floor strata to a depth of 100 m; 3 convergence ahead of the seam at a depth of 800 m; 4 limit of strata movement

compressive strain is generally one and a half times greater than the peak of tensile strain. Compressive strains of 5–15 mm/m are not uncommon in zones of strong curvature of the ground. In the profile along the strike, subsidence near the break line amounts to 10%–20% of full, or maximum possible subsidence, and directly over the face edge to 25%–33% of that maximum.

Over subcritical extraction areas the point of *maximum subsidence* lies at the intersection of the profile curves drawn from left and right (Fig. 160, right). The amplitude of the maximum subsidence occurring over a subcritical area is obtained from the reduction ratio of the two lengths l_M perpendicular to the stratification and intersected by the angles Wd :

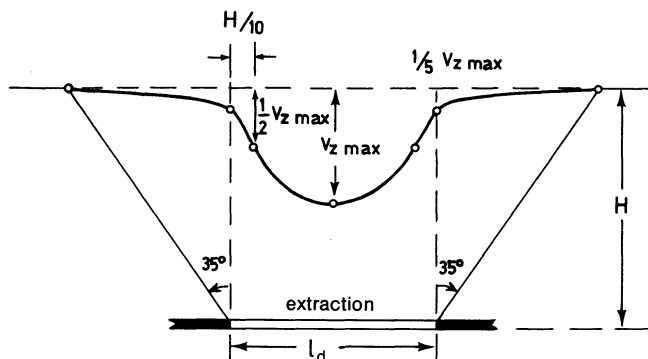
$$v_{z\max} = aM \frac{l_{M1}}{l_{M1} + l_{M2}}. \quad (136)$$

If the extraction area is also not of critical length in the other principal section, the reduction ratios $l_{M1} \div (l_{M1} + l_{M2})$ found for the two profiles are to be multiplied together and their product introduced into Eq. (136). The flank of the trough extends far over the marginal zone of a goaf. The subsidence profile points projected on to the ground-plan from the strike and dip section diagrams can be combined into lines of equal subsidence, and the subsidence values of given surface points obtained by interpolation (Loos, 1960).

8.2.2 The British Profile-curve Method

The British profile-curve method is an example of another empirical method in which, without using angles of intersection, the trough profile is drawn to an exaggerated scale for strata sections passing through the trough centre (principal

Fig. 161. The old British trough profile



sections) both along the strike and transversely. In this method the common form of the subsidence curve, as obtained from numerous measurements, is described by nomograms, from which the particular curve values for the working and the rock body under consideration are extracted. Up to the Second World War they were content in Britain to construct the subsidence profile from three values – the trough margin, half maximum subsidence, and maximum subsidence (Fig. 161). In these old profile curves the position of half maximum subsidence lies at a distance of approximately $0.1 H$ from the working face towards the extracted area.

In the post-war years it became possible to develop a more exact procedure from values obtained in countless observations. From three dimensionless diagrams (Fig. 162) the required subsidence of profile points can be found for widths of extraction l_d and point intervals l expressed in terms of depth. The construction of a curve may be explained using the example of an extraction panel 200 m wide and 300 m long at a depth of 400 m. First, maximum subsidence over the centre of the working is determined from diagrams 1 and 2. Assuming the same method of stowing throughout the panel and parallel mine roadways, the proportion of width of extraction to depth of workings $l_d/H = 0.5$ read off in diagram 1 gives a factor of 0.45 for caving, which, when multiplied by the thickness of seam worked $M = 200$ cm, gives a maximum subsidence of $v_{zmax} = 90$ cm for a face length of over $2R = 1.4 H$ (full, or critical, length). If, as in this case, the length of extraction l_s along the strike is less than $1.4 H$, and in fact only $0.75 H$, the maximum subsidence of 90 cm must, by diagram 2, be reduced by a factor 0.77 to 70 cm. The subsidence, expressed in terms of this maximum value, of other profile points at a constant distance apart of $l = 0.1 H$ is found from the nomogram for their distance from the transition point and for the given relation of $l_d/H = 0.5$ (National Coal Board¹).

The *transition point* is where the profile curve reaches its half-subsidence point. With very narrow subcritical-extraction areas, the transition point lies over the extracted area at a distance of up to $0.15 H$ from the face edge (the dashed line in Fig. 162, diagram 3). As the extraction becomes wider it moves

¹ These diagrams were further improved by the NCB in 1975 and have since been programmed for computer calculation.

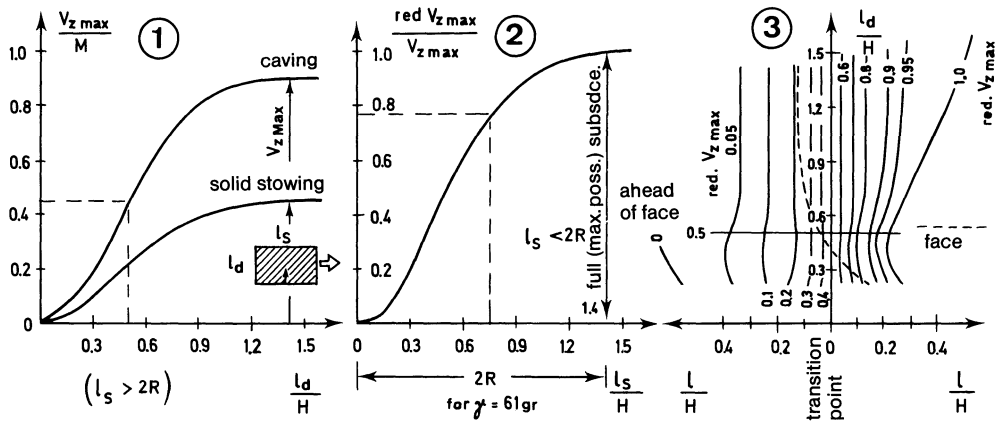


Fig. 162. Diagrams for constructing profile curves (after the U.K. National Coal Board, 1966). *Left:* maximum subsidence by reference to length of working front. *Centre:* reduction of maximum subsidence by reference to panel length. *Right:* lines of equal subsidence, related to distance l from transition point

over the face edge (at $l_d = 0.41 H$) and finally up to $0.14 h$ ahead of the face. With a dip of up to 20° , the distance between face edge and transition point read off from nomogram 3 is drawn from the place where a line erected on the face edge at right angles to the seam intersects the surface.

A half-profile curve constructed in this way (Fig. 163) is valid in flat-lying measures for any section along or across the strike, taken at right angles to the edge of the working. For every known point distance at right angles to the edge of the working therefore, the subsidence at that spot can be read off from the profile curve relating to that working. Alternatively, lines of equal subsidence can be drawn in on the ground-plan running parallel to the edges of the working and following round the four corners of the trough concentrically. In doing this it is assumed that the half-profile curve applies, unchanged in shape, all round the working, along the strike as much as down the dip. Ground waves (humps) in the trough outline, which may develop over central mine roadways, waste packs, and coal pillars, especially in near-surface workings, are entered in the profile curve separately. At a width of $0.1 H$ they may reduce subsidence in places to, for example, $0.8 v_{z\max}$ over a waste pack or $0.4 v_{z\max}$ over a coal pillar.

8.2.3 The Russian Profile-curve Method

Among empirical procedures in which the subsidence profile is obtained, not from diagrams, but from functional values are the profile-curve methods developed in the Donetz Basin, in Poland, in Hungary, and in the Ruhr District. In the first, the trough profile is described by the function

$$v_z = v_{z\max} \left[1 - \frac{x}{L} + \frac{1}{2\pi} \sin \left(2\pi \frac{x}{L} \right) \right] \quad (137)$$

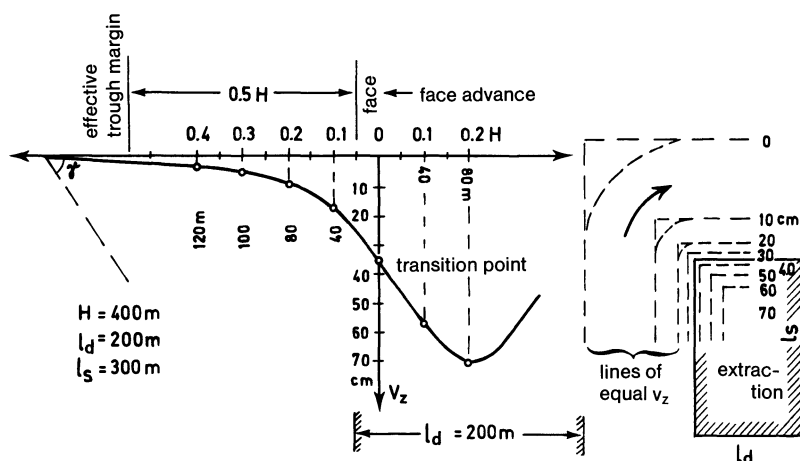


Fig. 163. Example for construction of a trough profile following the British procedure. *Right:* map with lines of equal subsidence

in which x signifies the horizontal distance of the calculation point P from the centre of subsidence Z , and L the distance from the same centre to where the trough margin is intersected by the profile line (VNIMI Institute, 1958). Equation (137) applies also to the profile line passing through Z in the direction of the strike and to the branch of the curve above the upper edge of the working. In the main section down the dip, the subsidence centre Z is defined by the trough angle ω erected at the centre of the working, which varies between 100 and 125 gr (Fig. 164). The subsidence peak value is arrived at by the formula

$$v_{z\max} = aM \cos \alpha \sqrt{n_1 n_2} \quad (138)$$

$$n_1 = 0.9 \frac{l_s}{H_m} \quad n_2 = 0.9 \frac{l_d}{H_m}.$$

This formula is derived from the empirically obtained ratio between extraction areas

$$\sqrt{\frac{A_{\text{sub}}}{A_{\text{crit}}}} = \frac{v_{z\max}}{v_{z\text{Max}}}. \quad (139)$$

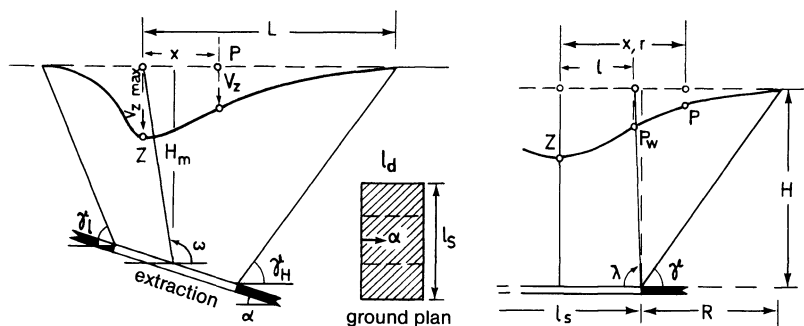


Fig. 164. Trough profiles derived from mathematical functions. *Left:* in the Donetz Basin, U.S.S.R. *Right:* in Hungary and Poland

If the square area of critical extraction has a length of side equal to $H/0.9$, as in the Donetz District, then the ratio between areas can be written as follows [see Eq. (138)]:

$$\sqrt{\frac{l_d l_s}{\frac{H}{0.9} \frac{H}{0.9}}} = \sqrt{0.9 \frac{l_d}{H} 0.9 \frac{l_s}{H}} = \sqrt{n_1 n_2}.$$

Should the length of the panel l_s along the strike, or the projection of face length in the direction of dip l_d , be greater than the mean depth of the workings H_m , then n_1 or n_2 as appropriate must be introduced into Eq. (138) at a value of 0.9.

To define the shape of the curve by this method therefore, a trough angle selected according to observed values is taken in addition to the otherwise usual limit angle. Convergence aM normal to the stratification is in Eq. (138) projected on to the vertical with $\cos \alpha$.

8.2.4 The Polish Profile-curve Method

For the subsidence profile in flat stratification obtained from numerous levellings in Upper Silesia, the applicable equation is

$$v_z = v_{zMax} e^{-nr^2}, \quad (140)$$

in which the exponent

$$n = \frac{v_{zMax}}{R^2 \bar{c}} \quad (141)$$

is derived from a surface trough which is equated to the settlement downfold in the roof beds (Kowalczyk, 1966). Here R signifies the radius of the critical area, r the distance from the panel centre of the profile point to be calculated, and $\bar{c}$ the average roof-bed settlement. In this method the incomplete roof settlement at the edge of the panel is taken into account, since $\bar{c}$ is not equal to aM or to C (see Fig. 10).

8.2.5 The Hungarian Profile-curve Method

In the Hungarian coal-mining districts the subsidence profile can also be represented by an exponential function²

$$v_z = v_{zMax} e^{-\frac{x^2}{2l^2}} \quad (142)$$

with x and l signifying the distances from the trough centre of the calculation point P and the transition point P_w respectively (Martos, 1958). With a subcritical extraction of only $l_s < 2R$, maximum subsidence amounts to

² See also Kosterin, 1972.

$$v_{z\max} = v_{z\text{Max}} \frac{l_s}{2R}. \quad (143)$$

The transition point's abscissa value l is calculated from the empirical transition-point angle λ (83 to 95 gr) and the depth H (Fig. 164, right). The introduction of the transition-point abscissa l was necessary because, even on small displacements of the transition point, the profile curve changes its shape markedly, and also because observations in Hungary indicate the transition point to be not over the face edge but over the margin zone of the stowed goaf.

8.2.6 The Programmed Profile-curve Method

A method of subsidence calculation in flat to steep stratification based on profile curves and programmed for electronic data processing installations first became known in 1962 (Niederhofer, 1962). It rests on an analytically obtained formula of approximation for the subsidence-profile curve, into which enter empirically obtained factors, together with the angle of break β and zenith angle ν as well as the limit angle γ , as critical factors. For the trough profile the equation chosen is ³

$$v_z = v_{z\max} \left[1 - \left(\frac{r}{R} \right)^2 \right]^2. \quad (144)$$

This equation is used to calculate the subsidence v_z of a profile point P , which travels from the trough's lowest point Z ($r = 0$) along the principal axes in the floor strata (r_L), roof strata (r_H), or the strike (r_S) to the line of break (n_1, o_1, p_1) or the trough margin (n, o, p ; Fig. 165).

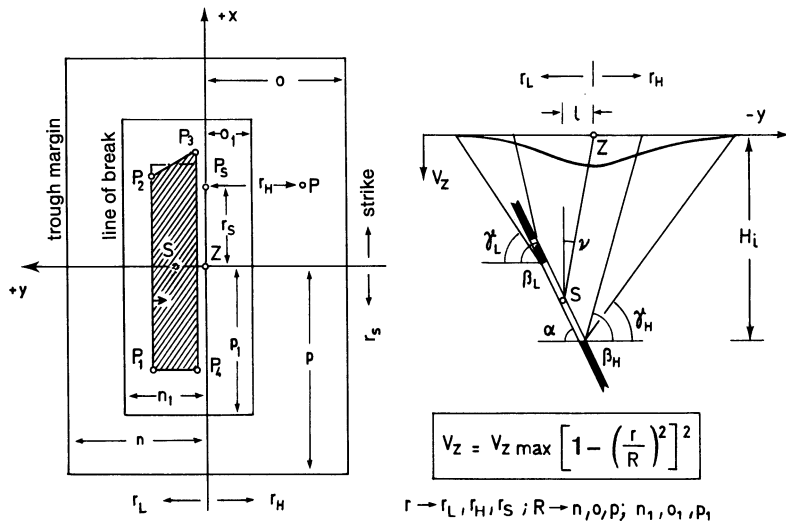


Fig. 165. Determining the area of influence over an extraction area using the corner points P_1 to P_4 , as seen (*left*) in plan and (*right*) in section down the dip (Niederhofer method, 1962)

³ This formula is also used in the Indian coal-mining industry (R = trough radius).

In order that the profile curve shall accord with observed subsidence, the weights g_{γ_L} , g_{γ_H} and g_{γ_S} are introduced for the area as a whole, and g_{β_L} , g_{β_H} and g_{β_S} for the outer area. Since $v_z \leq v_{zmax}$, these weights must add up to unity:

$$\begin{aligned} g_{\gamma_L} + g_{\beta_L} &= 1 \\ g_{\gamma_H} + g_{\beta_H} &= 1 \\ g_{\gamma_S} + g_{\beta_S} &= 1. \end{aligned} \quad (145)$$

For example, $0.7 + 0.3 = 1$. The superposition of the outer and inner curves gives the following formula for the principal axes through Z into the floor, the roof and along the strike respectively:

$$\begin{aligned} v_{z_L} &= v_{zmax} g_{\gamma_L} \left[1 - \left(\frac{r_L}{h} \right)^2 \right]^2 + v_{zmax} g_{\beta_L} \left[1 - \left(\frac{r_L}{n_1} \right)^2 \right]^2 \\ v_{z_H} &= v_{zmax} g_{\gamma_H} \left[1 - \left(\frac{r_H}{o} \right)^2 \right]^2 + v_{zmax} g_{\beta_H} \left[1 - \left(\frac{r_H}{o_1} \right)^2 \right]^2 \\ v_{z_S} &= v_{zmax} g_{\gamma_S} \left[1 - \left(\frac{r_S}{p} \right)^2 \right]^2 + v_{zmax} g_{\beta_S} \left[1 - \left(\frac{r_S}{p_1} \right)^2 \right]^2. \end{aligned} \quad (146)$$

The subsidence v_z of a given point P lying outside the principal axes can be determined in the following way. First, the subsidence v_{zS} of the base point P_S of P along the line of strike through Z is obtained by Eq. (146). This subsidence value is then introduced in place of v_{zmax} into the relevant Eq. (146) relating to the hanging-wall or footwall part of the trough according to the position of the point P.

The peak subsidence value v_{zmax} is derived from the condition that the volume of the subsidence trough at the surface must equal the volume of the mine excavation underground, using the equation

$$v_{zmax} = \frac{aM [y_1(x_2 - x_4) + y_2(x_3 - x_1) + y_3(x_4 - x_2) + y_4(x_1 - x_3)]}{1.138 \cos \alpha (g_{\gamma_L} n + g_{\beta_L} n_1 + g_{\gamma_H} o + g_{\beta_H} o_1) (g_{\gamma_S} p + g_{\beta_S} p_1)} \quad (147)$$

the numerator of which includes the coordinates of the four corners of the extraction area. In steep-lying stratification it is advisable to use the formula for the area of a trapezium:

$$\begin{aligned} v_{zmax} &= \\ &= \frac{aM(H_3 + H_4 - H_1 - H_2) [\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} + \sqrt{(x_4 - x_3)^2 + (y_4 - y_3)^2}]}{2 \sin \alpha 1.138 (g_{\gamma_L} n + g_{\beta_L} n_1 + g_{\gamma_H} o + g_{\beta_H} o_1) (g_{\gamma_S} p + g_{\beta_S} p_1)}. \end{aligned} \quad (148)$$

The location of this point of greatest subsidence v_{zmax} , the intersection point Z of the axes, lies at a distance of

$$l = \frac{H_1 + H_2 + H_3 + H_4}{4} \tan v = H_s \tan v \quad (149)$$

in the direction of dip from the centre point S of the extraction area, which is itself given by the arithmetical mean of the corner-point coordinates. The distances n , o , and p from the point Z to the trough margin, and n_1 , o_1 , and p_1 from Z to the surface projection of the angle of break, can be read off a sectional drawing, provided the size of the limit angle and the size of the angle of break are known; or they may be calculated by trigonometry, in which case the diagonal face shown in Fig. 165 may be adjusted to a right-angled side.

To effect the subsidence calculation by computer, the data are punched on cards, with separate cards for every extraction area and for every calculation point at the surface. A third punched card should be prepared for data characterizing the trough shape (weightings and the limit, break, and zenith angles). The extraction-area card should contain, besides the coordinates and depths of the corner points, also the thickness of seam worked and the stowage factor. The programme is fed into the computer by means of a punched tape.

The use of weight coefficients enables profile curves to be adjusted to values as measured locally and to any irregularities (e.g., the effects of repeated working) more comprehensively than is possible with other computer programmes. Enlargement of the programme to cover other components of ground movement can be effected without difficulty – for example, by taking the displacement v_x as being proportional to the tilt v'_z .

8.2.7 The Empirical Integration-grid Method

A last empirical method that should be mentioned is a graphical method for flattening measures which does without section drawings and mathematical formulae, and which, unlike the profile methods, can be applied also to non-rectangular, irregularly shaped extraction areas (Keinhorst, 1925; Korfmann, 1957). This is the integration-grid method, employing a grid (see Fig. 67) derived directly from observed subsidence movements. First, a critical extraction area with the radius $R = H \cot \gamma$ is drawn on transparent tracing-paper and divided into a network of annular zones, sectors or rectangular fields of any desired size. The influence of individual areas of this extraction "grid" on a surface point lying over the centre of the grid can be determined by laying the grid plan with its centre placed successively on the surface observation points P_i marked on the mine plan, and relating the segments A'_i of extraction area lying within the individual grid fields A_i to the subsidence measurements v_{zi} (end-state) observed at the relevant point P_i (Fig. 166). The required subsidence influence e_i of the individual fields in the grid is then obtained from the subsidence equations applicable to the measured final subsidence at each point (in this case ten fields and n points) with the aid of the coefficients $a_i = c_1 A'_i / A_1$, $b_i = c_2 A'_2 / A_2 \dots j_i = c_{10} A'_{10} / A_{10}$ (where c_i = convergence in the zones of the working):

[illegible]

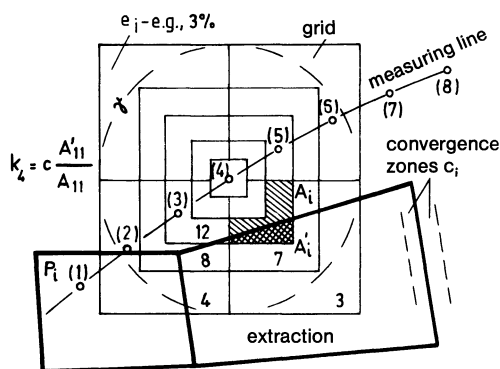


Fig. 166. Evaluating the influence of 20 grid fields. The segments of the extraction area having an influence on point (4) in the measuring line are those corresponding to grid fields 3, 4, 7, 8, 11 and 12

Contradictions in these equations arising through errors of measurement and irregular subsidence can be smoothed out by means of a Gaussian elimination procedure (by algorithm; Meyers and Drent, 1953). An integration grid evaluated in this way can be redrawn – if the subsidence half-value coincides with the ribside – by interpolation in ring zones of equal value, to lighten the task of counting up the shares of influence.

Evaluation of several grid fields in a single calculation process, as described, offers advantages over individual evaluation starting at the margin zone, because the considerable error which generally affects an evaluation of the outer zone is transmitted to the inner zones.

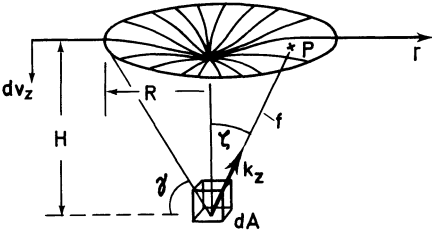
8.3 Methods Employing Influence Functions

It is common to all methods in this group⁴ that they are based on so-called influence functions, which are used to describe the amount of influence k_z exerted at the surface by infinitesimal elements of the extraction area. The element dA within a relatively large extraction area at depth H (Fig. 167) creates at the surface a very flat trough (an “*elementary trough*”), the deepest point of which lies vertically over the extraction element and the margin of which is determined by the limit angle γ ($R = H \cot \gamma$). The very small amount of subsidence dv_z of a point P moving radially within the elementary trough can be stated as a function $k_z(\zeta)$ of the zone angle ζ , or as a function $k_z(r)$ of the radial distance r from the centre of the trough, or as a function $k_z(f)$ of the oblique distance f from the extraction element to the surface point P . In some EDP programmes the elementary troughs of all extraction elements are added together according to their position.

Conversely, it is also possible to start from the surface point P and the critical area of an extraction at depth H (Fig. 168) and find out the contribution k_z , or de/dA , made by the individual extraction elements dA_i to the subsidence v_z of the

⁴ The group can be divided into graphical methods using integration grids, and numerical methods using tables of functional values or EDP programmes.

Fig. 167. The elementary trough created at the surface by the extraction element dA (after Neuhaus, 1976), assuming rotational symmetry



surface point P. In Fig. 168, for example, the extraction element dA_1 at the margin of the critical area will have less influence on the surface point P than the central element dA_2 , since it is at a greater distance and a greater angle ζ from the point P. This decrease in the influence k_z (in %) on the central surface point P exerted by an extraction element as the radial distance r increases – from a maximum 100% or 1 at $r = 0$ directly under P to zero at $r = R$ – is expressed by the bell-shaped distribution or *influence function* $k_z(r)$. For subsidence calculation

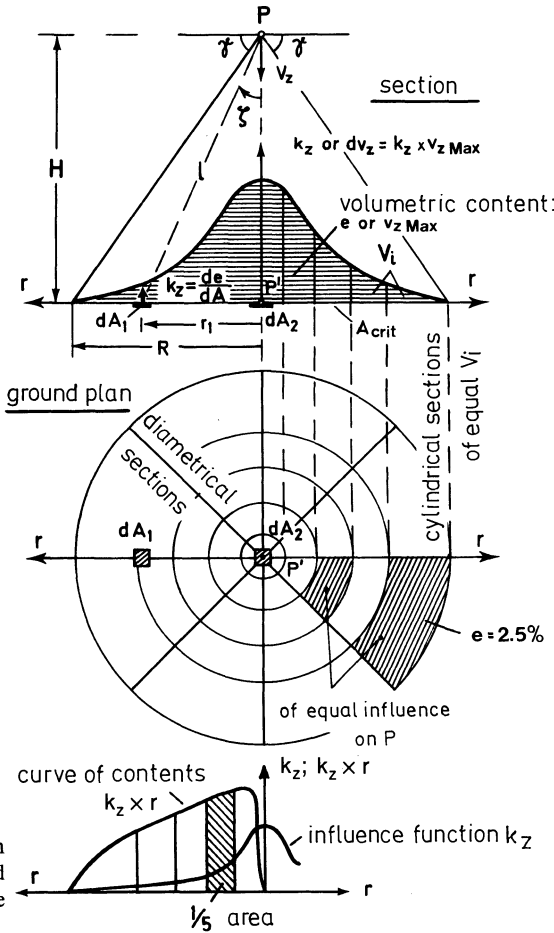


Fig. 168. Construction of an integration grid by dividing up the bell-shaped shares-of-influence body (*top*) on the basis of equal contents

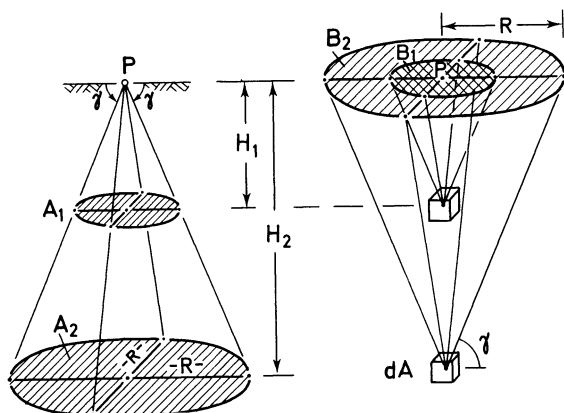


Fig. 169. The limit-angle principle. *Left:* extraction areas A_1 and A_2 acting on the point P. *Right:* zones of influence B_1 and B_2 of an extraction element dA at depths H_1 and H_2

by graphical methods, the infinitesimal contributions to subsidence are in most calculation procedures combined into ring zones of equal influence (e.g., $e = 20\%$), to produce an integration grid from which subsidence can be found, point by point, by simple estimation of areas.

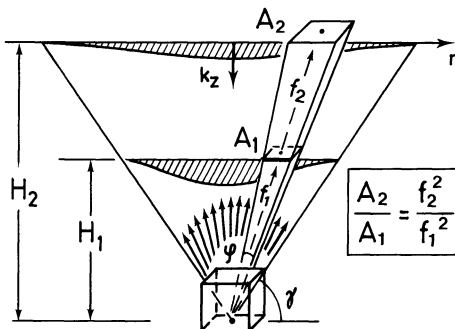
Methods employing influence functions – the first appeared in 1932 (Bals) – are up to now predominant in Germany and Poland. They have the advantage of not being confined to rectangular extraction areas like the profile-curve methods. Parts of the extraction area can be evaluated separately (margin zones, zones of reduced seam thickness, changes of stowing material); and calculations for surface points lying outside the principal sections on a diameter can be made directly, without interpolation.

8.3.1 The Theoretical Basis

The influence-function method is based on seven assumptions, which simplify the calculation process and make it universally applicable:

a) The limit-angle principle. In Sect. 7.21 it was explained, with the aid of Figs. 125 to 127, that the width of the trough over an extraction area is delimited by straight lines from the edge of the extraction, which are inclined at a constant angle γ , the empirical limit angle, regardless of the depth and size of the working. The relationship between the area of working and trough must apply equally, if the limit angle is independent of the size of working, to the smallest fraction of the trough – the surface point P – and of the extraction area – the extraction element dA (Fig. 169). The extraction areas A_1 , A_2 , etc. influencing equally the surface point P grow in size in linear relationship to the depth of working H_1 , H_2 , etc. within the limit-angle cone. At the same time the trough B_1 or B_2 produced at the surface by the extraction element dA grows larger with constant volume in proportion to the depth H_1 and H_2 , in such a way that A_1 is equal to B_1 and A_2 equals B_2 . From the straightness of the limit-angle lines, and from the fact that the apex angle of the influence cone ($200^\circ - 2\gamma$) is independent of the depth of

Fig. 170. The extraction element's radiating influence on the horizons H_1 and H_2



the extraction element, it follows that the influence itself radiates outwards according to the law of central projection (Fig. 170).

As with every force or radiation emanating from a central point (cf. the intensity of light), the influence of the extraction element on a constant area within the limit-angle cone decreases inversely with the square of the distance. That applies equally to each partial cone with an apex angle of φ . The influences k_{z1} and k_{z2} per 1 cm^2 on the areas A_1 and A_2 at the distances (i. e., horizons) H_1 and H_2 develop as follows:

$$\frac{k_{z1}}{k_{z2}} = \frac{A_2}{A_1} = \frac{f_2^2}{f_1^2}. \quad (151)$$

This *law of radiation*, with

$$\frac{1}{f^2} = \frac{1}{r^2 + H^2} = \frac{1}{H^2} \frac{1}{1 + \left(\frac{r}{H}\right)^2} \quad (152)$$

for the influence k_z of the extraction element, can be written in the form

$$k_z(r) = \frac{1}{H^2} g \left[1 + \left(\frac{r}{H}\right)^2 \right]^{-1} \quad (153)$$

(Neuhaus, 1976), where g is the still unknown function for the decrease in influence k_z , from the maximum $dv_{z\max} = 100\%$ for $\zeta = 0$ to zero for $\zeta = 100 \text{ gr} - \gamma$ (Fig. 167). Thus each influence function⁵ $g(r/H)^2$ fulfils the principle that the influence cone and its partial cones are independent of depth – i. e., confirms the constant apex-angle φ and thereby limit-angle principle.

b) The principle of equivalence. For all extraction elements dA_1 of equal depth H_1 or H_2 (Fig. 171), there is an assumption of equal influence cones and also of equal influence functions. All extraction elements of a seam are therefore equivalent in their influence on the surface. In the direction of a given zone angle ζ ,

⁵ For example, influence function $k_z(r) = \exp -(\lambda r^2/H^2)$ (after Neuhaus, 1972), with $\lambda/H^2 = 0.5$ (after Ehrhardt and Sauer, 1961).

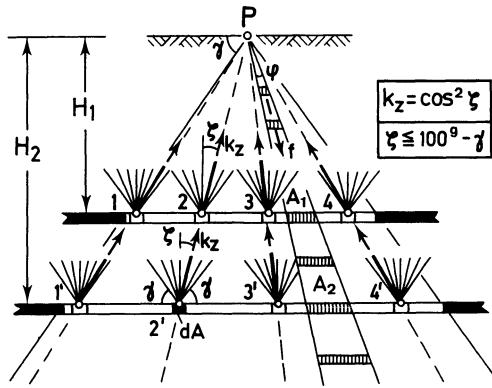


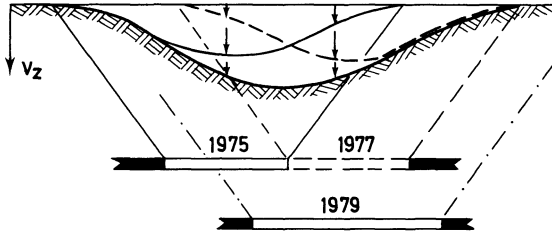
Fig. 171. The principle of equivalence. All the extraction elements 1 to 4, or 1' to 4', have the same radiating influence on the surface

the radiating influence $k_z(\zeta)$ is the same for all extraction elements. For example, the influence of the extraction elements 1 and 1' at the depths H_1 and H_2 on the point P is equal (in % of $dv_{z1\max}$ and $dv_{z1'\max}$ respectively), because the zone angles in the direction of P are equal. The same applies to the elements 2 and 2', 3 and 3', and 4 and 4'. Within a cone φ emanating from the surface point P, all extraction areas A_1 , A_2 , etc. act equally upon P, since they increase in size with the square of their distance f from P.

c) The principle of rotational symmetry. In a horizontally bedded and isotropic rock mass which is free of geological faults, the influence of an extraction element at every zone angle ζ can be taken as identical in all horizontal directions (Fig. 167). The elementary trough, described horizontally by a circle, arises through rotation of its profile curve or its k_z function (Fig. 170). Similarly, all extraction elements which lie at the same zone angle ζ from a surface point P have, in all horizontal directions, an equal influence on the point P (Figs. 168 and 171). From the rotational symmetry of influence it follows that subsidence over the centre of the ribside of a square critical area (points A, B, C, and D in Fig. 128) must be equal to half the critical (full) subsidence, since only 50% of the equivalent extraction elements act on this surface point. The bell-shaped figure in Fig. 168 representing shares of influence is halved on extraction up to P'.

d) The principle of superposition. For the sake of simplicity it is assumed that each element of the extraction area within the limit-angle cone acts upon the relevant surface point P unaffected by neighbouring elements (Fig. 171). The individual influence k_{zi} on the point P exerted by the extraction elements are superimposed on each other cumulatively to produce the final subsidence v_z . This applies also to several extraction areas of a single seam or of a number of seams. The trough is calculated for each working panel (e. g., 1975 and 1977 in Fig. 172), and the subsidence of each is added together to give a combined trough, which would have been obtained in the same size and shape if the subsidence for the 1975 and 1977 extraction areas combined had been calculated at one go. Even the order in which the extraction influences are superimposed has no effect on the final result (the law of commutation).

Fig. 172. The principle of superposition: the adding together of the subsidences of 1975 and 1977



e) The principle of reciprocity. From the foregoing assumptions it follows necessarily that the influence function coincides with the function of an elementary trough in the profile. Only the variables differ. From the point of view of the result it is therefore immaterial whether one calculates the three-dimensional shares-of-influence body from an influence function and constructs an integration grid (the graphical procedure, Fig. 168), or calculates the elementary trough belonging to an extraction element and combines the elementary troughs of all extraction elements into a complete trough (the numerical procedure, Fig. 199). The profile of the elementary trough is thus the mirror image of the influence function, as is evident from Fig. 173.

The extraction elements dA_1 to dA_5 of the critical extraction area act on the central surface point P with the values k_1 to k_5 in accordance with the influence function $k_z(P)$ shown at horizon (3), and do so at the zone angles ζ_1 to ζ_5 . On the principle of the equivalence of all extraction elements at horizon (2), the central extraction element dA_3 exerts the same influences k_1 to k_5 , at the zone angles ζ_1 to ζ_5 , on the points e, d, P, c , and b at the surface (1) as the extraction elements dA_1 to dA_5 have on the point P . The elementary trough at the surface is obtained by

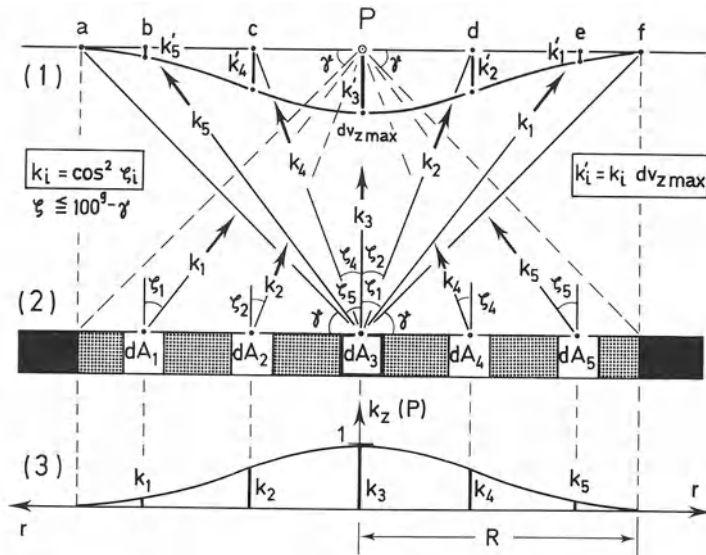


Fig. 173. The principle of reciprocity: deriving the elementary trough (1) for the extraction element dA_3 from the influence function (3) over the working panel (2)

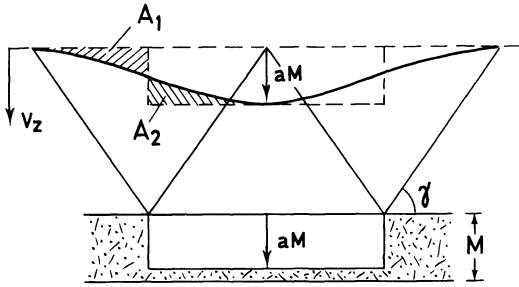


Fig. 174. The principle of volume constancy: trough volume = volume of convergence

marking off the subsidence values k'_i against the points e to b, equating $k_3 = 1$ with the largest amount of subsidence $dv_{z\max}$ in the elementary trough. At the points a and f, which lie towards the limit angle γ , elementary subsidence is nil. In this way a trough line similar to the influence function is derived from that function.

f) The principle of constancy of volume. By Eq. (16), subsidence and all other components of the trough stand in linear relationship to the thickness of seam mined M and to a factor a which depends chiefly on the manner of stowing. In addition, the cubic content of the trough at the surface has to be equal to the volume of convergence in the workings. Taking the critical extraction area as an example, it can easily be shown (Fig. 174) that this condition is met if uniform convergence of aM is assumed throughout the working, and if subsidence of the ground above the ribside, at the transition point of the profile curve, is equal to one-half of full subsidence aM . Given equal convex and concave curvature in the trough profile, A_1 will then be equal to A_2 .

g) The principle of transitivity. The trough over a critical extraction area must be derivable from the troughs over smaller, or subcritical areas; conversely, the troughs of subcritical areas must be derivable from the critical-area trough by the same law. In Fig. 175, using the normal subsidence curve (thick line) over a square critical-extraction area, the critical-profile line of the trough for the diametrical section 0–10–0 is drawn. From this critical subsidence trough the trough profiles for the subcritical areas up to strips II and V have to be derived graphically. The line emanating from point 3 at the limit angle intersects three extraction strips, the last of which has not been mined by stage II of the extraction programme. As compared with the critical extraction profile, this implies a reduction of the subsidence at point 3 corresponding to the subsidence at point 1 (normal subsidence) in the critical subsidence trough ($3 - II = 1$). By stage V of the extraction programme the three strips exerting an influence have been mined, and subsidence at point 3 is as much as in the critical-area trough. The reduction in subsidence then amounts to $5 - V = 0$. At stage II of the extraction, the three outer strips, Nos. III, IV and V, are not acting on point 5, as they would be in a critical-area trough. Critical subsidence at point 5 must therefore be reduced by the amount of the normal subsidence at point 3 (since $5 - II = 3$). At point 7 the influence of the five outer strips is absent at stage II, and of the two outer strips at stage V. Consequently, the critical subsidence trough at that point must be

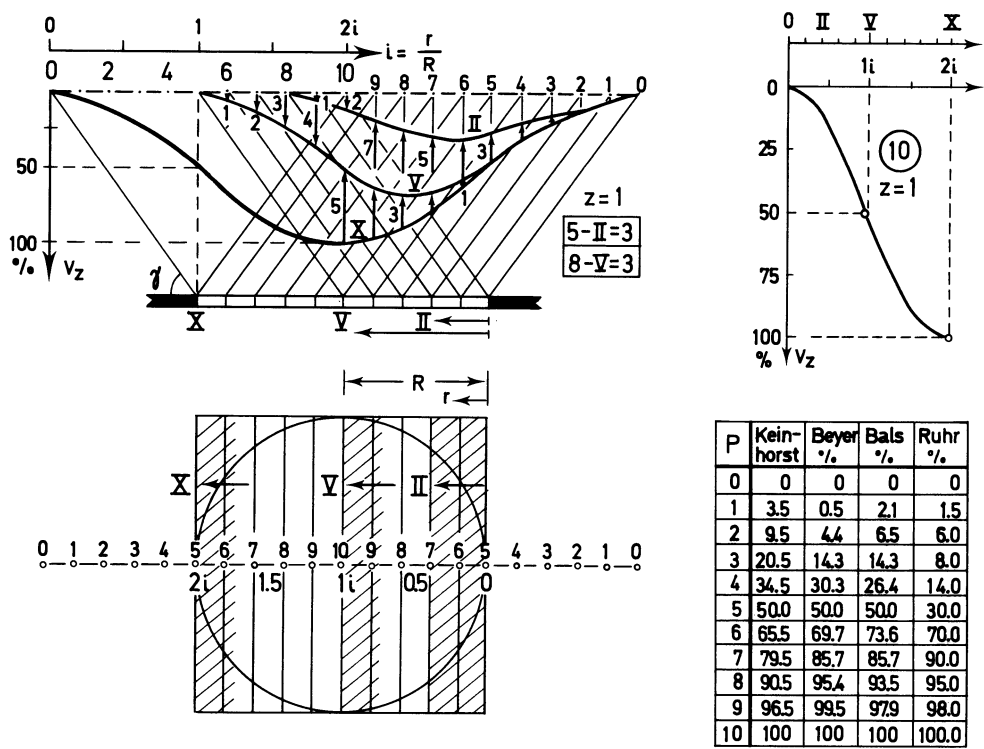


Fig. 175. The principle of transitivity: deriving subcritical-area troughs from the critical-area trough, and vice versa. *Top:* cross-section. *Bottom:* ground plan with extraction area and line of measuring points. *Right:* normal subsidence curve and table of values for it, expressed as percentages of critical subsidence

reduced by the amount of the normal subsidence at point 5 (i. e., $7 - \text{II} = 5$) or at point 2 (i. e., $7 - \text{V} = 2$); and so on. Subcritical troughs can thus be derived from the given curve of normal subsidence. In the same way, subsidence lines in the rock body can be derived from one horizon to the next (Fig. 39), and dynamic troughs projected from one time stage to another with the aid of a time factor.

In practice, these *seven assumptions* are not wholly correct. For example, the critical areas acting on the point *P* are delimited, according to the latest observations, not by straight lines but by lines curved in accordance with

$$R = c\sqrt{H} \tag{154}$$

(Szpetkowski, 1974, and Fig. 95), which correspond to influence functions of the form

$$k_z(r) = \frac{1}{H} \, g\left(\frac{r}{H}\right) \tag{155}$$

(Neuhaus, 1976). The principle of equivalence ignores the fact that convergence in the extraction element depends on convergence in the neighbouring element

and also on the size of the extraction panel, since vertical pressure increases towards the centre of the panel and with the size of the extraction area (Fig. 112). The partly plastic property of the fissured rock mass leads, even in horizontal stratification, to unsymmetrical troughs, depending on the direction of working or the direction of fissuring. Unimpeded superposition also does not hold true in repeatedly worked strata, as detailed in Sect. 6.5. Over the dividing line of two panels worked one after the other there remains, contrary to Fig. 172, a residual ground wave (see Fig. 150a). Even in an elastically isotropic continuum, the extraction of a second, equally thick seam does not produce double the depth of trough, according to the finite-elements method – a fact which is also clear from Fig. 150e for extractions A and B. Thus, from a linear material law after Hooke, it does not necessarily follow that the boundary conditions of the linear superposition of influences will apply. In the pressure shadow of an upper working, convergence in the lower panel is less great (Fig. 105), and so the commutative law also does not apply here. The principle of volume constancy ignores the fact that, in unloosened strata, the increase in volume from vertical expansion over the working can be greater than the decrease in volume from vertical compression over the unworked seam (Fig. 116, left), and secondly that, in already loosened strata, readjustment in the position of blocks of strata can lead to a trough volume which is greater than the volume of convergence in the working (Fig. 116, right). And because of the way that the rock fabric and the fields of stress alter during mining activity, transitivity is also, strictly speaking, not assured.

Despite these contradictions, the calculation method employing influence functions has proved itself in practice. With certain additional assumptions and weighting functions the method, which is based essentially on linearity, could be adapted to the non-linear behaviour of a fissured rock mass (Sect. 8.37).

8.3.2 The Graphical Method in Flat-lying Measures

The graphical method using an integration grid proceeds from the limit angle and the critical area of extraction defined with that angle [Eq. (161)]. The critical area, which in flat-lying strata is circular, is notionally divided into the smallest possible plane elements, so as to be able to apportion to those plane elements dA a subsidence influence k_z on the central surface point P which depends on their position (Fig. 168). Experience shows that this subsidence influence – written as the differential quotient $de/dA = k_z$ – decreases steadily with increasing distance r between the plane element dA and the centre P' of the critical area. This localized, rotationally symmetrical distribution of subsidence influence to the individual extraction elements within the critical area is expressed in the influence functions $k(r)$ of the various calculation procedures. The influence function is sometimes also stated in the form $k_z(i)$ or $k_z(\zeta)$, as a function of the relation $i = r/R$ or of the zonal angle ζ , where

$$\tan \zeta = \frac{r}{H}. \quad (156)$$

If the differential quotients de/dA , calculated in accordance with the influence function, are plotted as prisms at the appropriate position on the critical extraction area A_{crit} , a bell-shaped figure will be formed (see Fig. 195), which coincides with the rotational figure of the diametrical distribution curve (Beyer, 1944). Its volumetric content corresponds to the *factor of influence* or effect $e = 1$, thus:

$$\int_{A_{crit}} \frac{de}{dA} dA = 1. \quad (157)$$

As with this figure depicting influence, a figure depicting shares of subsidence, enlarged by the standard factor v_{zMax} , can be formed from the subsidence differentials dv_z of the plane elements dA , on account of

$$\frac{dv_z}{dA} = k_z v_{zMax}. \quad (158)$$

The volumetric content of this figure can be equated to the subsidence of the point P:

$$\int_{A_{crit}} \frac{dv_z}{dA} dA = v_{zMax}. \quad (159)$$

The influence function k_z thus also defines the infinitesimal change in subsidence dv_z in the vicinity of the central surface point P (Fig. 168) – and therefore the amount of tilt dv_z/dx [see Eqs. (158) and (166)] if the very narrow working ($dA = dx dy$, with $dy = 1$) advances along the x-axis by dx .

Accordingly, in Fig. 195, the hatched area A represents the tilt v'_z at the point P given an extracted area $x_2 - D - E - x_1$, which can be obtained by integration of the subsidence differentials dv_z along the rectilinear working face $x_1 - x_2$. For the single extraction element, on the other hand, it is not the functional value $k_z(r)$ but the tangents dk_z/dr which give the slope of the subsidence – i. e., the tilt at P given an imaginary infinitesimal displacement of the point P by the amount dr . In sum, the integration grid represents the ground plan of the influence figure, as divided up by vertical cylindrical sections and diametrical sections into zones and sectors of equal volumetric content V_i (Fig. 168).

To draw such an *integration grid*, it is usual to present the annular increase in the cubic content of the influence figure, taken from the centre outwards, by an integral curve (line of summation)

$$K_z = 2\pi \int_0^R r k_z(r) dr \quad (160)$$

letting the upper integration limit run through all values from zero to the critical area radius R ⁶. A mathematical solution of this problem, for an exponential distribution function, can be found in the relevant literature (Bräuner, 1961; Ehrhardt and Sauer, 1951). In the one example, the volumetric content of the

⁶ $K_z = e = \int_{A_{crit}} \frac{de}{dA} dA$ with $de/dA = k_z(r)$ and – see Fig. 222 – $dA = dr(r d\delta)$.

shares-of-influence figure is obtained by summation of infinitely small rings ($2\pi r dr$) of width dr and height k_z , using Eq. (160); in the other, the volume of a shares-of-influence figure divided up by cylindrical sections is calculated from a cylindrical base ($\pi r^2 k_z$) and the bell-shape outline resting on it, with the bell shape regarded as being made up of infinitely thin horizontal slices of height dk_z and ground area πr^2 . In a third example (Neuhaus, 1974), the area enclosed by the curve $rk_z(r)$ – namely, the influence function multiplied by the radii r_i – and the r -axis, which corresponds to the volume of the shares-of-influence figure, is divided up graphically into five sections of equal size (Fig. 168, bottom).

The K_z -diagram thus gives, on the ordinate, the finite subsidence influences e_i of the annular extraction areas on the central surface point P , as a function of the radius r or of the progress in an extraction operation being conducted concentrically outwards from P' to the edge of the critical area (Fig. 176). An integral curve for an extraction of the critical area in strips from left to right can similarly be derived from the distribution function. If the ordinate is divided into, for example, five equal sections of $\Delta e = 20\%$ each, then the zonal radii $i = r/R$, related to the critical-area radius, can be read off the diagram as the appropriate abscissa values for an integration grid with five equivalent influence zones and then converted to the scale of the mine plan. To facilitate the estimation of areas, the zones are divided into eight sectors with equal angles of 50° at the centre (Fig. 168).

The mining of a large zonal sector at the margin of the critical area thus causes the same amount of subsidence ($2.5\% aM$) at the surface point P as the mining of a comparatively small sector in the inner zone. Because of their appearance, integration grids are also known as “spiders’ webs”. In the sectional drawing (Fig. 176, right), the terminal points of radii plotted at the depth H_1 are linked with the surface point P by zonal rays, to facilitate the finding of zonal

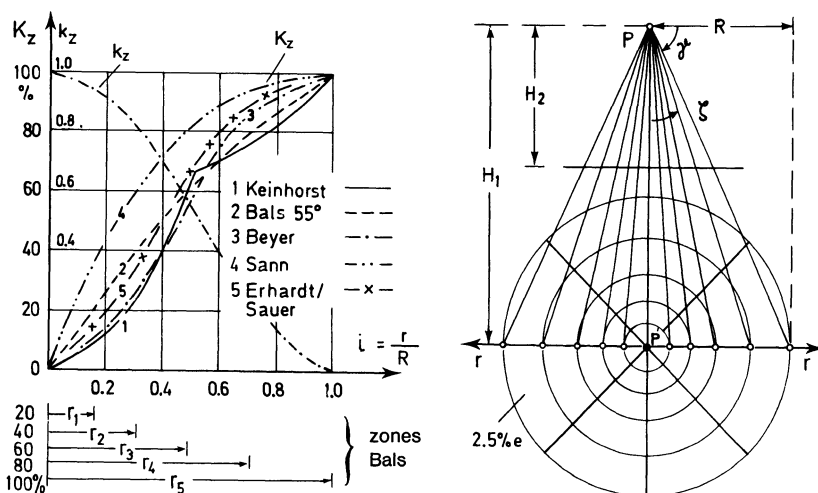


Fig. 176. Left: diagram of the influence function k_z (Beyer) and of various integral curves K_z . Right: construction of an integration grid (Bals) for the depth H_1

radii for given depths (e.g., H_2). The outer radius of the grid bears the following proportion to the mining depth H :

$$R = H \cot \gamma. \quad (161)$$

In the process of calculating subsidence, the integration grid, drawn on transparent paper, is laid with its centre point P' covering the surface point P as projected on to the ground plan of the mine (Fig. 67). The parts of the working area covered in whole or in part by individual zonal sectors of the grid are then counted, and each of the 40 zonal sectors is allocated a share of influence amounting to $\Delta e = 100\% \div 40 = 2.5\%$ of full subsidence. On extraction of the whole critical area, e will equal 100%, and the central surface point P will be lowered by the greatest possible amount, i.e. by full or *critical subsidence* – the product of the subsidence factor a and the thickness of seam mined M [see Eq. (16)]:

$$v_{z\text{Max}} = aM. \quad (162)$$

On extraction of a smaller area – a so-called subcritical area – subsidence of the central surface point in the integration grid, corresponding to the ascertained influence factor $e = 1$, will amount to

$$v_z = eaM. \quad (163)$$

The *subsidence factor* a is defined as the relationship between full subsidence and the thickness of seam mined. Strictly speaking, thickness of seam mined M must here be taken as meaning the height, perpendicular to the stratification, of all beds (coal, partings, country rock) in their state when first opened up – i.e. before initial convergence – in other words, everything that is loaded out from the face. Material selected at the face for use as wastes for stowing is not included (see Fig. 5).

Dynamic subsidence $v_{z\text{dyn}}$ before the final state of ground movement (ground stability) can be derived with an empirical time factor $z \leq 1$ from the final value as follows:

$$v_{z\text{dyn}} = z v_{z\text{end}} = eaMz. \quad (164)$$

An example of *graphical calculation* with the integration grid has already been given in Sect. 3.31 (Fig. 67), and Fig. 177 gives an example of calculating subsidence in mining salt by chamber-and-pillar working. Workings lying within the limit-angle cone $\gamma = 60^\circ$, with equal safety coefficients n , are designated as areas A_1 , A_2 , and A_3 and distinguished by different types of hatching. For panels with the same safety coefficient n (i.e. pillar limit-stress divided by specific pillar load), the waste of mineral, the type of stowing, and the average working heights are generally the same and can consequently be grouped together for purposes of calculating subsidence influence. The subsidence factors a_i of 0.3 to 0.6 applicable to these areas A_i take account of the manner of stowing (in this case caving) and of the waste of mineral (38% – 46%). The influence factors e_1 , e_2 , and e_3 for the three areas are estimated with the aid of the integration grid after Bals. Each ring zone contributes a share of $e = 20\%$ to the full subsidence of the

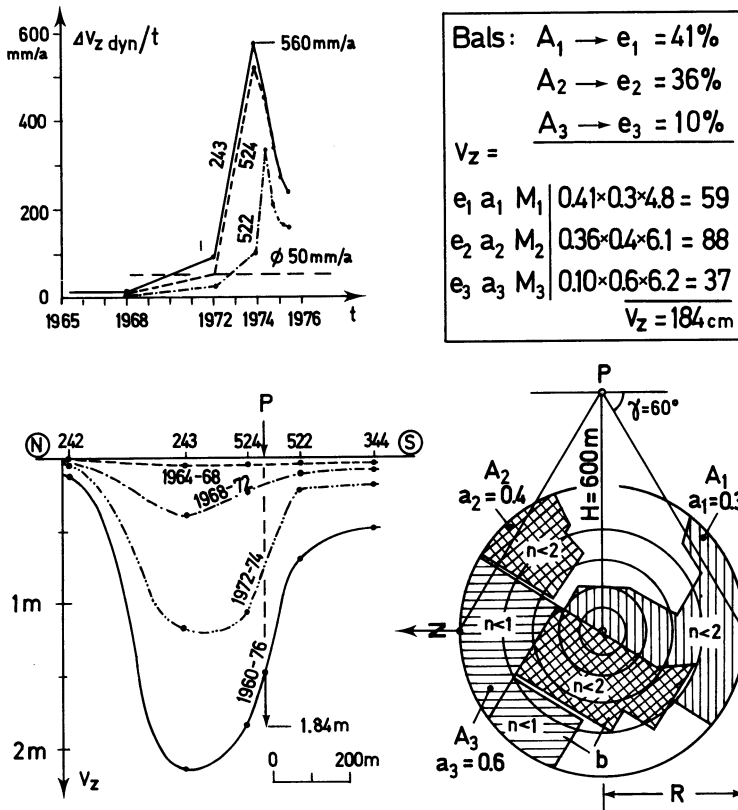


Fig. 177. Left: rate and amplitude of ground subsidence over an initially undersized chamber-and-pillar working in hard salt (sylvite). (After Bonk and Raue, 1976). Right: calculation of final subsidence at the point P by the Bals method. *b* subsequent hydraulic stowing

central point P. The heights of the chambers, corresponding to the thickness mined M , are read off from the mine plan. All the factors a_i , e_i , and M_i are now known, and consequently, by Eq. (163), the subsidence of 59 cm, 88 cm, and 37 cm induced at point P by the three working panels can be calculated and added together to give the total subsidence $v_z = 184$ cm on the return of ground stability ($z = 1$). In the trough profile last measured in 1976 this final situation at the point P had already been reached to 80%.

In Table 12 the zonal radii of a few integration grids in flat-lying stratification are compared. For these grids the following *influence functions* $k_z(r)$ or $k_z(\zeta)$ are applicable for the subsidence influence of infinitesimal portions of extraction area in the diametrical seam section, as also are their K_z equations for the increase in ultimate subsidence influence on the surface point P, given concentric working of the critical area from the centre outwards:

$$\left. \begin{aligned} k_z &= \cos^2 \zeta \\ K_z &= 0.25 (\sin 2\zeta + 2\zeta) \end{aligned} \right\} \text{ (Bals, 1932)} \quad (165)$$

Table 12. Zonal radii for annular areas of the integration grid having equal influence ($R = 1$, limit angle 55°)

Zone (from the centre outwards)	Subsidence				Displacement		
	Bals	Beyer	Sann	Ehrhardt, Sauer	Flaschen- träger, Perz	Beyer	Bräuner
1	0.16	0.25	0.09	0.22	0.38	0.34	0.33
2	0.30	0.39	0.18	0.33	0.56	0.46	0.49
3	0.49	0.50	0.30	0.45	0.71	0.58	0.63
4	0.70	0.63	0.45	0.59	0.85	0.71	0.80
5	1.00	1.00	1.00	1.00	1.00	1.00	1.00

$$\left. \begin{aligned} k_z &= k(1 - i^2)^2 \\ K_z &= 2\pi k R^2 \left(i^2 - i^4 + \frac{i^6}{3} \right) \end{aligned} \right\} \text{ (Beyer, 1944)} \quad (166)$$

$$\left. \begin{aligned} k_z &= 2.256 \frac{1}{r} e^{-4r^2} \\ K_z &= 2\pi 2.256 (i - 1.33i^3 + 1.6i^5 - 1.523i^7 + 1.185i^9 - \dots) \end{aligned} \right\} \text{ (Sann, 1949)}^7 \quad (167)$$

$$\left. \begin{aligned} k_z &= 0.1392 e^{-0.5r^2} \\ K_z &= 2\pi (1 - e^{-0.5r^2}) \end{aligned} \right\} \text{ (Ehrhardt and Sauer, 1961)} \quad (168)$$

Influence functions are either derived analytically from observations or are based on assumptions. At the Emscher Association, for example, the conclusion was reached that the outer ring of the critical extraction area, lying between the limit angle and the angle of break, accounted for one-third of the total subsidence influence e , and the circular zone within the cone described by the angle of break ($\beta = 78^\circ$ or 70°) for two-thirds (the two-zone grid of Keinhorst). With other methods it is assumed that an element of extraction area acts on the central surface point with a force in accordance with the Newtonian gravitational law ($1/f^2$ in Fig. 170, after Bals, 1932) or in accordance with other selected functions (Beyer, Sann et al.). Finally, the subsidence process can be imagined as consisting in the surface point P, together with an upper plane A of the limit-angle cone, being subsided by the full critical amount 1, this plane A being subsided as to one-half each by the two equal-sized conical bases in the next lower plane B, which in its turn is caused to subside as to one-half each by the conical bases in plane C, and so on (Fig. 178, left) — a process which continues to the mining horizon by a Gaussian distribution function (Ehrhardt and Sauer, 1961). Conversely, if one starts from an extraction element with a subsidence volume of

⁷ If the exponent of r in the e function ($e = 2.718$) is raised from 2 to 4 or 6, the inner zones of the integration grid become larger (Fig. 180). The element position is expressed as $i = r/R$.

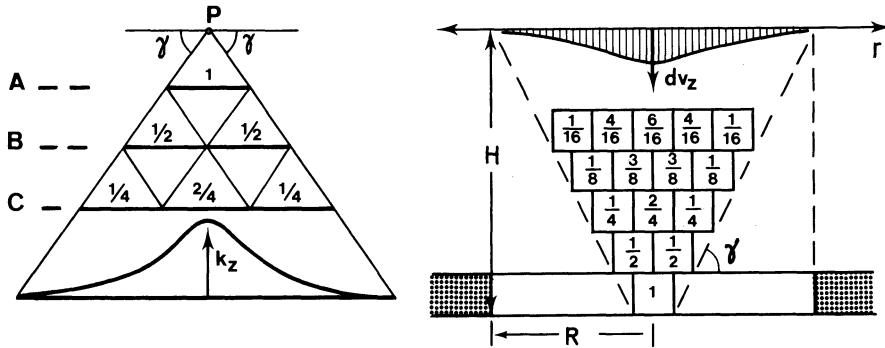


Fig. 178. The stochastic theory of subsidence distribution. Bell-shaped influence curve (left) and elementary trough (right)

1 and assumes half the subsidence movement from each of two overlying strata elements, as in the stochastic theory (Fig. 73), one obtains a bell-shaped profile curve of the elementary trough at the surface as a mirror image of the influence function (Fig. 178, right).

The influence functions must fulfil the *boundary condition* that, for values of

$$\zeta = 100 - \gamma, \quad \text{or} \quad i = 1 \quad (\text{margin}) \text{ and}$$

$$\zeta > 100 - \gamma, \quad \text{or} \quad i > 1 \quad (\text{supercritical area})$$

the functional value shall have a minimum of 0, and for values of

$$\zeta = 0, \quad \text{or} \quad i = 0 \quad (\text{centre}),$$

the functional value shall have a maximum of 1. For example, function 166 yields, for values of $i > 1$ (supercritical area), an influence capacity of the extraction elements rising from zero with $i = 1$. It is therefore only valid for i -values from 0 to 1. Other functions, such as Eq. (168), follow an asymptotic course towards the exterior, with the result that a critical-area extraction produces only 96%–99% of full subsidence at the central surface point. A “practical limit angle”, on the basis of which a working produces, say, 99% of full subsidence, must therefore be selected.

The comparison in Fig. 179 of normal subsidence arising over a square critical-extraction area in the principal x -axis along the strike, as calculated by means of the various grids, shows a great similarity between the curves which are antimetrical to the half-subsidence and transition point. On extraction of a subcritical area, on the other hand, the trough profiles produced by the various calculation procedures differ considerably. Thus, for example, over a working 0.4 R wide and 2 R long, which at a depth of 700 m corresponds to a panel of approximately 200 m by 1000 m, subsidence over the ribside varies between 35% and 45% of critical subsidence, and between 63% and 71% of maximum subsidence. Equation (167) (Sann) gives a deep subcritical trough, and Eq. (166) (Beyer) a shallow one. A marginal zone showing less compaction of stowing material was not considered in these comparisons. Figure 180 compares the zones into which several integration grids are divided.

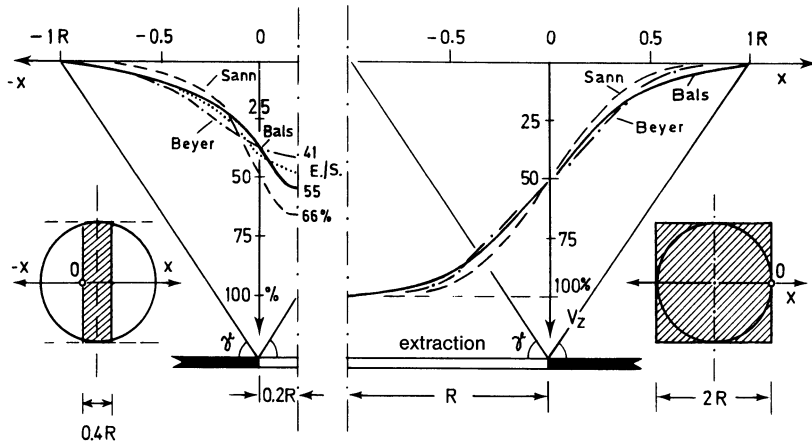


Fig. 179. Subsidence over a subcritical area (*left*) and a critical area (*right*) as calculated by various methods. E/S: Ehrhardt/Sauer method

8.3.3 Methods for Adapting to Distorted Superposition

If occasionally it is questionable whether integration grids are applicable to strata that have been intensively worked through, or perhaps even to mining conditions in European coalfields today ($H > 500$ m, $v_t > 2.5$ m/d, caving, and subcritical extraction), it should be remembered that these grids were developed for a medium depth of mining and for solid-stowed panels, and also for minimal working of strata and a slow, single-seam mining operation. Consequently, if these conditions no longer hold good, there is no alternative to adapting the calculation procedure to the new circumstances.

It has long been known that the point of half-critical subsidence occurs not over the ribside of an area of critical extraction – the location deduced with every integration grid from the assumption of rotational symmetry in shares of influence – but over the marginal zone of the panel. The reason for this is seen in the incomplete compaction of that marginal zone, as illustrated in Fig. 181. To take account of this *zone of reduced convergence*, the following procedures are proposed:

- a) The apex of the limit angle is placed in the middle of a marginal zone of width $2b$ (Fig. 181 a). Along the margin, beside the unworked seam, a strip of width b is

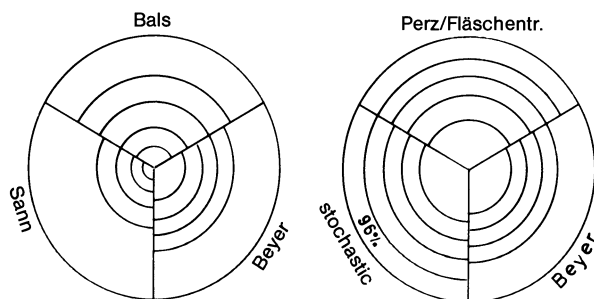


Fig. 180. Comparison of zones in several integration grids for the calculation of subsidence (*left*) and of displacement (*right*). Each zone contributes 20%

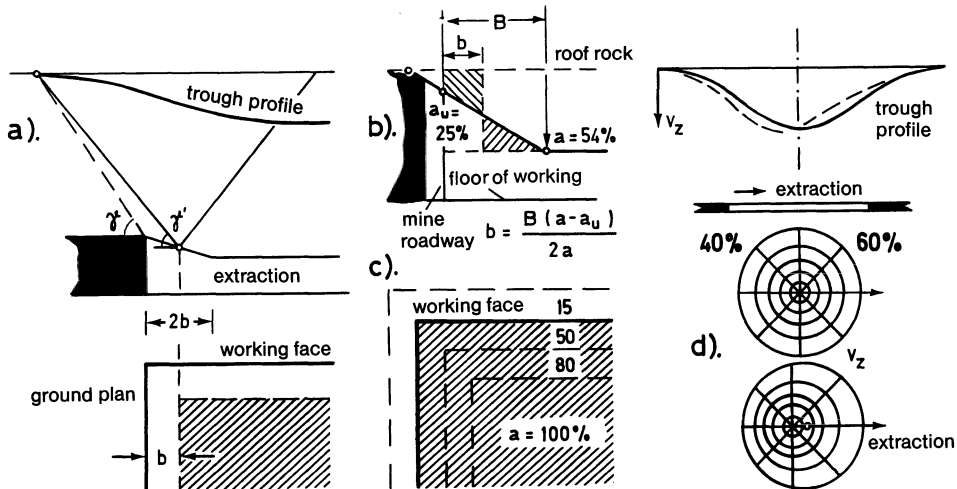


Fig. 181. Refining the subsidence calculation by introducing a marginal zone b (diagrams *a* and *b*) or several marginal strips (diagram *c*), and by assigning unequal values to the two halves of the grid or using eccentric zones (diagram *d*)

marked off and is ignored in ascertaining the mining influence. The inner half of the marginal zone is given its full value (Beyer, 1944).

b) By bringing in subsidence ahead of the face a_u , the width b of the marginal strip that is to be separated off from the panel can be calculated with the formula for subdividing areas given in Fig. 181 b (Fläschenträger, 1956). If mining takes place alongside an old panel, the working area on that side is to be widened in the evaluation plan on account of delayed settlement in the goaf (see also Fig. 104).

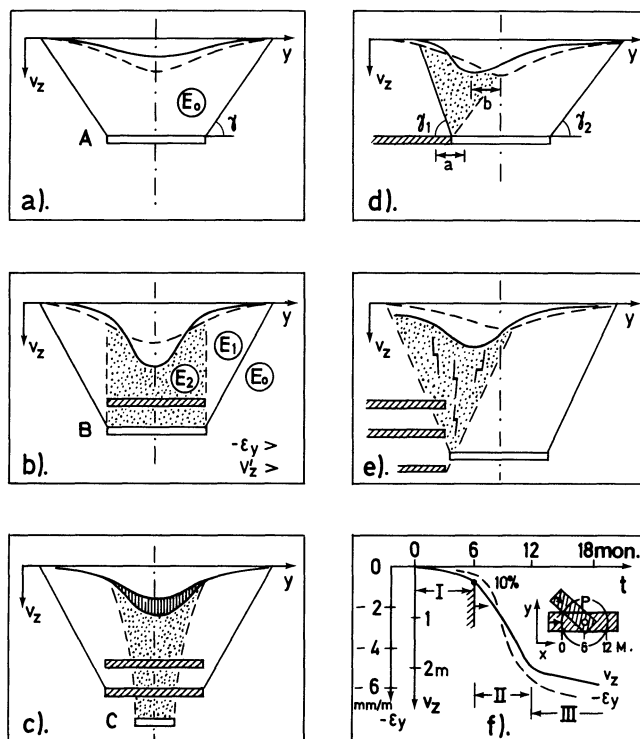
c) The increase in roof settlement in the region of the ribside (Fig. 103) can be represented in a sectional drawing by a series of steps, and in a plan drawing by strips given varying values (Bals, 1956). This method certainly reflects local roof settlement most nearly but entails a larger calculation effort than other methods, as the influence of each strip has to be ascertained separately (Fig. 181 c).

Delayed settlement in an old working at a higher level (Fig. 108) can also be calculated by method (c), provided the amount of convergence is known or can be estimated.

The main reason why the acute-angled, *unsymmetrical trough profiles* (Fig. 102) found today in some coal districts deviate from the normal profiles illustrated in Fig. 179, right, can be found in the fracturing and breaking up of strata that have been repeatedly worked through. Although with pneumatic stowing the deviation from normal is slight, the "steep" trough sides discussed in Sect. 6.5 may be observed over workings adopting caving. These trough shapes which give difficulty in calculation are compared in Fig. 182.

The first working A (with caving) in virgin strata, or in strata that have been worked previously but have reconsolidated, produces at the surface a flat trough (the continuous line) which is less deep than the calculated normal trough (the dashed line in diagram a). The second caving operation B, directly under A,

Fig. 182. The effect on the trough shape, as seen in a cross-sectional direction, of mining through a block of strata several times rapidly (multi-seam working)



causes an excess of subsidence in the centre of the trough as compared with the calculated profile, with considerable compressive strain and tilt (diagram b). By being concentrated on the same block of strata, the workings have severely broken up the roof rock (as represented by the smaller Young's modulus E_2), whereas in the marginal region (E_1) the fabric of the rock mass is little altered. A third, but narrow, caved working C induces in the fractured strata a particular shape of trough resembling that produced when material is drawn off in a silo (diagram c), with a curved line at the limit of strata movement which is steeper than the limit angle in the first working. Where a seam is removed near an old panel (shown hatched in diagram d), the bottom of the trough is displaced by the amount b towards the old panel (the continuous trough line). Delayed subsidence in the marginal zone of the old panel and the movement patterns set in advance by the first working have thus led to an unsymmetrical trough profile. The limit angles at each end of the new panel consequently differ in size. Where a new working proceeds along the edge of a block of three or more old workings, a deep trough will be produced which will be displaced towards the edge of that old block. This is because the cracks penetrating high above the old ribsides have created a zone of weakness, which attracts the subsidence potential of the new working alongside (diagram e). In strata that have been intensively worked, the surface point P drops rapidly after the passage of the face (phase II), and compression over the panel also increases rapidly. Subsidence in phase III following on the halting of the face in the 12th month is slight (diagram f).

Taking as an *example* a caving panel of $250\text{ m} \times 800\text{ m}$ at a depth of 400 m , Fig. 183 examines how the trough profile on the cross-sectional line A – A alters if the parameters of the Eqs. (161) and (163) are altered. The normal trough shape for a limit angle of $\gamma = 60^\circ$, with a subsidence factor of $a = 0.95$ and an extraction area without a marginal zone of reduced compaction, is represented as a continuous line. Altering the limit angle γ from 60° to 75° produces a narrower and deeper trough. Over the ribside, however, subsidence in both cases stands at approximately half the maximum amplitude: which does not agree with the subsidence amplitudes of $20\% - 30\% v_{z\max}$ observed over the margins of sub-critical areas in repeatedly mined strata. If the subsidence factor is reduced from 0.95 to 0.5 , the result is minimal subsidence over the ribside, but the trough is too flat. If, on the other hand, the inner zone 1 of the integration grid is given a value higher than 20% , the desired effect – a deep, acute-angled trough and little subsidence over the ribside – is obtained. A marginal strip of extraction area with a width of $b = 50\text{ m}$ and evaluated at $a = 0.5$ results, as might be expected, in less subsidence over the ribside but also in a less deep trough. If the calculation point P in the integration grid is displaced from Z to Ex, the trough profile is correspondingly displaced and the trough bottom is no longer over the centre Z of the

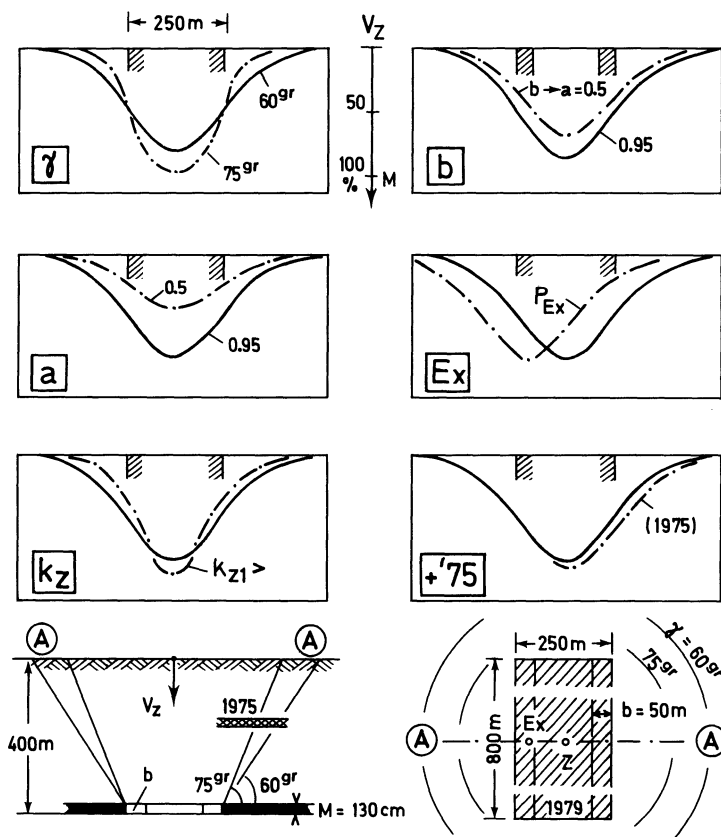
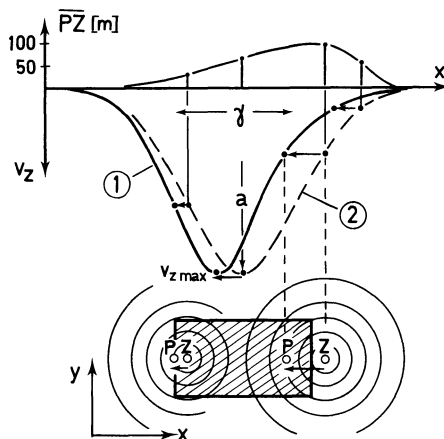


Fig. 183. Trough shapes on varying the parameters of calculation. *Bottom*: section and ground plan of the panel being calculated. Ex = eccentric

Fig. 184. Adjustment of the calculated profile (2) to the measured trough profile (1) by varying the subsidence factor a , the limit angle γ , and the amount of eccentricity PZ of the calculation point in the integration grid



working, but the trough profile remains symmetrical. If delayed convergence in the old panel of 1975 higher up in the strata is taken into account, the right-hand side of the trough becomes deeper, but the profile becomes somewhat unsymmetrical.

Varying the limit angle and the subsidence factor is therefore what has the greatest influence on the trough shape. One course, as proposed in Fig. 184, is to adjust the subsidence maximum and the trough width in the calculated profile (2) to the values in the observed profile (1) by altering the subsidence factor and the limit angle, and then, by shifting the individual calculation points from Z to P in conformity with an empirical local function, to make the calculated profile coincide with the observed profile (Chilian, 1976). This change in the position of numerical values could also be achieved by having a movable, eccentric calculation point P in the integration grid, or by means of a programmed *correction function* $PZ(x)$.

Increasing the limit angle reduces the area of the trough, and this can lead to misleading conclusions in assessing the chances of subsidence damage at the margin of the zone of influence. This objection does not apply to the procedure of assigning a higher value to the inner than the outer zones of the grid, or of reducing their diameter while keeping the zone values equal (at 20%) – in other words, seeking an influence function appropriate to the “degree of previous working” (Chap. 6) and the amount of reworking that has taken place. Unsymmetrical profiles in flat-lying measures (Fig. 181 d) can be calculated with integration grids of which the two halves are unequally weighted, or with elliptical grids, or eccentric circular zones. Elliptical grids are also appropriate in flat stratification when account has to be taken of differences in limit angle found in heavily reworked strata – for example, 60° over the stationary face edge and 50° over the advancing face.

Adjustment can also be effected with a non-linear correction function (Dzegniuk, 1977):

$$f(e) = 1 - k \sin(\pi e)^q. \quad (169)$$

As a function of the influence factor e , the calculated provisional subsidence aMe is refined according to the relation

$$v_z = aMef(e) . \quad (170)$$

The purpose of this operation is above all to take account – through $f(e)$ – of the delay in subsidence over small extraction areas, to reduce the subsidence value over the ribside (parameter k), and to shift the transition point of the trough profile in critical-area extraction into the range of $0.62 v_{zMax}$, in accordance with observations made (parameter q) – a procedure which leads to asymmetrical subsidence curves and entails a deviation from the principle of volume constancy. Other subsidence functions that have been extended through adjustment parameters have been given by Walther (1972) and Greń (1979).

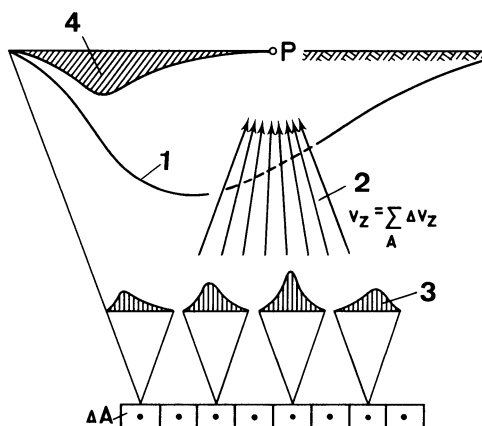
If it is desired to deal with steep and asymmetrical trough shapes by influence functions alone and not by a results correction, there are two ways to proceed:

a) The principle of undistorted superposition is abandoned and a *non-linear influence operator* is introduced into Eq. (163) in place of the influence factor, taking account of the mutual interaction of the extraction elements – i. e., of the vicinity influence. Clearly, on the removal of an extraction element, support is withdrawn from the undermined roof strata. In addition to an elementary trough, this produces an “abutment” pressure on the neighbouring extraction elements. The removal of a second, neighbouring extraction element thus means the loss of an even greater supporting force than with the first extraction element, and this gives rise not only to a further increase in the ribside abutment pressure (the support load) but also to a deeper, second elementary trough and delayed subsidence over the first extraction element – and so on until the critical area is reached.

An operator defining these relations can consist of a linear term that is the same for all extraction elements – for example, the orthodox influence function $k_z(r)$ for the approximate influence of the surface – and a quadratic term which reflects, in several iterations, the interaction of the extraction elements as the extraction area is progressively enlarged (Neuhaus, 1978). The elementary trough of each extraction element lying within the width R behind the extraction front thus alters with each step in the face advance.

b) Alternatively, each extraction element is given a different influence function, and the internal dead-weight force of the rock mass is replaced notionally by external forces, which act on the roof rock over each extraction element and create a subsidence trough over the mine excavation – an imaginary excavation formed not step by step but all at once. Then it is possible furthermore to superimpose the individual subsidence forces, which are independent of each other, or the elementary troughs, and thus to employ the method of superposition. To be able to calculate given trough shapes, however, the individual elementary troughs or influence functions must vary according to the position of the element concerned within the extraction area (Fig. 185). Their shape thus also depends – by analogy with the span of a bridge – on the size of the extraction area for which the subsidence trough is to be calculated (Kratzsch, 1979).

Fig. 185. The calculation of unsymmetrical trough profiles (1) while maintaining undistorted superposition (2), by using unequal influence functions (3) for the individual extraction elements ΔA . (4) Unsymmetrical elementary trough



This course (b), which is simpler in terms of calculation, is the one followed in the EDP programme EFLUX (Neuhaus, 1978). The elementary troughs or influence functions pertaining to the extraction elements are unsymmetrical in the marginal zone of the extraction area, and their maxima vary if the trough profiles to be calculated are unsymmetrical and steep.

The known adjustment procedures are listed in Table 13.

8.3.4 Integration Grids in Steep-lying Measures

The circular, concentric integration grids appropriate to flat-lying measures can be used up to seam dips α of 20 gr approximately. With steeper angles of dip the area of critical extraction, which is determined by three limit angles, deviates too

Table 13. Procedures for adjusting numerical values to observed quantities

Trough shape	Method of adjustment
Deep hollow	Increase the subsidence factor a Give more weight to inner zones of integration grid
Steep sides in troughs	Increase limit angle from 60 gr to 75 gr Introduce 50 m marginal zones of reduced convergence (50%) Superimpose subsidence curves of varying limit angles, with differentiated weighting according to locality Non-linear correction function related to influence factor e
Reduced subsidence at margin	Displacement of calculated local values according to empirical characteristics (adjustment function)
Asymmetrical trough	Unsymmetrical influence functions Eccentric calculation point in integration grid Localized correction of calculated subsidence (adjustment function)
Any given trough shape	Empirical correction function, derived from difference between calculated and observed values

much from the circular, and the bottom of the trough is no longer at the centre of the critical area (cf. Fig. 164). In an inclined working the influence k_z of the extraction elements is distributed unsymmetrically (see Fig. 187, left). For steep stratification therefore, special grids have to be prepared which take account both of the dip and of the difference in depth at the upper and lower working faces.

The distribution function of Eq. (165) can be modified separately for steep working in the lower $[-\alpha$ in Eq. (171)] and upper $(+\alpha)$ parts of the seam from point P, to the extent that a vertical influence force proportional to the depth attaches to the individual extraction element (Schleier, 1937):

$$\left. \begin{aligned} k_z &= \frac{dA}{H \cos \alpha} \cos(\zeta \pm \alpha) \cos \zeta \\ K_z &= \frac{1}{2} \left[\left(\frac{1}{2} \sin 2\zeta + \zeta \right) \cos \alpha \pm \sin^2 \zeta \sin \alpha \right] \end{aligned} \right\} \quad (171)$$

The *focal point* of all vertical influence forces is represented by an eccentric point Z in the critical area (see Fig. 158). The K_z values and the zone angles for each angle of dip α in the main section down the dip and through the point Z can be read from a nomogram (Fig. 186).

If the *zone angles* ζ are related, not to the vertical, but to the seam normal P–N (Fig. 187)

$$\eta = \zeta \pm \alpha, \quad (172)$$

the proportions of final subsidence can be obtained from the simple relation

$$K_z = \sin \eta \quad (173)$$

(Perz, 1940). On this basis the subsidence influence on point P is found for the upper part N–A and the lower part N–B of the working in the main section A–B, as follows:

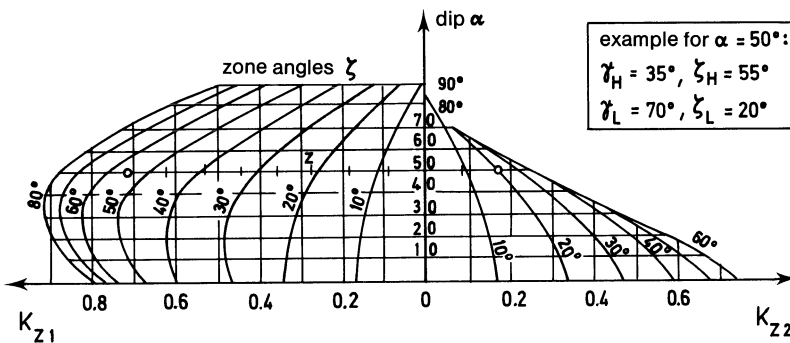


Fig. 186. Zone-angles diagram for Eq. (171). *Example:* the distance between $K_{z1} = 0.72$ and $K_{z2} = 0.17$ is divided into 10 equal sections, to enable the zone angles in the dip section to be read off. (After Schleier, 1956)

Table 14. The calculation of zone angles for an integration grid in inclined measures (Perz method)

Dip section		Strike section	
Sine values	Zone angles gr	Sine values	Zone angles gr
$K_{z1} = 0.2334$	15.0 (η_H)	$K'_{z1} = 0.6375$	44.0 (σ)
$K_{z1} - 1K = 0.1248$	0.8	$K'_{z1} - 1K' = 0.5100$	34.1
$K_{z1} - 2K = 0.0162$	1.0	$K'_{z1} - 2K' = 0.3825$	25.0
$K_{z1} - 3K = -0.0924$	-5.9	$K'_{z1} - 3K' = 0.2550$	16.4
$K_{z1} - 4K = -0.2010$	-12.9	$K'_{z1} - 4K' = 0.1275$	7.8
$K_{z1} - 5K = -0.3096$	-20.0 ($\alpha - \mu$)	$K'_{z1} - 5K' = 0.0000$	0.0
$K_{z1} - 6K = -0.4182$	-27.5	Division into sectors: $\tan y = \frac{\cos \alpha}{\cos (\alpha - \mu)}$ $= \frac{\cos 45}{\cos 20} = \frac{0.7604}{0.9511}$ $y = 42.9 \text{ gr}$	
$K_{z1} - 7K = -0.5268$	-35.3		
$K_{z1} - 8K = -0.6354$	-43.8		
$K_{z1} - 9K = -0.7440$	-53.4		
$K_{z2} = -0.8526$	-65.0 (η_L)		

$$\left. \begin{aligned} K_{z1} &= \sin (100 - \gamma_H - \alpha) \\ K_{z2} &= \sin (100 - \gamma_L + \alpha) \end{aligned} \right\} \quad (174)$$

The zone angle $\eta_z = \alpha - \mu$ to the focal point of influence Z can be calculated by the formula:

$$Z = \frac{K_{z1} - K_{z2}}{2} = K_{z1} - 5K = \sin \eta_z. \quad (175)$$

The influence values of the ten zonal sections are

$$K = \frac{K_{z1} + K_{z2}}{10}. \quad (176)$$

The required zone angles η are obtained from the sine values for K_{z1} , $K_{z1} - 1K$, $K_{z1} - 2K$, ... $K_{z1} - 9K$ and K_{z2} (see Table 14).

On the basis that the influence figure in steep stratification resembles a tilted circular cone, the *integration grid* seen in plan can be represented as ellipses grouped around the centre of influence Z (Fig. 187, right). The horizontal distance between their vertices on the small or large axis AB is read from the main section down the dip into which the limit and zonal angles are marked. The intersection of the limit-angle line γ_S with the horizontal through P' gives the two other points C and D of the surrounding curve along the strike. To obtain the limit points for the zonal ellipses on the line of strike through Z, the diagonal line PZ of the main section is projected on to the horizontal above Z in the dip axis and the angle σ at the point P'' read off with a protractor. The end-points F and G of the horizontal through Z are first determined as ellipse points of the grid surround by the so-called paper-strip method, using the small and large axial

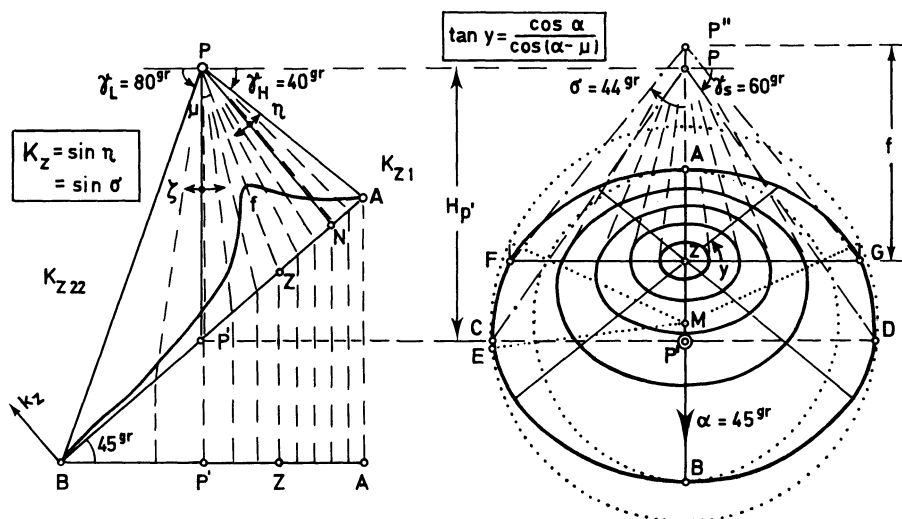


Fig. 187. Constructing an integration grid in steep-lying measures. *Left*: division into zones in the dip section. *Right*: division into zones along the strike line FZG, as done from P''. (Method of Perz and H. Schröder)

circles given by the axial centre point M and by the points A and B or E (constructed). From Eq. (173) $K'_z = \sin \sigma$, the five zonal angles along the strike are then calculated (Table 14). Finally, the diagonal angles

$$\tan \gamma = \frac{\cos \alpha}{\cos (\alpha - \mu)} \quad (177)$$

for the division of sectors must be calculated and the ellipses for the inner zone drawn.

In the surveying office, web-like subsidence grids such as these are conveniently prepared on transparent paper at a scale of 1 : 5000 in depth stages of 50 m (H_p) and for dip angles of 30, 40, 50, 60, and 70 gr. Unlike flat-lying stratification, grids of varying size must be used for each calculation point over the working area in steep-lying measures, as the seam depth H_p , under each surface point is different. The grids must furthermore be oriented towards the direction of dip. Thus, in comparison with flat-lying strata, the subsidence calculation is much more time-consuming.

Apart from these analytical methods, there are *projection methods* in which, for example, an integration grid for flat-lying strata is redrawn by reproducing the circular ring zones centrally on a diagonal seam plane (Fig. 188, right) or, on the assumption that all overlying beds go down perpendicularly, by reproducing them as elliptical grids (Bals, 1953). As with flat-lying measures, so also with steep-lying, a network of fields of influence can be derived from observations, with the very small fields grouped around the centre of influence. In the Netherlands it is common practice to project the zonal divisions pertaining to flat stratification centrally on to the steeply inclined seam line of the main section, and to trace the zone boundaries in the ground plan as eccentric circles (Fig. 188, left).

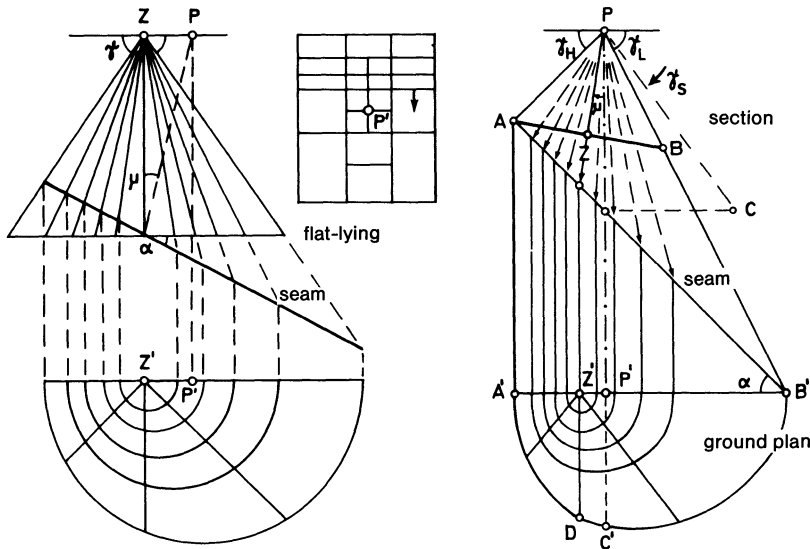


Fig. 188. Re-drawing an integration grid for horizontal stratification by projection. *Left:* equal limit angles (after Wessel). *Right:* unequal limit angles (after Kratzsch). *Centre:* empirically evaluated fields-of-influence grid

For the accuracy of the integration-grid method in steep-lying measures, the crucial factor is not so much an exact knowledge of the size of the limit angles as the correct choice of the nadir angle μ , which determines the position of the focal point Z in the integration grid. In some mining districts this nadir angle is of the order of

$$\mu \approx \frac{1}{2}(\gamma_L - \gamma_H), \tag{178}$$

as Table 15 shows. It can, however, be much larger, particularly if the sum of the limit angles in the dip plane

Table 15. Order of magnitude of limit and nadir angles in inclined to steep-lying measures

Dip	Limit angle			Nadir angle
gr	gr			gr
α	γ_H	γ_L	γ_S	μ
40	55	65	60	5
50	50	70	60	10
60	45	75	60	15
70	40	80	60	20
80	35	85	60	25

Note: $\gamma_L + \gamma_H = 2\gamma_S$
 $\mu = \frac{1}{2}(\gamma_L - \gamma_H)$

$$\gamma_L + \gamma_H \approx 2\gamma_S \quad (179)$$

differs significantly from double the value of the limit angle along the strike (see also the limit-angle curve in Fig. 154, left). In Upper Silesia the nadir angle is equivalent to 0.7α , and in the Netherlands to 0.5α .

In addition, for calculating subsidence by Eq. (163) it is important to know the correct *subsidence factor* a . A guide to the choice of subsidence factor, depending on seam dip and on pressure normal to the stratification, was given in Table 4. Another way of determining the subsidence factor in steep stratification, namely from measured maximum subsidence at the surface, from thickness of seam worked, and from seam dip, is provided by the formula (Kosterin, 1972):

$$a = 1 - \frac{v_{z\max}}{M \cos \alpha}. \quad (180)$$

In cases where the upper limit angle is smaller than the angle of dip of the seam ($\gamma_L < \alpha$) and consequently a critical area cannot be constructed – which can occur when the extraction influence extends to the footwall strata (Fig. 155) – it is sufficient to prepare the integration grid for the overlying strata. Applying the grid as a mirror image to the footwall of the deposit enables movement on that side of it also to be deduced (Schleier, 1964).

8.3.5 Semi-graphical Methods, and Methods Employing Analogues

Credit for being the first proposal for ascertaining the influence factor e semi-graphically, with tabulated values, must go to the K_z -table for a strip-type extraction of the critical area (Beyer, 1944), which can be used in place of zonal grids in case of straight and parallel extraction fronts. This table of constants is drawn up for a surface point P over the centre of the critical area by ascertaining the influence factor $e = K_z$, for various stages $i = r/R$ in the extraction of a critical area, with the aid of a finely subdivided integration grid, or by stripwise integration of the differential quotient $k_z = de/dA$ (Fig. 189, left). The normal subsidence curve can be obtained in this way – for example, for the point 10 in

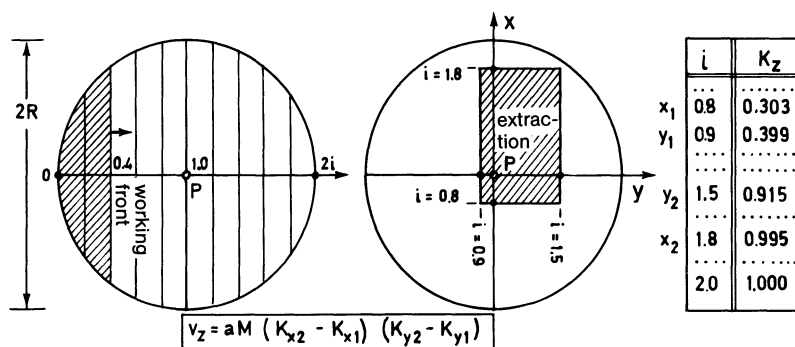


Fig. 189. Calculating subsidence at the point P with the aid of the table of constants taken from the i -values of working faces. (After Beyer, 1944 and Spettmann, 1962)

Fig. 175 when extraction proceeds from I to X, or from 0 to 2i (time factor $z = 1$). In extracting a critical area the normal subsidence curve is identical with the half-trough profile. Normal subsidence values for several influence functions, and for the kind of trough profile observed in the Ruhr today on intensive reworking of strata, can be read off from Fig. 175.

Subsidence can be calculated from the product of differences in tabulated values if a transparent sheet, marked with the critical-area circle, the centre point (i.e., the calculation point P), and a system of rectangular coordinates with decimal values of the critical-area radius R, is used to determine the i values of the extraction fronts by orienting the system of coordinates parallel to them and reading the relevant K_z values from the table of constants:

$$v_z = v_{z\text{Max}}(K_{x2} - K_{x1})(K_{y2} - K_{y1}). \quad (181)$$

This graphical-numerical procedure is in principle similar to the diagrammatic profile construction illustrated in Fig. 175.

For the example in Fig. 189, right, the calculation is as follows:

$$v_z = 120 \text{ cm} (0.995 - 0.303) (0.915 - 0.399) = 21.1 \text{ cm}.$$

In *Poland*, from empirical curves of normal subsidence, the subsidence influence k_z of infinitesimal extraction elements in flat-lying strata for the point over the ribside ($x = 0$) has been derived from the tilt values dv_z/dx — i.e. the subsidence differentials — giving an influence function after *Gauss* (Knothe, 1957):

$$\frac{dv_z}{dx} = v_{z\text{Max}} \frac{\lambda}{\sqrt{\pi}} \exp(-\lambda^2 x^2). \quad (182)$$

The parameter λ , which determines the height and shape of the curve, can be expressed by the statement that the critical-area radius R is equal to the subtangent of the normal subsidence curve at the transition point over the ribside (Fig. 190)

$$R = \frac{v_{z\text{Max}}}{\tan \psi} \quad (183)$$

and, at the same time, that the maximum tilt value $\tan \psi$ at that position ($x = 0$) can be equated to the Gaussian distribution curve

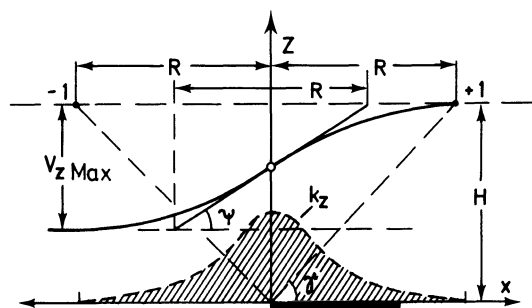


Fig. 190. The subtangent at the normal subsidence curve over the ribside, and the influence function k_z . (After Knothe, 1957)

$$\tan \psi_{x \rightarrow 0} = v_{z\text{Max}} \frac{\lambda}{\sqrt{\pi}} \quad (184)$$

where

$$\lambda = \frac{\sqrt{\pi}}{R}. \quad (185)$$

With the influence function

$$k_z = \frac{1}{R} \exp \left(-\frac{\pi}{R^2} x^2 \right) \quad (186)$$

the equation for the normal subsidence profile then is

$$v_z = v_{z\text{Max}} \frac{1}{R} \int_{-\infty}^x \exp \left(-\frac{\pi}{R^2} x^2 \right) dx. \quad (187)$$

The subsidence of surface points is calculated by this formula with the aid of tables as in Fig. 189. For a programmed calculation, the exponential function (185) must be introduced into Eq. (196) (Dzegliuk, Pielok and Śroka, 1980).

In another Polish pre-calculation method (Kochmanski, 1958), the distribution of the subsidence influence in extracting a horizontal seam is likewise expressed by an *exponential function*

$$k_z = \frac{1}{C} \exp \left(-\frac{r}{r_0} \right)^b \quad (188)$$

in which r_0 and b are constant parameters which have to be determined empirically for the working in question. The strata parameter or unit radius r_0 in metres varies between 18 m in soft rock and 200 m in solid strata. In Upper Silesia r_0 reaches 50 to 80 m. The parameter b depends on the depth of mining according to the following relation:

$$\left. \begin{aligned} b &= 2.05 - 0.50 \log H & (H > 400 \text{ m}) \\ b &= 2.69 - 0.75 \log H & (H < 400 \text{ m}) \end{aligned} \right\}. \quad (189)$$

The constant C , which is a function of b , results from the boundary condition that, in integrating the shares of subsidence produced by a critical extraction area, the critical subsidence value must be maintained. Summation of the shares of subsidence within a rectangular extraction panel leads to an equation of the form

$$v_z = aM \frac{\int_{x_1}^{x_2} dx \int_{y_1}^{y_2} \exp \left(-\frac{r}{r_0} \right)^b dy}{C} \quad (190)$$

where

$$C = \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} \exp \left(-\frac{r}{r_0} \right)^b dy.$$

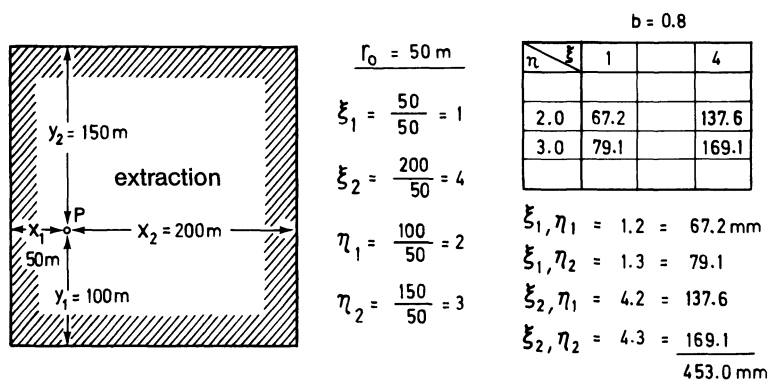


Fig. 191. Calculating subsidence for the surface point P when full ("critical") subsidence is 1 m. (After Kochmanski, 1958)

The integrals $\phi(\zeta, \eta)$ of Eq. (190) are tabulated for the integration limits 0 to ζ or η in the numerator and 0 to ∞ in the denominator (cf. the Beyer method, Fig. 189), and the coordinates of the panel boundaries are related on a dimensionless basis to the strata parameter r_0 , thus:

$$\xi_1 = \frac{x_1}{r_0}, \quad \xi_2 = \frac{x_2}{r_0}, \quad \eta_1 = \frac{y_1}{r_0}, \quad \eta_2 = \frac{y_2}{r_0}.$$

Subsidence of the point P in Fig. 191 represents the sum of the four subcritical areas $x_1 y_1$, $x_1 y_2$, $x_2 y_1$, and $x_2 y_2$. With $r_0 = 50$ m, the values $\xi_1 = 1$, $\xi_2 = 4$, $\eta_1 = 2$, and $\eta_2 = 3$ are obtained.

From Kochmanski's Table IV the following subcritical subsidence values can be obtained for $aM = 1$ and $b = 0.8$:

$(\xi_1, \eta_1) \dots (1, 2):$	67.2 mm
$(\xi_1, \eta_2) \dots (1, 3):$	79.1 mm
$(\xi_2, \eta_1) \dots (4, 2):$	137.6 mm
$(\xi_2, \eta_2) \dots (4, 3):$	169.1 mm
	<u>453.0 mm</u>

With a critical subsidence of $aM = 0.80$ m instead of 1 m, subsidence for the point P decreases to $453 \times 0.8 = 362$ mm.

The procedure described can be extended to cover inclined to *steep-lying strata* on the basis that

a) the influence of an extraction area dipping at an angle of α is transmitted to the surface at a principal influence angle of

$$\mu = 0.7 \alpha \quad \text{and} \quad (191)$$

b) that the subsidence profile curve represents the sum of those elementary subsidence troughs which arise at various depths of an inclined panel when narrow (horizontal) strips of seam are worked along the strike (Ochab, 1959).

subsidence it is sufficient to convert the panel into three horizontal strips in the direction of the strike, which are displaced in accordance with the principal influence angle μ in relation to the roof ($A \rightarrow A'$, $B \rightarrow B'$).

The displaced profile $A' - B'$ is traced into the nomogram between the lines of depth for 130 m and 200 m, from the upper to the lower ribside, repeating the process afresh in relation to each calculation point 1, 2, ... to 9. For the example presented in Fig. 192 the following initial values are found, given $\mu = 15^\circ$, $r_0 = 50$ m, and $\alpha = 25^\circ$:

$$\text{Displacement of the upper ribside } \Delta x = 35 \text{ m}, \quad \delta_0 = 35/50 = +0.70$$

$$\text{Displacement of the lower ribside } \Delta x = 52 \text{ m}, \quad \delta_u = 52/50 = +1.07.$$

For point 1 the values are:

Unit distance of the upper ribside A' from point 1:

$$\xi_{A'} = \frac{x_A}{r_0} + \delta_0 = \frac{100}{50} + 0.70 = +2.70;$$

Unit distance of the lower ribside B' from point 1:

$$\xi_{B'} = \frac{x_B}{r_0} + \delta_u = \frac{250}{50} + 1.07 = +6.07.$$

These distances are marked off against the depth lines of the ribsides ($H_i = 130$ and 200 m) in the nomogram. For the middle points of the three sections into which the stretch $A' - B'$ is divided, the subsidence values 1.0, 0.4, and 0.2 are then read off, so that for point 1 a subsidence function of

$$\phi = 2 \times 5 (0.2 + 0.4 + 1.0) = 16.0$$

is formed. The factor 5 results from the increase in the horizontal lengths of extraction from $AB = 150$ m to $A'B' = 168$ m

$$k = \frac{\overline{A'B'}}{AB} = \frac{168 \div 50}{150 \div 50} = \frac{3.37}{3.00} = 1.123$$

and from the interval underlying the nomogram on the ζ -axis of $0.2k = 0.2 \times 1.123 = 0.225$, i.e., of $3.37/0.225 = 15$ sections. If instead only three sections of length are chosen, the result must be multiplied by $15/3 = 5$. The factor 2 takes into account the fact that the extraction panel on both sides of the calculation line has a dimension of

$$\eta = \frac{y}{r_0} = \frac{50}{50} = 1.0.$$

The point subsidence is obtained by multiplying the subsidence function ϕ by the critical subsidence value

$$v_z = \frac{aM}{\cos \alpha} \phi. \quad (193)$$

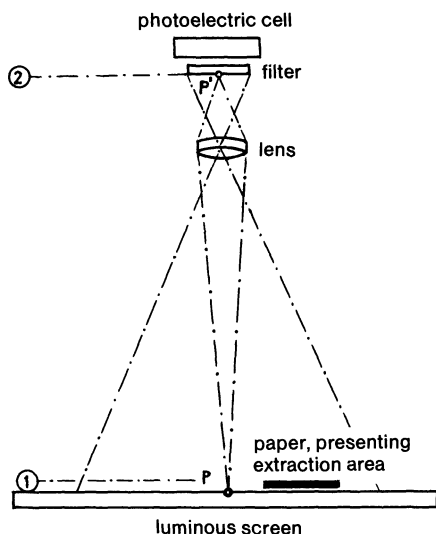


Fig. 194. The principle of the optical analog computer for calculating subsidence and displacement. (After Kratzsch, 1963)

With a subsidence factor of $a = 0.8$ for caving, a face height of $M = 1.5$ m, and a dip of $\alpha = 25^\circ$, the final result is

$$v_{z1} = \frac{0.8 \times 1.5}{0.91} \times 16.0 = 1.32 \times 16.0 = 21.1 \text{ mm}.$$

Subsidence values for the other profile points are calculated in the same way, shifting the calculation point $P(0)$ of the nomogram in Fig. 192 from point 1 to point 2, and so on. Nomograms have also been prepared for wider extraction areas of $\eta = 1.4, 2.0, 3.0, 4.0, 6.0$ and ∞ .

The subsidence calculation can also be simplified and speeded up by the use of an *analogue*. Graphical tabulation of shares of influence can, for example, be simulated photoelectrically (Kratzsch, 1963). In a radiation meter, a luminous surface 1 is projected by means of a lens on to an optical filter 2, placed in front of a photoelectric cell (Fig. 194). The rings of polaroid foil and film in the filter attenuate the intensity of radiation reaching the photoelectric cell in a manner which corresponds to an integration grid in value terms. With zones of equal value (e.g., after Bals), the partial radiation admitted by each filter zone will produce a photoelectric current of equal strength. If a transparent evaluation plan of the mine, in which the working areas are covered with black paper, is moved about on the luminous screen in such a way that the calculation points, plotted to scale on the plan, are brought to cover one by one the centre point of projection marked on the luminous screen, then a reflection galvanometer coupled to the photoelectric cell will immediately indicate the amount of subsidence in centimetres. If a differential cell is used, it will also show the amount of displacement. The full, or "critical", subsidence value must, of course, be set on the galvanometer in advance.

For calculations in steep-lying measures, the central projection method is used, in which a radiation meter with a wide-angle lens is tilted at the nadir angle

and the opaque paper representing the extraction area is set at the angle of seam dip (Kratzsch, 1964). In Poland, a photoelectric analogue computer has likewise been developed, with which subsidence profile curves can be calculated and at the same time drawn (Greń, 1967). For the radiation filter a rotating disc, with zonally ordered slits, is used.

8.3.6 Programmed Calculation of the Influence Factor

The main task in calculating subsidence by the influence-functions method consists in determining the influence factor e for the extraction area acting on the surface point P . In the methods described up to now this factor has been ascertained with – leaving aside the analogue computer – an integration grid or with tabulated functional values. Since electronic data processing (EDP) installations have become available to mining companies for a wide range of technical calculations, the adding together of elementary troughs and the integration of areas can be added as further possibilities.

No programmed calculation of subsidence using functional values of normal subsidence curves in accordance with Fig. 189 or Fig. 191 has yet come to notice. Areal integration for subsidence calculation in flat-lying measures and a programmed grid method for steep-lying measures have been used by the Ruhrkohle AG combine for some years, as well as recently the summation of elementary troughs (the EFLUX programme).

a) The areal-integration method for flat-lying measures. The standard *programme of Ruhrkohle AG* for subsidence calculation in flat-lying measures is based on the influence-function equation [Eq. (168)], expanded to include the parameters c and k and written thus:

$$dv_z = c \exp \left(- \frac{x^2 + y^2}{2k^2} \right) dx dy \quad (194)$$

where dv_z signifies the differential share of subsidence at the surface point $P(0,0)$ of an extraction element $dx dy$ (Maassen, 1961). These shares of subsidence dv_z may be imagined as prisms dv_z/dA erected on the critical area A_{crit} , as in Fig. 168, in such a way that a bell-shaped shares-of-influence body is formed, the volumetric content of which corresponds, by Eq. (159), to the critical subsidence aM . If only a part CDEF of the critical area is mined, then the task of subsidence calculation for the central surface point P consists in determining the volumetric content of the shares-of-influence body above the extraction area (Fig. 195). For this purpose the shares-of-influence body is divided into narrow, vertical strips parallel to the x -axis, and their volumetric contents, from the left-hand side at y_1 to the right-hand side y_3 at the edge of the critical area, are added together to obtain the required subsidence v_z of the point P . The volumetric content V of the strip is obtained from its width Δy and the area A of the shares-of-influence body, as sectioned, for example, at the position y_2 . This area consists of a rectangle and an upper portion of the bell corresponding to the influence function between the position x_1 and x_2 in Fig. 195.

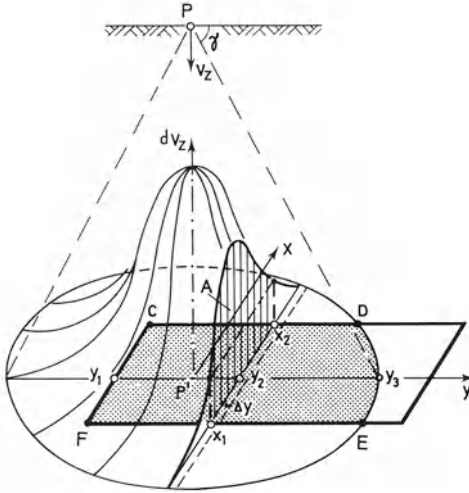


Fig. 195. Bell-shaped distribution of the shares of each extraction element in the subsidence at the central surface point P within the limit-angle cone. CDEF: extraction area

General integration of the exponential influence function (194) reads, in terms of Cartesian coordinates:

$$v_z = c \iint \exp \left(- \frac{x^2 + y^2}{2k^2} \right) dx dy \quad (195)$$

or, since with $H = \text{const.}$, $k = \text{const.}$ also:

$$v_z = c \int \exp \left(- \frac{x^2}{2k^2} \right) dx \int \exp \left(- \frac{y^2}{2k^2} \right) dy. \quad (196)$$

Integration over a polygonal working, which is divided up to the y-axis, as in Fig. 196, into trapezia with positive and negative counting direction, in such a way that the hatched areas outside the working cancel themselves out, is undertaken for individual trapezia, according to Simpson's rule:

$$A = \frac{y_2 - y_1}{3(2m)} (x_1 + 4x_2 + 2x_3 + 4x_4 + \dots + 4x_{2m} + x_{2m+1}) \quad (197)$$

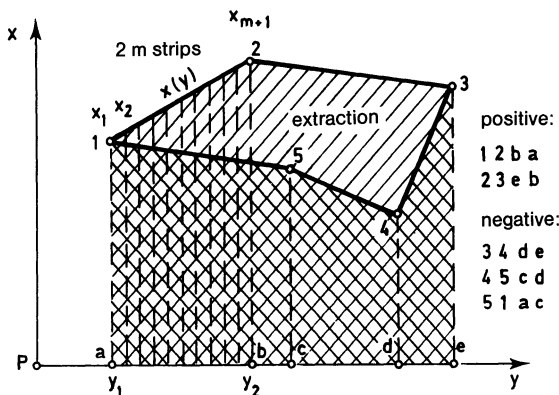


Fig. 196. Integration over a working of polygonal shape (plan), divided into trapezia. (After Maassen, 1961)

with the trapezium being subdivided at 2m intervals by the unit width $y_2 - y_1/2$ m.

In this connection

$$x(y) = ay + b \quad (198)$$

with

$$a = \frac{x_2 - x_1}{y_2 - y_1} \quad \text{and} \quad b = x_1 - \frac{x_2 - x_1}{y_2 - y_1} y_1. \quad (199)$$

The integral of the shares of subsidence over the trapezium area, as related to the surface point P(0,0), is as follows, by Eq. (196):

$$v_z(\text{trapez}) = \int_{y_1}^{y_2} \left[\int_0^{x(y)} \exp\left(-\frac{x^2}{2k^2}\right) dx \right] \exp\left(-\frac{y^2}{2k^2}\right) dy. \quad (200)$$

Tabulated and programmed, the integral in the square brackets of Eq. (200):

$$\int_0^{x(y)} \exp\left(-\frac{x^2}{2k^2}\right) dx = \sqrt{2}k \frac{\sqrt{\pi}}{2} \psi\left(\frac{x(y)}{\sqrt{2}k}\right) \quad (201)$$

appears in the form

$$\psi(\lambda) = \frac{2}{\sqrt{\pi}} \int_0^\lambda \exp(-t^2) dt$$

if substitutions are made as follows:

$$\lambda = x(y), \quad t = \frac{x}{\sqrt{2}k} \quad \text{and} \quad dt = \frac{1}{\sqrt{2}k} dx.$$

Eq. (200) thereby becomes:

$$\begin{aligned} v_z(\text{trapez}) &= \int_{y_1}^{y_2} \left[\frac{k}{2} \sqrt{2\pi} \psi\left(\frac{x(y)}{\sqrt{2}k}\right) \exp\left(-\frac{y^2}{2k^2}\right) \right] dy \\ &= \int_{y_1}^{y_2} x_i dy, \end{aligned} \quad (202)$$

with x_i standing for the terms in Simpson's rule.

The integrals of the individual trapezia must then be added together and multiplied by the factor c (in cm). The depth of working H is computed with programme control from the coordinates and depths of the given points of the extraction area, together with the dip and strike. The parameter c can be determined from integration of Eq. (195), rewritten in polar coordinates, for a very large extraction area, in which the term $\exp(-r^2/2k^2)$ becomes 0 and $v_{z\text{Max}}$ becomes $2\pi ck^2$, with

$$c = \frac{aM}{2\pi k^2}. \quad (203)$$

Assuming a practical limit-angle or critical-area radius R_{pr} , which is measured at $0.99 v_{zMax}$, it is found from $\exp(-R_{pr}^2/2k^2) = 0.01$ that

$$k = 0.3295 \frac{H}{\tan \gamma}. \quad (204)$$

b) The programmed grid method for steep-lying measures. This method of electronic subsidence calculation depends on defining the outlines of the *grid zones* mathematically, with the grids composed of two semi-ellipses and the area of influence as in Eq. (173) ($K_z = \sin \eta$) subdivided into five zones (Kastenbein, 1970). At the outer margin of the grid the semi-ellipses meet on the line of strike at the height of P' (Fig. 197), and in the inner zone at the height of Z . The zone lines are calculated by the ellipse equation from the previously determined semi-axes a and b . From Eqs. (174) and (176), the semi-axes of the ellipses for the five zones are obtained in the direction of dip⁸ from P' from the relation

$$a_i = H \sin \alpha \cos \alpha \left[1 - \frac{(c_1 K_{z1} + c_2 K_{z2}) \cot \alpha}{\sqrt{1 - (c_1 K_{z1} + c_2 K_{z2})^2}} \right] \quad (205)$$

and in the direction of strike from Z , from

$$b_i = f \frac{c_3 \cos \mu}{\sqrt{1 - c_4 \cos^2 \mu}}, \quad (206)$$

when the following values are introduced for the factors c_i taken separately (see Fig. 197):

$c_1 = -0.9$	$c_2 = 0.1$	for a_{H2} ;	likewise $c_3 = 1.0$	and $c_4 = 1.0$	for b_{Z1}
-0.8	0.2	a_{H3}	0.8	0.64	b_{Z2}
...	...	...	...	0.36; 0.16	-
-0.1	0.9	a_{L2}	0.2	0.04	b_{Z5}

For the oblique distance f from P to Z , and for the angle μ to the strike section passing through Z , the determining equations are as follows:

$$f = \frac{H \cos \alpha}{\sqrt{1 - 0.25(K_{z1} - K_{z2})^2}} \quad (207)$$

and

$$\cos \mu = \frac{b_{Z1}}{\sqrt{f^2 + b_{Z1}^2}}. \quad (208)$$

Division into sectors of equal value, which in steep stratification would entail evaluating surface elements more highly in the direction of the dip section than in the strike direction, is not considered necessary with this method because of its minimal influence. It could, however, be considered for the influence value of

⁸ The x-axis lying in the direction of dip was selected here, rather than the y-axis, in order to fit in with the available computer programme.

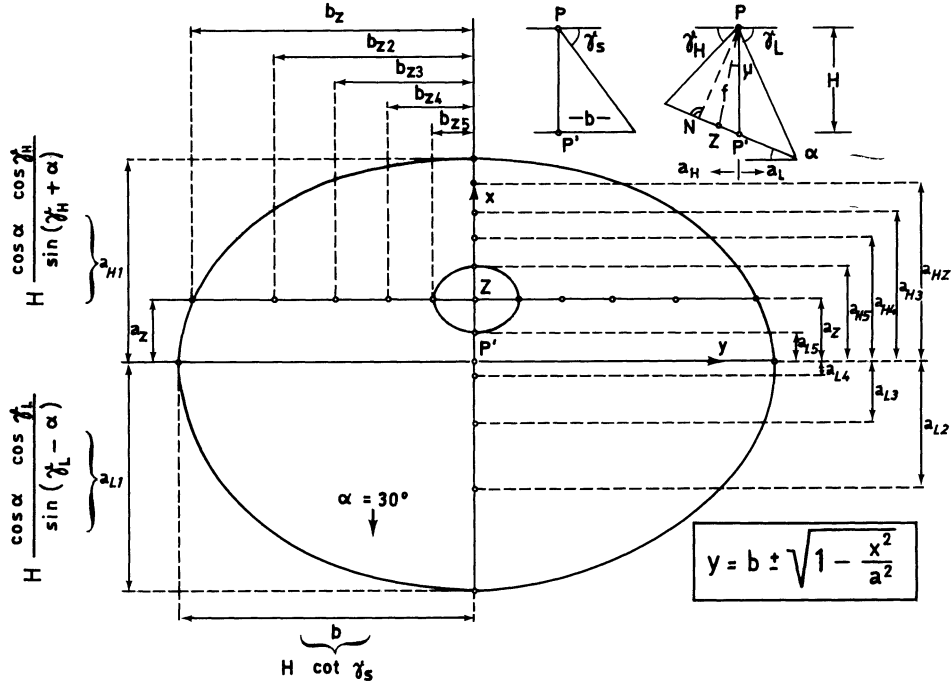


Fig. 197. Calculation of the zone line by the ellipse equation. (After Kastenbein, 1970)

the respective zonal sectors when calculating influence factors. The depth of influence H can be calculated from the coordinates of the calculation point and from the upper and lower limits of the working.

The *influence factor* e [Eq. (157)] is found in the following manner. From the equations for ellipses in the system of polar coordinates

$$r = \frac{ab}{\sqrt{a^2 \sin^2 \varphi + b^2 \cos^2 \varphi}} \quad (209)$$

the radius vectors r_i of all zones are calculated for each 5° of polar angle, beginning with $\varphi = 0$ in the direction of dip (Fig. 198). Since within the same polar angles the radius vectors R for the extraction-area boundary between the points $P_1(x_1, y_1)$, $P_2(x_2, y_2)$, etc. also are known through the function

$$R = \frac{x_1 y_2 - y_1 x_2}{(y_2 - y_1) \cos \varphi - (x_2 - x_1) \sin \varphi} \quad (210)$$

it is also possible, after checking whether R_i is greater or smaller than r_i , to determine zonally, at an apex angle of $\Delta \varphi = 5^\circ$, the zonal sector areas covered in whole ($e = 0.266\%$) or in part by the working, with their respective radii R_i or r_i , in accordance with either

$$A_{\text{zone1}} = \pi(r_{1M}^2 - r_{2M}^2) \frac{\Delta \varphi}{360^\circ} \quad R > r_1$$

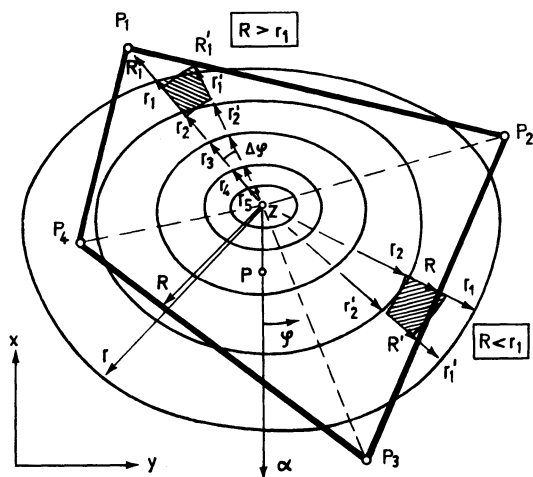


Fig. 198. Programmed calculation of the influence factor e through a circulating sector. (After Maassen, 1970)

or

$$A_{\text{zone1}} = \pi(R_M^2 - r_{2M}^2) \frac{\Delta\varphi}{360^\circ} \quad r_2 < R < r_1. \quad (211)$$

In Eq. (211), r_{1M} , r_{2M} , and R_M are the mean values of $r_1 + r'_1$, $r_2 + r'_2$, or $R_1 + R'_1$ in the circular sector. The extraction areas exerting an influence in the other zones 2 to 5 are similarly obtained. Calculation of the share of extraction area is undertaken zonally within the segments P_1zP_2 , P_2zP_3 , etc. The influence factor e is found by summation of all shares of extraction area, when φ has become equal to 2π . The apex angle of the circular sector must be kept sufficiently acute (5°) as to minimize the area error which arises from substitution of the arc of ellipse by an arc of circle.

The method can be progressively improved by adjusting the results of calculation v_z to the results of observation $\bar{v}_z$ with a correction factor $k = \sqrt{\bar{v}_z/v_z}$. In doing this, either the limit-angle values, which are usually not known exactly in inclined to steep stratification, are modified in accordance with

$$\begin{aligned} \sin \gamma_{H,L\text{cor.}} &= \frac{H - a \tan \alpha k}{\sqrt{(H - a \tan \alpha k)^2 + a^2 k^2}} \\ \sin \gamma_{S\text{cor.}} &= \frac{H}{\sqrt{H^2 + b^2 k^2}} \end{aligned} \quad (212)$$

with a for a_{H1} and a_{L1} ; or alternatively, the influence factors e_i of the individual zones constituting $\bar{e}_i = e_i k^2$, as found for a point subsidence v_z , are re-evaluated (non-equivalent zonal division) in accordance with

$$\bar{e}_i = 0.2 e_1 k^2 + 0.2 e_2 k^2 + \dots + 0.2 e_5 k^2 \leq 1. \quad (213)$$

c) The programmed summation of elementary troughs. If the influence function $k_z(r)$ and the limit angle γ are known, then the shape of elementary trough produced at the surface by an extraction element will also be known (see Fig.

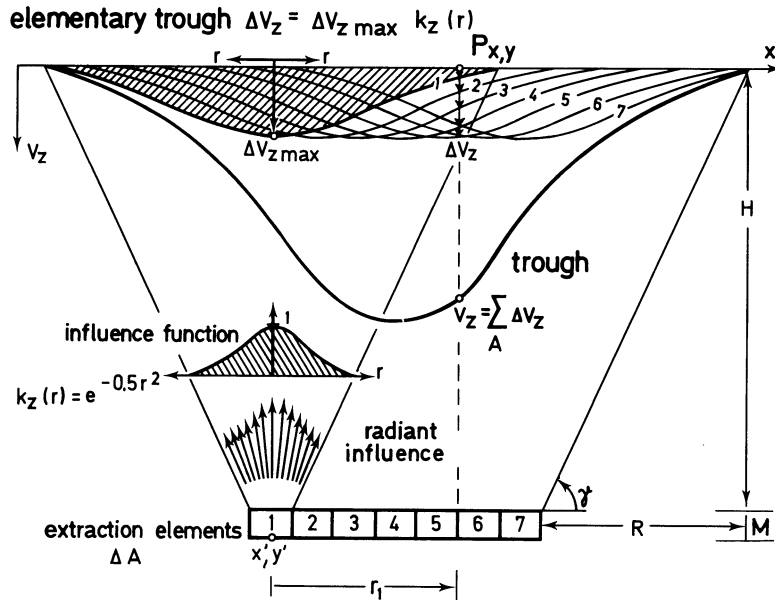


Fig. 199. Programmed summation of elementary troughs

173). Given the size of extraction area ΔA , thickness of seam mined M , depth of working H , limit angle γ , and subsidence factor a , it is possible to calculate the depth $v_{z \max}$ of the elementary trough for a finitely large extraction element by integration of the influence function, in view of the boundary conditions of constancy of volume (convergence in the working = trough volume) and rotational symmetry. If it is assumed, as is usual, that in flat-lying stratification all elementary troughs are of equal size — an assumption which presupposes an isotropic rock mass and an exertion of influence by each separate extraction element undisturbed by its neighbouring element — then the elementary troughs produced by extraction elements will superimpose themselves additively to form a subsidence trough (Fig. 199).

The computer therefore needs only to calculate the horizontal distances r_i of the individual extraction elements $A_i(x'_i, y'_i)$ from the relevant surface point $P(x, y)$ from the differences in coordinates, using Pythagoras's theorem:

$$r_i = \sqrt{(x - x'_i)^2 + (y - y'_i)^2}. \quad (214)$$

With these distances, and with the influence function selected

$$k_z(r) = e^{-\lambda r^2} \quad (215)$$

for $0 \leq r \leq R$ (the critical-area radius), the finite elementary-subsidence values

$$\Delta v_z = \Delta v_{z \max} k_z(r) \quad (216)$$

for each extraction element and surface point can be determined, as in Fig. 199. Finally, these subsidence values can be added together to give the required subsidence of the point P :

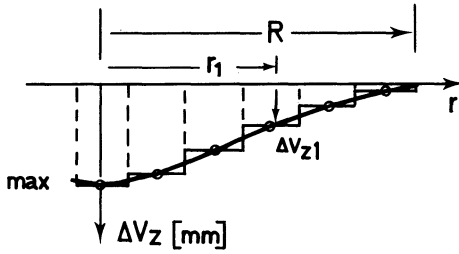


Fig. 200. Discretized elementary trough

$$v_z = \sum_A \Delta v_z. \quad (217)$$

All that requires to be done in advance is to divide up the extraction area into finite elements ΔA , tabulate in the computer memory by means of the given influence function (discretation) the subsidence values Δv_z for individual depth-related distances (e. g., $r_i = 200 \text{ m} \rightarrow 3 \text{ mm}$ subsidence), and calculate the largest subsidence value $\Delta v_{z\max}$ of an extraction element.

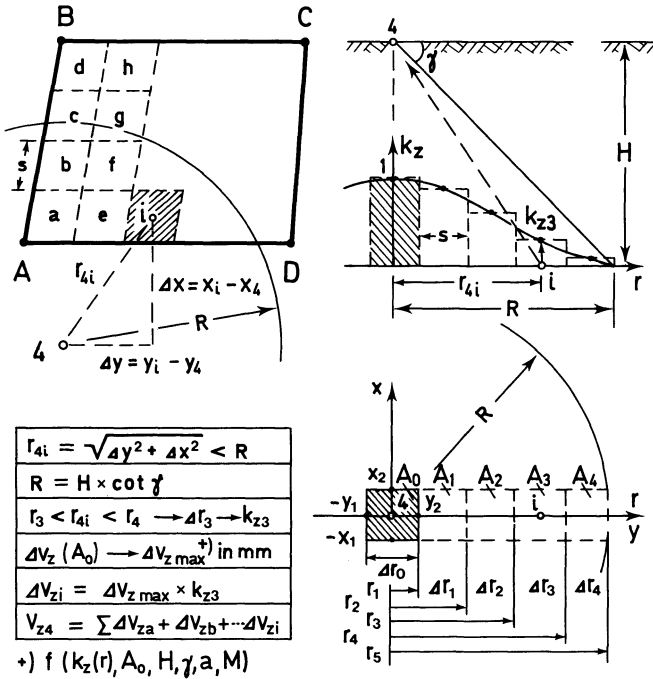
For example, for the horizontal distance r_1 of the extraction element 1 from the surface point P in Fig. 199, the elementary subsidence value Δv_{z1} in mm which the extraction element 1 causes at the point P can be derived from the discretized (sectionalized) elementary trough in Fig. 200.

The development of an elementary trough like this can be imagined as follows, bearing in mind Fig. 73. The convergence volume 1 of the extraction element C is distributed as to half each to the two strata elements A and B lying symmetrically above it. This process is repeated until at the surface an elementary trough with the profile of a bell-shaped curve is formed (see also Fig. 178, right).

The adding together of the shares of subsidence k_z or alternatively elementary subsidence values Δv_z is shown in Fig. 201 for the elements a, b, c, etc. (with length of side s) into which the extraction area ABCD has been divided, and all calculation steps are there summarized. The extraction element i is at a horizontal distance r_{4i} from the surface point 4 — a distance which is intermediate in size between the radii r_4 and r_3 and which, in the discretized influence function $k_z(r)$, corresponds to the subsidence influence k_{z3} in the step Δr_3 . The subsidence influence k_{z3} , multiplied by the largest elementary-subsidence value $\Delta v_{z\max}$, gives the required elementary subsidence Δv_{z3} at point 4 resulting from extraction element i. The largest elementary subsidence value $\Delta v_{z\max}$ depends on the values shown bottom left in Fig. 201. A further condition is that the volume of the elementary trough must agree with the convergence volume of the extraction element. The influence function is found from the shares of subsidence k_{zi} produced at surface point 4 by the finite extraction elements A_i of size Δr_0^2 .

Integration of elementary subsidence values as described above is used in the *EFLUX programme* of the Berlin University of Technology (Neuhaus, 1978). Taking into account the points discussed in Sect. 8.33 (Fig. 185), the extraction area is divided up into concentric zones and triangular elements, starting from the focal point S, in order to be able to assign a special influence function to each zone. As is demonstrated by a model of roof strata constructed from a truss, the lower nodal points of which are loaded in turn with a vertical point-force $F = 1$, elementary troughs forming at its upper surface become increasingly less sym-

Fig. 201. Programmed calculation of subsidence at the surface point 4 as a result of extracting the area ABCD, with a given influence function k_z or elementary trough



metrical the nearer the point-force comes to the edge of the working. This point-force corresponds to the subsidence force of the extraction element. In a preliminary programme, the respective influence functions $k_z(r)$ are calculated from the given points of the selected elementary troughs, and the functional values g_{ik} (i surface point, k extraction element), which by Eqs. (214) and (215) depend on the distance r , are combined into an influence matrix $[G]$, consisting of m lines (the number of points) and n columns (the number of elements). If this influence matrix $[G]$ is multiplied by the column vector $\{q\}$ of the largest elementary-subsidence values $\Delta v_{z \max}$ of the extraction elements or extraction zones, the subsidence value of all m surface points is obtained by simple matrix multiplication (Fig. 202).

If the individual extraction zones are given varying $\Delta v_{z \max}$ values (weights), the subsidence troughs certainly become deeper, but steep and unsymmetrical trough shapes are obtained only with special influence functions showing an increasingly tilted distribution towards the outer zone (Fig. 185). When several seams are extracted, the weight $\Delta v_{z \max}$ of the extraction elements must be reduced for the first working and increased for the working in the second seam (see Fig. 182a and b).

From the subsidence values calculated with all programmes, the degrees of tilt and the curvature between neighbouring points can be ascertained in any desired direction, for the lines of a regular points grid or with the aid of a rotatable cross of points (Fig. 214). If it is assumed that displacement is proportional to tilt, then extension and compression strains can also be derived with these programmes.

Nevertheless, with flat stratification, Ruhrkohle AG in its computer programme calculate also tilt and curvature at the point P directly with influence

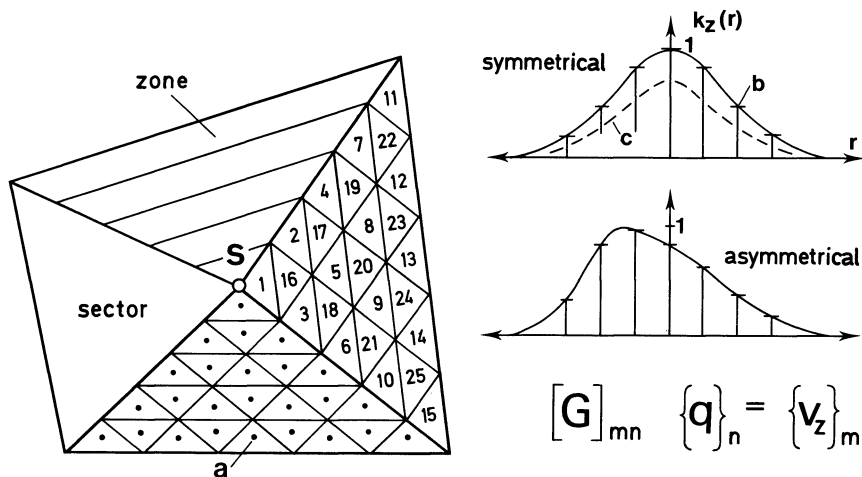


Fig. 202. Subdivision of the extraction area into triangular elements from the focal point S outwards (left); determination of the influence function with given points (right) (after Neuhaus, 1978). *a* Focal points of individual elements with coordinates x, y ; *b* given support points; *c* influence function with reduced weighting

functions, by differentiating the subsidence function in Eq. (194) once and then again (see Sect. 9.4). This shortens the calculation process, because then no subsidence movements for the neighbouring points $P_1 - P_4$ (Fig. 214) need be calculated (see Appendix II).

8.4 Methods Employing Theoretical Models

In calculation procedures employing theoretical models, the sagging of the whole overlying rock body must be calculated with the aid of mechanical or statistical laws, to derive information on ground movement from deformation occurring in the surface of the model. Material for the model is either a cohesionless, stochastic, or elastic medium – in one case even a plastic medium – and the strata model is generally constructed homogeneously from this material. With a two-layer model consisting, for example, of a stochastic medium over the workings and an elastic upper zone, deformation through caving in the lower roof can be overlain by elastic deformation in the upper strata (Gil, 1966).

8.4.1 The Stochastic Model

The statistical method for the development of movement in a *cohesionless model-medium*, dealt with in Sect. 3.34, led to Eq. (74) for the elementary trough $w(r)$ created in a higher horizon by an extraction element. If this exponential function – to which Eq. (76) applies for the strata-fabric value $\phi(h)$ – is intro-

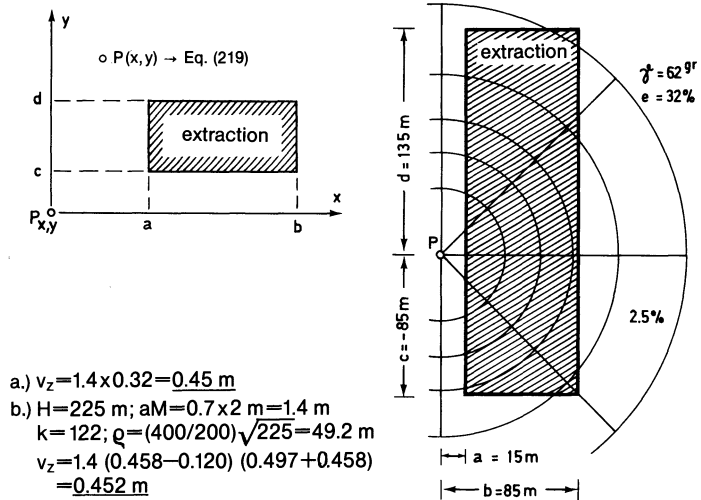


Fig. 203. The calculation of subsidence for the point P. *a* Using a grid after Bräuner (*right*); *b* using tabulated values after Szpetkowski (*left*)

duced into the general Eq. (160), we obtain – after solving the integral expression and converting with $aM = 1$ – the equation

$$r = R \sqrt{\frac{1}{\pi} \ln \frac{1}{1 - v_z}} \quad (218)$$

for the calculation of zonal radii in an integration grid (Bräuner, 1961). The zonal radii r_i , given by Eq. (218) for the subsidence values $v_z = 20\%$, 40% , 60% , etc. respectively, agree on the whole with those given by Eq. (166) (Beyer) (Fig. 203). The infinitely large fifth zone (for $v_z = 1$, r becomes ∞) can be limited to R without the calculated subsidence value for points in the interior of the trough being out by more than 4% .

The fundamental equation of the stochastic method, Eq. (72), can also be solved for tabular evaluation of rectangular working panels in flat-lying measures (Szpetkowski, 1972). After conversion for the subsidence of a surface point $P(x, y)$ when working an area of seam (a, b, c, d) at a depth of H , we obtain an exponential function

$$v_z = \frac{aM}{4\pi BH} \int_a^b \exp \left[-\frac{(x - \xi)^2}{4BH} \right] d\xi \int_c^d \exp \left[-\frac{(y - \eta)^2}{4BH} \right] d\eta \quad (219)$$

in which the symbols ξ and η apply to the working, and the strata constant B can be calculated from

$$\frac{1}{\sqrt{2B}} = \frac{k\sqrt{2\pi}}{1000}$$

where k is a characteristic quantity of the strata. If the calculation point P is taken as being at the origin of the coordinates ($x = 0$, $y = 0$), Eq. (219) can be simplified as follows:

$$v_z = aM A_x A_y \quad (220)$$

with

$$A_x = \Theta(v_2) - \Theta(v_1) \quad \text{and} \quad A_y = \Theta(v_4) - \Theta(v_3)$$

also

$$v_1 = \frac{a}{\rho}; \quad v_2 = \frac{b}{\rho}; \quad v_3 = \frac{c}{\rho}; \quad v_4 = \frac{d}{\rho}$$

$$\rho = 400 k^{-1} H^{\frac{1}{2}}.$$

The values of the function Θ in Eq. (220)

$$\Theta(v) = \frac{1}{\sqrt{2\pi}} \int_0^v e^{-\frac{t^2}{2}} dt$$

are listed in Table 16.

An example calculated on the basis of Eq. (220) and Table 16 is given in Fig. 203. For instance, for the first functional value we obtain:

$$b/\rho = 85/49.2 = 1.73; \quad \Theta = 0.458.$$

The value of the characteristic strata factor

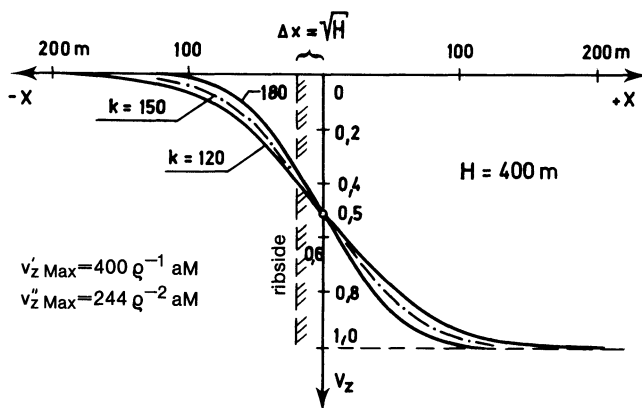
$$k = \frac{v'_{z\max}}{v_{z\max}} \sqrt{H}; \quad k = 122 + 55 \frac{H_1}{H} \quad (221)$$

can be derived from the greatest degree of tilt $v'_{z\max}$ in the subsidence profile or from the thickness H_1 of the caprock. In Upper Silesia this factor is on average 122, and it rises to values of around 180 if the Carboniferous is covered by a thick layer of caprock. It can be seen from Fig. 203 that subsidence values arrived at with an integration grid agree closely with those obtained from tables based on Eq. (220).

Table 16. Values of the integral function θ for calculating subsidence on the basis of the stochastic theory – Eq. (220) (after Szpetkowski)

$\frac{n}{\rho}$	$\theta(v)$	$\frac{n}{\rho}$	$\theta(v)$	$\frac{n}{\rho}$	$\theta(v)$	$\frac{n}{\rho}$	$\theta(v)$
0.0	0.00 000	1.0	0.34 134	2.0	0.47 725	3.0	0.49 865
0.1	0.03 983	1.1	0.36 433	2.1	0.48 214	3.1	0.49 903
0.2	0.07 926	1.2	0.38 493	2.2	0.48 610	3.2	0.49 931
0.3	0.11 791	1.3	0.40 320	2.3	0.48 928	3.3	0.49 952
0.4	0.15 542	1.4	0.41 924	2.4	0.49 180	3.4	0.49 966
0.5	0.19 146	1.5	0.43 319	2.5	0.49 379		
0.6	0.22 575	1.6	0.44 520	2.6	0.49 534		
0.7	0.25 804	1.7	0.45 543	2.7	0.49 653		
0.8	0.28 814	1.8	0.46 407	2.8	0.49 745		
0.9	0.31 594	1.9	0.47 128	2.9	0.49 813		

Fig. 204. Normal subsidence curves for various k values (after Szpetkowski, 1972). Formulae for tilt in mm/m and curvature in $1/\text{km}$



The influence of the strata characteristic k on the steepness of the normal subsidence curve is evident from Fig. 204: the higher the k value, the steeper the curve. The ribsides stands at a distance of

$$-\Delta x = \sqrt{H} \quad (222)$$

from the zero point of the abscissa in the subsidence profile curve, since the point of half-maximum subsidence lies within the projection of the extraction panel in the Upper Silesian coalfield as elsewhere.

8.4.2 The Elastic Model

On the assumption that the cavity effect of an infinitesimal particle of extraction can be represented by a resulting force F at the point S over that infinitesimal element (Fig. 70), an elastic medium yields a function

$$w_0 = \frac{F}{4\pi G} \left(\frac{1}{R} + \frac{H^2}{R^3} \right), \quad (223)$$

for the elementary trough at the surface of the model if, in Eq. (60), the surface with $z = 0$ is introduced for the calculation horizon, the depth of the working with $c = H$ for the depth at which the force acts, and R for the expression $\sqrt{r^2 + H^2} = R_1 = R_2$ (Bräuner, 1959). Equation (223), which does not fulfil the boundary conditions mentioned in Sect. 3.32 – namely, $w = 0$ at the mining horizon ($z = H$) and at the limit-angle line above ground ($r = H \cot \gamma$) – applies to uninterrupted propagation of subsidence in an elastic medium. A comparison with other influence functions [e.g., Eq. (168)] shows that, for practical subsidence calculation, it will suffice to limit the function value at the position $r = H \cot \gamma$ to 10% or 20% of the maximum value. It is best to take a factor of $k \approx 4.5$ ($k^2 = 20$) for the radial variable r , so that Eq. (223) becomes

$$w_0 = \text{Const.} \cdot \frac{1}{\sqrt{20r^2 + H^2}} + \frac{H^2}{\sqrt{20r^2 + H^2}^3}. \quad (224)$$

This function pays regard to the concept of the practical limit angle marking the point where ground movements are, if not at end, then at least harmless. Subsidence at the lower horizon ($z > H$), which by the other boundary condition would have to be zero, can be left out of consideration in a movement calculation limited to the surface.

Equation (224) is introduced directly into Eq. (160) if, after superposition of all elementary troughs created at the surface by extraction elements lying within the critical area, the formula

$$v_z = \pi \frac{r^2}{\sqrt{20r^2 + H^2}} + C \quad (225)$$

for the construction of an integration grid requires to be stated. With a limit angle of 60 gr, zonal radii for the elastic model calculated by Eq. (225) become $r_1 = 0.28$, $r_2 = 0.44$, $r_3 = 0.64$, $r_4 = 0.81$, and $r_5 = R = 1.00$. This integration grid shows great similarity with Beyer's grid (Table 12). Through the higher value placed on the fifth zone – it is narrower than in other methods – the calculation error stemming from premature limitation of the area of influence is cancelled out. These investigations have demonstrated that elastic theory can be applied to the area effects of strata movement.

8.4.3 The Plastic Model

For a rock body taken as being stiffly plastic, only the end-movements of points at the surface of a vertical semi-plane can at present be justifiably calculated (Cernyi, 1966). In doing this it is assumed that

- a) the amount of the incompressible medium passing through a given horizon above the horizontal extraction opening is constant and equal to the volume

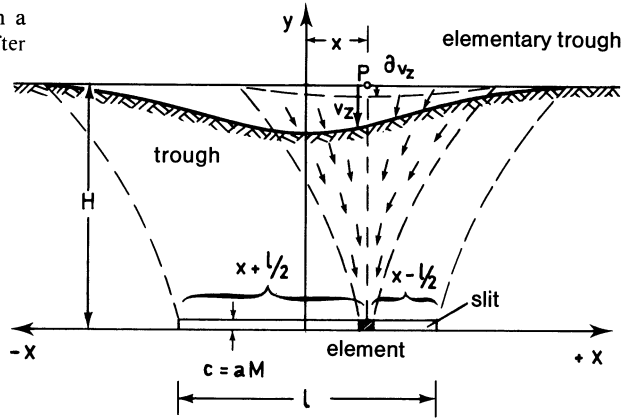
$$V = -2 \int_0^{\infty} c(x) dx \quad (226)$$

- of the convergence c in the mine excavation;
- b) throughout the length l of the extraction opening, movement of the roof bed amounts to $c = aM$ vertically and to $v_x = 0$ horizontally;
- c) displacement throughout the vertical centre-axis y of the model amounts to $v_x = 0$ (Fig. 205);
- d) movement gradually dies out as the coordinates x and y tend to infinity;
- e) the movement of points in the medium over each extraction element is independent of the movement over neighbouring elements.

Leaving out intermediate calculations, Eq. (67) gives the vector

$$\partial\psi = -\partial v_z \partial x = -\frac{aM}{2} \tanh\left(\omega \frac{x}{H}\right) dx \quad (227)$$

Fig. 205. Stiffly plastic flow in a semi-plane slit horizontally. (After Cernyi, 1966)



for the subsidence component of movement, corresponding to the elementary trough at the surface ($y = H$). Here x is the running coordinate of the element of volume, $\tanh$ the hyperbolic tangential function, and

$$\omega = \frac{v_{zm}}{4\eta} \quad (228)$$

a characteristic value for the rheological properties of the rock body, which depends on the average rate of subsidence v_{zm} and the kinematic stiffness of the medium η .

With integration over the length of the extraction, Eq. (227) yields the subsidence end-value

$$v_z = -\frac{aM}{2} \left[\tanh \left(\omega \frac{x - 0.5l}{H} \right) - \tanh \left(\omega \frac{x + 0.5l}{H} \right) \right] \quad (229)$$

for the running profile coordinate x at the surface. The shape of the curve and the position of the trough margin can be adjusted to observed profile curves by the choice of an appropriate ω value. In the Lemberg coal district this ω value is 4.

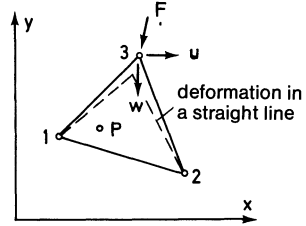
By using a model, a similar *hyperbolic function* has been obtained for the surface subsidence of a slit gelatine body (King and Whetton, 1957).

Full, critical subsidence over the centre of the extraction ($x = 0$) is attained, according to Eq. (229), theoretically only with infinitely long panels. Nevertheless, maximum subsidence over an extraction $l = 1000$ m long and 800 m deep is as much as 98.6% of critical subsidence aM . For a subcritical area with $l = 250$ m, $H = 800$ m and $\omega = 4$, subsidence values amount to 42.4% of critical subsidence over the ribside, and to 55.0% over the centre of the extraction. Thus in subcritical extraction, the point of half-maximum subsidence – in this case 27.5% – lies over the zone ahead of the face (see Fig. 179). With increasing width of extraction it moves towards the ribside, and over the margin of a critical area reaches a value of 50% v_{zMax} , as in integration-grid methods. At the trough margin, subsidence at a distance of $x = 500$ m and $x = 650$ m ($\gamma \approx 60$ gr) decreases to 2.1% and 0.45% respectively.

Table 17. Schematic for computation with finite elementsField-of-movement coefficients α for elemental and nodal points:

$$\left. \begin{aligned} u_i &= \alpha_1 + \alpha_2 x_i + \alpha_3 y_i \\ w_i &= \alpha_4 + \alpha_5 x_i + \alpha_6 y_i \\ \{\delta\} &= [K] \{\alpha\} \end{aligned} \right\} \text{Compatibility condition}$$

$$\text{Coordinates matrix } [K] = \begin{pmatrix} 1 & x_1 & y_1 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 1 & x_1 & y_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}_{6.6}$$

Strains in interior of element resulting from nodal-point movement δ : triangular element

$$\{\epsilon\} = \begin{pmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{pmatrix} \{\delta\} = [D] \{\delta\} \quad \text{Structural matrix } [D] = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{pmatrix} \cdot [K]^{-1}$$

Stresses in element resulting from strain or from nodal-point movement:

$$\{\sigma\} = [C] \{\epsilon\} = [C] [D] \{\delta\} \quad \text{Elasticity matrix } [C] = \frac{E}{1-\nu^2} \begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{pmatrix}$$

Nodal-point forces F resulting from external influence and internal stress:Virtual internal work = virtual internal strain $\times$ internal stress

$$\bar{W}_i = \iint \{\epsilon\}^T \{\sigma\} dx dy = \iint \{\delta\}^T [D]^T [C] [D] \{\delta\} dx dy$$

Virtual external work, equivalent to nodal-point travel

$$\bar{W}_a = \{\delta\}^T \{F\}$$

From equilibrium between virtual external and internal work there follows, with $\bar{\delta} = 1$

$$\{F\}_6 = \iint [D]^T [C] [D] dx dy \{\delta\} = [S] \{\delta\}$$

Stiffness matrix $[S] =$

$$\iint [D]^T [C] [D] dx dy$$

Travel/force relationship

Dead weight p (in g/cm^3) of the element, converted to nodal-point forces and added to the external work:

$$\{F\} = \frac{1}{3} p A \{1 \ 1 \ 1 \ 0 \ 0 \ 0\}^T$$

Elemental surface A , thickness 1

Additive assembling of separate elements into a system:

$$\begin{aligned} \{F\}_{\text{Syst.}} &= [S]_{\text{Syst.}} \{\delta\}_{\text{Syst.}} \\ \{\delta\}_{\text{Syst.}} &= [S]_{\text{Syst.}} \{F\}_{\text{Syst.}} \end{aligned}$$

System stiffness matrix

 $[S]_{\text{Syst.}}$ 2n lines and columns with n nodes

matrix determine the magnitude relationship between each nodal-point force and nodal-point movement.

Applied to the rock mass, investigations into the choice of element form and boundary conditions have established the following.

With triangular elements, irregularly formed systems can be satisfactorily simulated. A close sequence of elements must be adopted, however, as stresses and strains vary only from element to element. Rectangular elements, on the

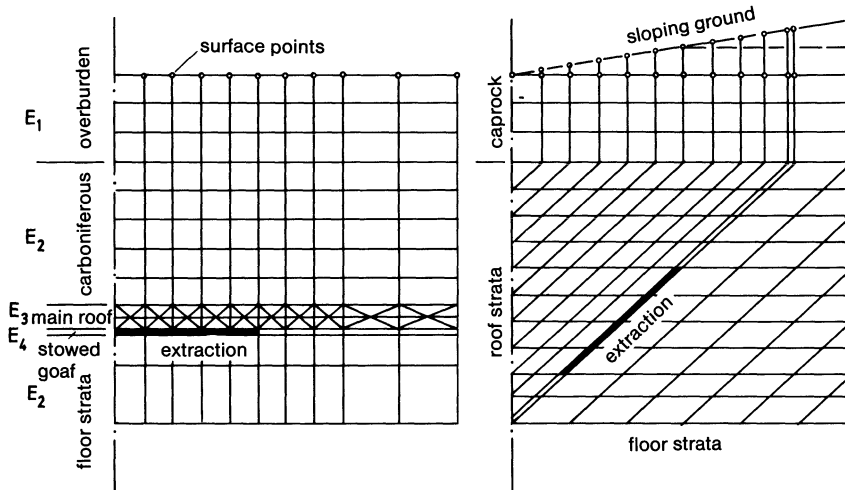


Fig. 207. Grid of triangular or quadrangular finite elements for flat (*left*) or steep (*right*) stratification. E = modulus of elasticity

other hand, because of the linear change in the interior of an element, may exhibit larger measurements, with the result that fewer elements are needed. A combination of triangular and rectangular elements is thus found to be the optimal arrangement (Fig. 207). With thin, weak strata, slip planes in the rock mass can be simulated. The elements grid can be limited in the forefield where no further extraction influences are to be expected in the rock body, or where movements – for example, in the roof or floor stratum – are known and given in advance. If the movement process is symmetrical to the vertical central axis of the working, only one half of the grid need be calculated. The nodal points of the axis are then maintained in a horizontal direction. In calculating, the measurements of the original system in centimetres are used – i. e., without conversion via a model scale.

For a vertical section of strata 1500 m long and 500 m high, 500 elements will be sufficient, with measurements of 150×100 m in the upper strata and 25×10 m in the main roof. As material values, an elasticity modulus (E modulus) of 1000 MN/m^2 for compressive stress and 10 MN/m^2 for tensional stress can be adopted for the caprock, 10,000 or 100 MN/m^2 for Carboniferous strata, including seams, and 6 MN/m^2 for stowage material in the extraction area. Poisson's ratio for the caprock is between 0.3 and 0.4, and for the Carboniferous 0–0.3. The varying elasticity modulus for compression and tension necessitates up to three computer runs, using the Gauss-Seidel iteration method. In these, the grid is first worked through with the elasticity modulus for compression. For those elements in which tensional stresses are established, the relevant E modulus for tension is introduced on the second and third run. It is not necessary to prepare for each numbered nodal point a punched card with the coordinate values of the whole system, and with the limitations on movement; nor a card for each element with details of number, respective nodal-point number, and material code. It will be sufficient to describe only the start and finish point of

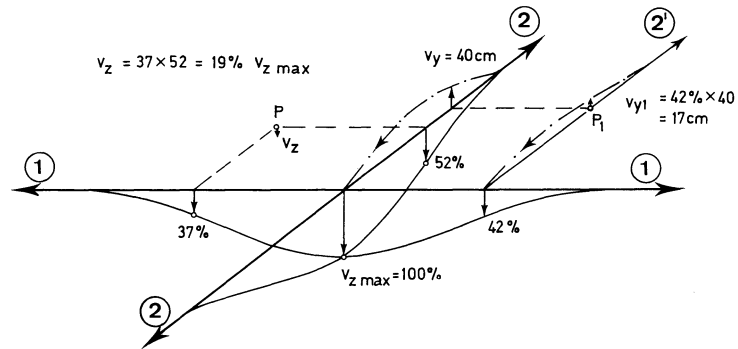


Fig. 208. Interpolation of subsidence and displacement values for points lying aside from the diametrical profile curves (1) and (2)

each grid row or column, since the programme⁹ will automatically calculate the intermediate points on the basis of equal spacing. The code number for each material (sand, shale, sandstone, etc.) its E modulus, Poisson's ratio, and dead weight ($2 - 2.5 \text{ g/cm}^3$ or $20 - 25 \text{ MN/m}^3$) are entered on a further punched card. The results are obtained in tabular form.

Before the actual calculation, a *vertical grid* with no mine excavation is worked through, to determine the model's dead-weight subsidence in the undisturbed state. The differences on calculating the slit model, in which the deformations in overlying strata lead to additional stresses and strains in the elements as well as to the development of external nodal-point forces, are then the extraction influences that it is required to find. The magnitude of the movement is influenced less by the moduli of elasticity — these alter the state of stress in the elements grid — as by the transverse-strain (Poisson's) ratios. This is because the sagging of the main roof is limited by the volume of extraction, and additional deformation is possible only by an escape of the material in a horizontal direction. Thus, for example, the trough becomes narrower if the calculation is made, as in soil mechanics, with a material free from transverse strain ($\nu = 0$ for Carboniferous rock).

Subsidence and displacement at surface points lying outside the section plane are calculated by an *interpolation method*. Two intersecting, diametrical sections 1 and 2 are laid across the field of extraction (Fig. 208). Subsidence values calculated for these sections do not as a rule agree at their point of intersection. Consequently, a mean value $\bar{v}_{z \max}$ has to be taken and all subsidence values v_{zi} in both sections adjusted in accordance with the relation

$$v_{zi \text{ cor.}} = v_{zi} \frac{\bar{v}_{z \max}}{v_{z \max}}. \quad (231)$$

The subsidence of a side-point P is obtained from the respective "subsidence coordinates" of the sections

$$v_z = v_{z1} v_{z2} \bar{v}_{z \max}, \quad (232)$$

⁹ E. g. the programme of Wilson, Berkeley, USA.

with the "coordinates value" v_{z1} or v_{z2} (in %) being found from the linearly interpolated subsidence values of the neighbouring profile nodal points. This programmed interpolation method is thus based on the assumption that, on parallel displacement of a subsidence profile curve, all subsidence values alter to the same extent as those at the respective points of intersection with the other profile curve (see also Fig. 165).

The displacement components of side points are established in the same way. The displacement curve 2', which has been shifted in parallel as far as the point P_1 , is reduced to the same degree as subsidence at the point of intersection with the other profile curve 1 has decreased. In rectangular extraction areas, the curve 2' retains its original diametrical length l . The same applies to the displacement curve parallel to section 1. The total displacement v of the side point is the resultant of the two parallel curve components.

The effect of the line of sag of the roof beds on the shape of the ground-movement curve is represented in Fig. 209. Where strata sag freely, deeper and steeper troughs arise at the surface than with a predetermined roof settlement, beginning 25 m ahead of the face. The example shows that with the finite-elements method it is possible to reproduce realistic roof settlement and floor heave. Delimiting the zone of influence with a limit angle is unnecessary, since deformation and movement in the model, from the lowest extraction element to the surface elements, develop according to mechanical laws approximating closely to those in the rock mass.

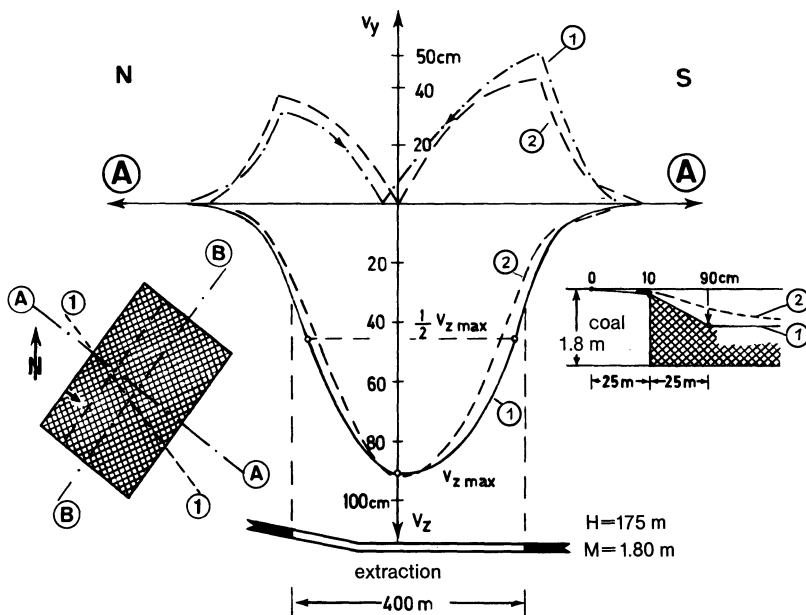
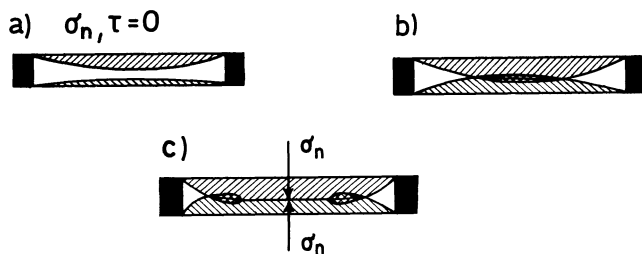


Fig. 209. Subsidence and displacement movements calculated by the finite-elements method. (1) With predetermined roof settlement, and (2) with free sagging of the roof beds (after W. Müller, 1973). Measured maxima on the profile line 1-1: subsidence 91 cm, displacement 43 cm in the south, 31 cm in the north

Fig. 210. Closing up the mine excavation when using the finite-elements method (after Neuhaus, 1978). **a** Working area still open; **b** overlapping; **c** equal perpendicular stress in roof and floor



In addition, calculation of the model can be adapted both to non-linear and to rheological material behaviour. In doing this, the loading is increased by stages, and a check is made after each change in load to see whether the deformation energy of the individual elements has exceeded a certain value; if so, the modulus of elasticity for the element in question is lowered in a second computer run. The curved, inelastic deformation characteristic is approximated to by a line of traverse (Voigt and Dahl, 1969).

A particular problem is presented by the boundary conditions for the lines delimiting the mine excavation. Where settlement of the roof is not given, it can happen with the elements model that the roof bed sinks down through the floor line and overlaps with the lifted floor (Fig. 210). If roof settlement and floor lift are given – and here the sum of absolute settlement and absolute lift must equal the seam thickness mined M or the product aM – then contact stresses arise at the line where they meet, which are difficult to handle in calculation. Whereas previously the perpendicular stresses and shear stresses at the free limit planes of the mine excavation were equal to zero, now the perpendicular stresses in the immediate roof and floor must be of equal magnitude and pass uniformly through the line of contact. The weight of the rock mass must therefore be temporarily reduced in accordance with the principle of relaxation, and the line of contact shifted gradually up or down until these conditions are fulfilled, except at the left- and right-hand margins of the mine excavation (Neuhaus, 1978).

Another problem, which can be solved only approximately, is friction on the rupture surfaces of geological faults or joints and on the horizontal limit planes of strata (slip planes). If stresses normal to the rupture plane are constant, shear stress τ on movement of the upper body A develops as in Fig. 211 a. If the contact

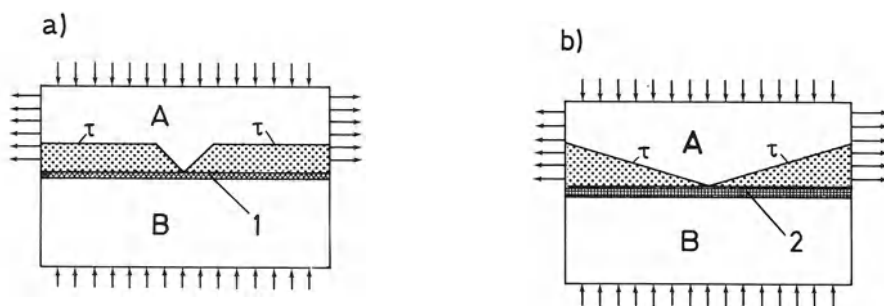


Fig. 211. Friction on the fracture surfaces of geological faults (after Neuhaus, 1978). (1) Contact surface; (2) weak intermediate layer

surface is replaced by a weak intermediate layer of the order of magnitude of the expected displacement, then the stress patterns tend to look like those in Fig. 211b. Shear stresses in the elements model 211b do not become zero if displacement ceases, since the intermediate layer is elastic and blocks A and B remain bound to each other at the nodes. To achieve a plastic displacement of the blocks, the nodes would have to be progressively loosened during the displacement; but calculation of the coupling conditions is complicated.

Investigations with the strata model from finite elements have shown that the asymmetrical trough shape can be ascribed to the interim, irreversible shear movements of the strata limit planes, and to the peak of stowage-material pressure arising over the stationary face as extraction proceeds; and that the steep, acute-angled trough shape can be ascribed to fracturing of the rock mass over the ribside at right angles to the stratification, and to the reduced bending strength of the roof in repeatedly worked strata.

This plastic aspect of deformation in intensively worked strata, which should not be overlooked, can be allowed for as follows in the finite-elements method of calculation (Fig. 212):

Between strata of varying hardness there are inserted weak slip layers which increase in thickness, wedge-like, in the direction of the face advance. The gravity force g_i of the strata elements is varied from place to place in such a way that vertical strata pressure σ_z becomes greater on the side of the stationary face edge than on the side of the advancing face. The funnel shape of the trough develops by virtue of long, perpendicular cracks – reproduced with weak, thin layers. The result of this fissuring is that perpendicular shear movement is favoured, rather than bend deformation. A similar effect can be achieved by the

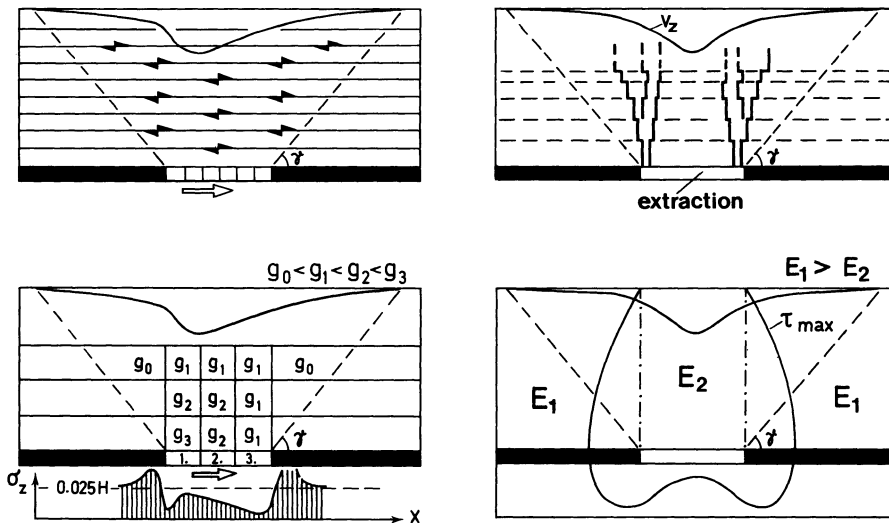


Fig. 212. The rock-mechanics explanation for the asymmetrical trough profile (*left*) and the funnel-shaped trough profile (*right*). Strata model with horizontal slip planes (*top left*); model from finite elements with differing fields of gravity (*bottom left*), with vertical cracks (*top right*), and with bent lines of slip and varying moduli of elasticity (*bottom right*)

choice of different moduli of elasticity for the solid zone ahead of the face (E modulus high) and the weak, easily bent mine roof (E modulus low). The zone boundaries must be determined empirically or as lines of slip ($\tau_{\max}$ trajectories) from the finite shear resistance of the strata model (Kratzsch, 1979).

The elements method can also be used for inclined and steep stratification, by placing the rows of elements at an angle. Even surface rock lying discordantly and a sloping or hilly surface can be accommodated in the grid of elements (Fig. 207, right).

9. Pre-calculation of Ground Displacement

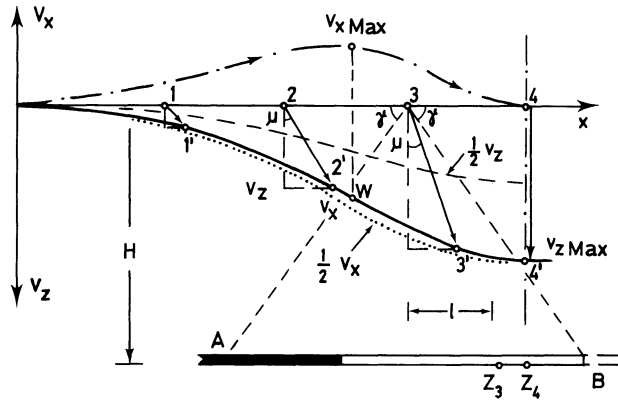
9.1 Special Features of Displacement Calculation

Pre-calculation of the horizontal components of ground movement causes more difficulty, even today, than the calculation of subsidence. There are a number of reasons for this. For one thing, the measurement of linear change along lines of markers, unlike the more frequent operation of taking levels, can rarely be pursued over an extended period, as it requires a great deal of time and trouble, and many markers become lost in the course of time as a result of earth works. The values obtained in practice cannot yet therefore be regarded as statistically sound for all shapes and sizes of extraction area. Above all, the magnitude of critical displacement v_{Max} , which cannot be deduced as easily as critical subsidence from mining concepts such as seam thickness and type of stowage, is known only approximately. This is because a half or whole critical area, over the ribside of which this peak displacement value occurs, is frequently interrupted at greater depths by fault zones. Depending on the direction of the line being measured, the displacement component v_x or v_y measured is larger or smaller, and it agrees with the resultant displacement v only when observation lines run diametrically across the working (Fig. 121). This fact seriously complicates the comparison and analysis of measurement results. By contrast, the subsidence of points is independent of direction. A further difficulty is that displacement values are very small, not only at the trough margin but even at its centre, and are thus particularly sensitive to errors in measurement or to being overlain by secondary movements of the marker pegs, unconnected with mining.

All investigations so far indicate that the amount of displacement depends very much more on the structure of the overlying rock mass – particularly on the horizontal slip planes in the upper surface strata – than on the amount of subsidence, which alters only slightly from place to place with the petrographically determined trough shape. The horizontal component of point movement has practically no influence on the shape and volume of the trough. If, for example, displacement values are reduced by half, the tilt of the ground over the ribside alters significantly less than if subsidence is reduced by half (Fig. 213). An error in displacement calculation is therefore not so patently noticeable in the trough shape as a faulty subsidence calculation.

In many mining districts in the plains, where ground water is found 2–3 m under the surface, and where only 30% of the costs of mining damage can be ascribed to linear changes in the ground, the calculation of displacement is economically less important than the calculation of subsidence. As, however, larger structures and the typically elongated installations of transportation and public

Fig. 213. Geometrical relationship between the subsidence and the displacement of point 2 to position 2'. A – B: area affecting point 3



utilities are particularly affected by extension and compression damage, there is both a safety and a construction interest in having as reliable a prediction as possible of ground displacements.

The methods, known from subsidence calculation, of empirically found nomograms and integration grids, of influence functions, and of elastic, stochastic, and plastic theoretical models – all are applied to displacement calculation. What is novel is the mechanical interpretation of shares of displacement assigned to extraction elements, which are determined either according to the centre-of-gravity position of the extraction element and its subsidence influence, or from the curvature of the upper strata. In addition, displacement can be derived with a proportionality factor from the amount of tilt, which resembles it in both magnitude and direction. For steep-lying stratification, displacement grids are redrawn in elliptical form by various methods of projection. Because of the mentioned difficulties in measuring displacement, investigations by the finite-elements method and into graphical strata models have become of great importance for the horizontal component of ground movement in recent years.

9.2 The Focal-Point Method

As points undergo a change of position $\vec{V}$ in the direction of the working, the subsidence and displacement components of their movement stand in the following relation as shown in Fig. 121:

$$v = v_z \tan \mu. \quad (233)$$

The extraction area A – B lying within the limit-angle cone in Fig. 213 “attracts” the surface point 3 towards the *focal point* Z_3 – i.e., towards the influence centre (Keinhorst, 1925). If the respective focal-point position Z_i is known, it is possible, from the given point coordinates $P_i(x, y)$ and $Z_i(x, y)$, the depth of working H , and the calculated subsidence v_z , to determine the resultant point displacement v by Eq. (233). In another direction the displacement components amount, by Fig. 121, to:

$$\begin{aligned} v_x &= v \cos \varphi_0 \\ v_y &= v \sin \varphi_0. \end{aligned} \quad (234)$$

If it is assumed that all surface points in the trough have a common focal point Z lying under the centre of the working (see Fig. 225, left), at the point of intersection of the limit-angle lines as produced downwards (Kowalczyk, 1966), then – from the diagrams in Fig. 164, left – the displacement amounts to

$$v_x = v_z \frac{xR}{LH}. \quad (235)$$

As the length of the working increases, so the point Z moves farther down.

Practical displacement calculation by the focal-point theory is usually effected in such a way that, in conjunction with the subsidence determination, the focal points of extraction for the ring sectors of the integration grid being used are determined at the same time.

This method is also used in programmed displacement calculation for inclined and *steep-lying measures* (Fig. 198) in the Ruhr District (Kastenbein, 1970). In the computer programme, the focal points determined for the separate ring sectors of the grid are weighted according to the respective subsidence value of the ring sector and then combined on the basis of coordinates into a common focal point for the complete extraction area within the grid. For the focal point I of a circle sector, it holds good that it must lie on the bisector of the sector angle at a distance of

$$r_I = \frac{2}{3} r \frac{\sin \Delta \varphi / 2}{\Delta \varphi / 2} \quad (236)$$

from the centre of the grid. As can be seen from Fig. 214, the focal-point coordinates

$$\begin{aligned} x_I &= r_I \sin (\varphi + \Delta \varphi / 2) \\ y_I &= r_I \cos (\varphi + \Delta \varphi / 2) \end{aligned} \quad (237)$$

can be determined trigonometrically for each zone of the sector [see Eq. (211)]. Since the determination of proportions is effected from the centre Z of the

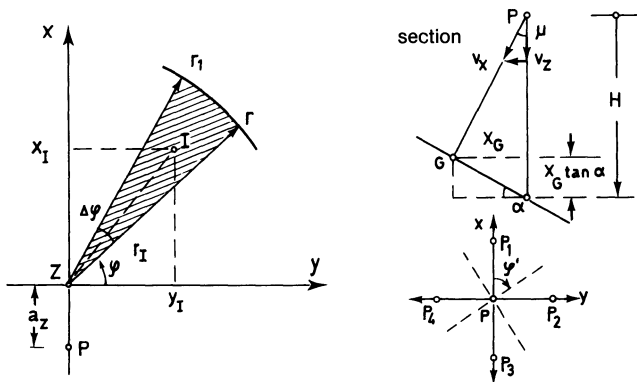
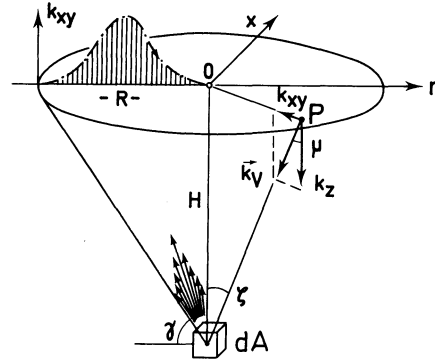


Fig. 214. Determination of coordinates for the focal point I of a sector (*left*). Auxiliary points P_1 to P_4 for the calculation of linear change (*bottom right*)

Fig. 215. Influence of the extraction element dA on horizontal displacement k_{xy} of point P in the direction of the centre O . (Influence function *shaded*)



ellipse, whereas displacement has to be related to the calculation point P , the abscissa of the focal point must be corrected by the amount a_z before the weighted focal point G for the extraction area can be arrived at. Finally, Eq. (233) leads to the components of displacement:

$$\begin{aligned} v_x &= v_z \frac{x_G}{H \pm x_G \tan \alpha} \\ v_y &= v_z \frac{y_G}{H \pm x_G \tan \alpha} . \end{aligned} \quad (238)$$

From the displacements for the four auxiliary points P_1 to P_4 (Fig. 214), which are likewise calculated, the tension and compression strains for the point P can be derived. If it is required to find the displacements and linear changes in a preferred direction φ' , the cross of auxiliary points is rotated by this angle.

9.3 The Method with Influence Functions

Displacement, like subsidence, can be calculated with the aid of influence functions and integration grids. In doing so, the extraction area affecting the surface point P is divided, not into any desired sections as with the focal-point method, but into sections exercising an equal displacement force. For the critical area, this gives a displacement grid of such a type, divided into zones and sectors. For a subcritical extraction it is then only necessary to count up the number of grid areas worked, to be able to determine the displacement components in any direction.

As with subsidence calculation, it is assumed that each extraction element dA at the depth H exercises an influence (a force) on points at the surface within the limit-angle cone, as a result of which the surface point P , for example, in Fig. 215, is displaced towards the extraction element by the amount $k_{\bar{v}}$ at the nadir angle μ (see also Fig. 66). The influence vector $k_{\bar{v}}$ can be analysed into the orthogonal components k_z (subsidence) and k_{xy} (horizontal, radial displacement). This horizontal component k_{xy} becomes zero both at the margin of the limit-angle

cone, where $k_z = 0$, and vertically over the extraction element at the point 0, where $\zeta = \mu = 0$. Between the margin and the centre point 0 is found the greatest value $k_{xy\max}$. On either side of the vertical line there is thus a radiating influence, so that even the influence function $k_{xy}(r)$ has two branches to its curve. A further difference from subsidence, and one adding to the difficulty of calculation, is the fact that the horizontal component of the influence vector is no scalar like subsidence but indeed a vector, leading in the direction of the centre point 0 of the field of influence. The principles of the influence method dealt with in Sect. 8.31 apply also to displacement, except that the principle of constancy of volume has no practical significance here.

9.3.1 Deriving the Grid from the Focal-point Formula

In Fig. 216, $P'BC$ is a narrow sector of the critical area related to P and requires to be divided into annular zones of equal displacement influence. Within this sector, which in all accounts for a share amounting to $\phi/2\pi$ of full, critical subsidence, the area of the hatched, differential zone

$$dA = \phi r dr \quad (239)$$

is responsible, by Eq. (158), for subsidence amounting (Perz, 1948) to:

$$dv_z = \phi a M r k_z(r) dr. \quad (240)$$

Given a sufficiently small angle ϕ at the centre, the focal point I of the area dA can be taken as being at a distance of

$$r = H \tan \mu \quad (241)$$

from P' on the x -axis. Consequently, by Eq. (233), the share of displacement accounted for by the zonal area dA comes to:

$$dv_x = dv_z \tan \mu = \frac{\phi a M}{H} r^2 k_z(r) dr. \quad (242)$$

Since each zonal sector $dA(r)$ accounts for the same $\phi/2\pi$ share of the respective critical area's subsidence, the share of displacement of the sector $P'BC$ can be calculated by integration within the limits $x = 0$ and $x = R$:

$$K_{x\text{sector}} = \frac{\phi a M}{H} \int_0^R r^2 k_z(r) dr. \quad (243)$$

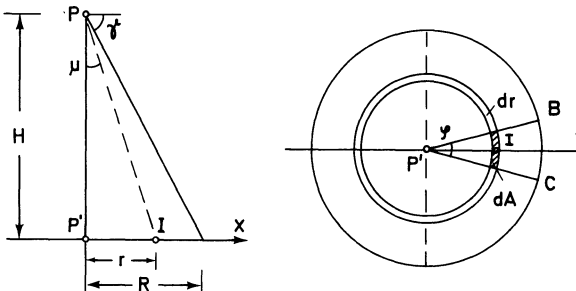


Fig. 216. Division of the critical area into zones of equal influence for the calculation of displacement, after Perz

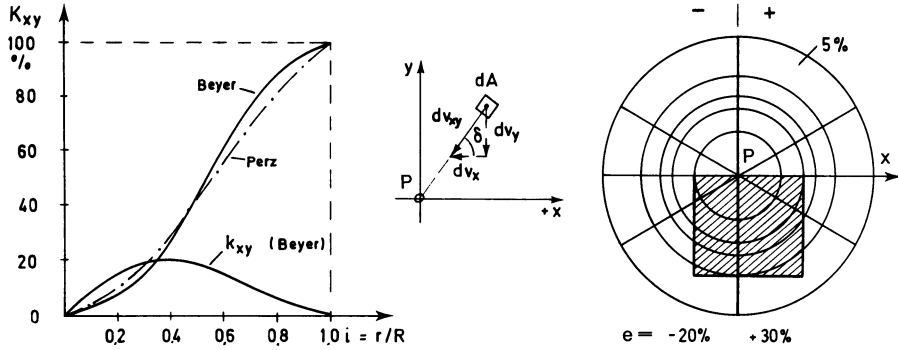


Fig. 217. Construction of an integration grid for the calculation of displacement

If a distributive function of the Eqs. (165) to (168) is introduced into Eq. (243) for $k_z(r)$, and the integral curve K_x is drawn (Fig. 217, left), then the radii for dividing the critical area into zones of equal influence can be read from the diagram (Table 12) in the same way as when constructing a subsidence grid. This is because, owing to the rotational symmetry of the shares of influence and the diametrical direction of the displacement resultants, the same zonal division on the x -axis applies to each differently lying sector.

For example, starting from the zonal angles and letting $\mu = \zeta$, Eq. (165) (Bals), in conjunction with Eq. (242), gives an influence function¹ for the infinitesimal shares of displacement

$$k_x = c \cos^2 \zeta \tan \zeta d\zeta = c \cos \zeta \sin \zeta d\zeta = c \sin 2\zeta \quad (244)$$

and an integral function derived in the plane of section

$$K_x = c \int_{\zeta_0}^{\zeta_g} \sin 2\zeta d\zeta = c \frac{1}{2} \sin^2 \zeta \quad (245)$$

for the division into zones (Flächenträger, 1938; Perz, 1948).

If from a division into zones such as this it is desired to obtain the displacement in a particular direction (e. g., in $+x$ — see Fig. 217, centre), and if the deviation from this direction of the diametrical influence-direction for an extraction element located to one side is designated as δ , then

$$dv_x = dv_{xy} \cos \delta \quad (246)$$

and

$$v_x = v_{xy} \sin \delta \quad (247)$$

will be the components of the shares of displacement in the direction concerned of lateral extraction elements [see Eqs. (263) and (265)].

For the calculation of displacement, with a given direction of determination, zonal rings must therefore be divided in accordance with a sine function into

¹ If the x -axis passes through the centre of the trough, k_{xy} and K_{xy} may be written for short as k_x and K_x . The subscript xy is intended to make clear the radial direction towards the centre and is omitted with dv and v , as in Fig. 121.

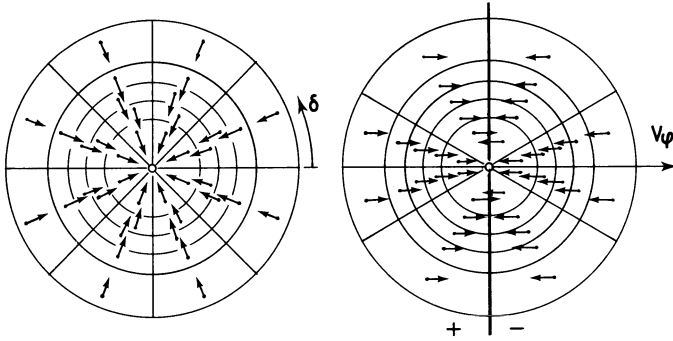


Fig. 218. Influence of zones and sectors on the direction of displacement of the central point

sectors of equal displacement effect. To divide the quadrant into two sectors or three, the following sector angles δ are reckoned from the x-axis:

$$\begin{aligned} \sin \delta &= 1/2 & \sin \delta &= 1/3; 2/3 \\ \delta &= 33.3 & & 21.6; 46.4 \text{ gr.} \end{aligned} \quad (248)$$

The critical area is usually subdivided into five zones and eight sectors, as in subsidence calculation. Figure 218 shows a critical area subdivided into zones of equal influence. In dividing the zones into sectors of equal angle at the centre (Fig. 218, left), as with subsidence grids, the displacement of equal magnitude which each extracted mesh of the grid causes at the central point is directed radially inwards. Only a division of sectors in accordance with Eq. (248) gives the components of displacement in the direction of the v_ϕ -axis caused at the central point by each grid mesh (Fig. 218, right).

In comparison with the subsidence grid (Fig. 180) it is noticeable that in the *displacement grid* the inner zone is larger and the third zone is narrower (Fig. 217, right). Extraction of half the critical area alone causes the greatest amount of displacement v_{Max} , and further extraction in the other half ($-x$) reduces the displacement of the surface point P. Each of the 20 grid meshes in half a critical area thus has an influence value of $e = 100/20 = 5\% v_{\text{Max}}$.

The transparent grid is placed with its centre on the calculation point P of the mine plan and rotated until the $+x$ arrow coincides with the measuring line (or railway trace, building axis, etc.). For this direction the displacement value is obtained by determining the influence factors for each half of the grid separately and then adding them together. Multiplying the sum of positive and negative influence factors with the critical (full) displacement v_{Max} gives the required point displacement in the direction of the x-axis:

$$\pm v_x = \pm e v_{\text{Max}}. \quad (249)$$

For example, for the shaded extraction area in Fig. 217, right, we obtain an influence factor in relation to the central surface point P of

$$e = +30\% - 20\% = +10\%.$$

If the observed critical displacement from working a 2 m thick seam by caving is 80 cm – i.e., approx. 45% of critical subsidence – then the calculation gives a point displacement in the x-axis of

$$v_x = +0.10 \times 80 = +8 \text{ cm}.$$

9.3.2 Deriving Displacement from Curvature

By analogy with the model proposition for a flexed bar or beam, displacement can be derived from curvature of the surface layer of ground (Beyer, 1944). On the upper side of the bar illustrated in Fig. 219, bottom, with a trough-like deformation, compressive stress will be found, and on the underside² tensile stress. In between, down the middle of the bar, lies the stress- and strain-free neutral fibre. If perpendiculars are erected at the points a and b on the neutral fibre, they will intersect at o, and the lines $\overline{ao}$ and $\overline{bo}$ will represent the radius of curvature ρ_z . Before the bar was flexed, the perpendiculars ran parallel and passed through the surface points 1 and 2. If it is assumed, in accordance with *Bernoulli's theorem*, that cross-sections of the bar which were vertical before deformation will remain in a plane after bending and at right angles to the neutral fibre, then it will be seen from Fig. 219 that the upper surface of the bar from 1' to 2' has shortened by the distances 1'–3 and 2'–4 approximately. That implies a horizontal displacement v_x from 1 to 1' and from 2 to 2' by a distance of

$$v_x = \frac{d}{\rho_z} l, \quad (250)$$

which results from the aspect ratios of the similar sectors omb and b2'4. Given constant curvature $v_z'' = 1/\rho$, compression between c and 2' of the upper surface of the bar will amount to

$$-\varepsilon = \frac{v_x}{l} = d v_z'' . \quad (251)^3$$

With a variable curvature v_z'' of the elastic surface zone of the ground, the displacement of a trough point P_1 (Fig. 220) can on this basis be determined from the infinitesimal linear changes $\varepsilon = dv_x/dx$ between the trough margin ($x = 0$) and $P_1(x)$, constantly added together

$$v_x = \int_0^x \varepsilon dx = \int_0^x d v_z'' dx \quad (252)$$

provided the respective distance d from the surface of the ground to the neutral fibre of the surface zone is known. This distance will be zero at the trough margin and reach its greatest value at the trough centre. Integration of curvature $v_z'' = dv_z'/dx$ in Eq. (252) from the trough margin up to the point P_1 leads to the

² Sliding friction at the limit planes of the beds is here ignored (Fig. 38).

³ In Eq. (251) v_x signifies Δv_x between the points 1 or 2 and the centre c (distance l), because the point c is not displaced: $v_x(c) = 0$ (see Fig. 219).

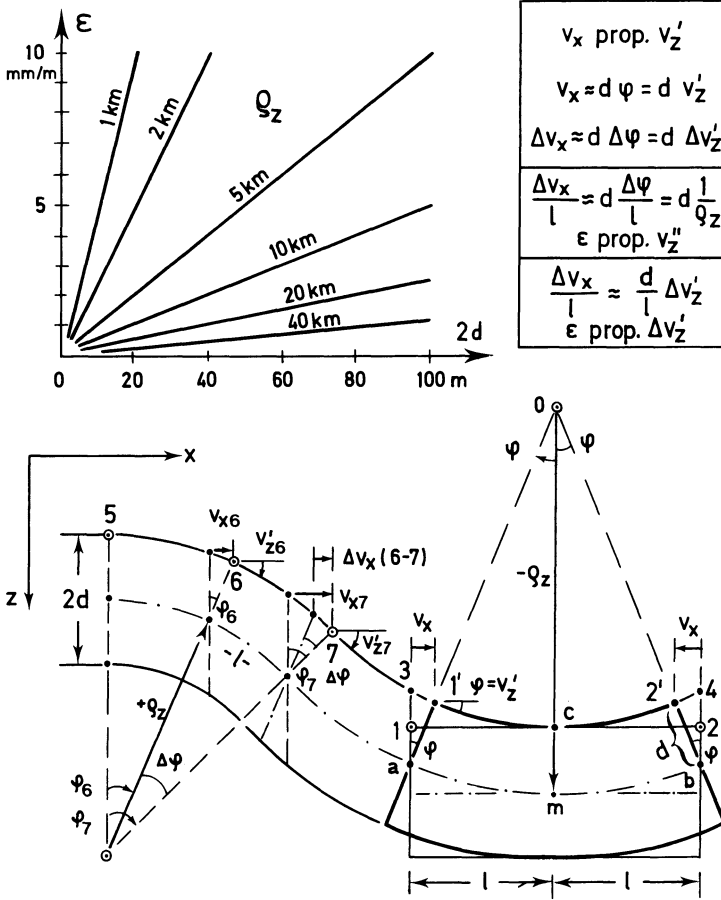


Fig. 219. Deriving linear change and point displacement in a flexed beam (*bottom*), after Bernoulli's theorem. *Top*: relationship between bed thickness $2d$, radius of curvature ρ_z , and linear change $\pm \epsilon$ at the upper side of the beam, as in Eq. (251)

amount of tilt v'_z at point P_1 , since – by Fig. 137 – curvature is the infinitesimal alteration of tilt.

To be able to calculate the displacement of a surface point P_1 directly by this method, with an integration grid – i.e., without the detour via the subsidence-profile curve – it is first necessary to determine the differential shares dv'_z in the tilt of the locality around P_1 exerted by the extraction-area elements dA acting on P_1 . The product of this tilt differential dv'_z and the local deformation depth d at the point P_1 gives the share of displacement

$$k_x = dv'_z d \quad (253)$$

of the extraction element at the point P_1 (Fig. 220) acting in the direction of investigation x . Since the first derivative of subsidence function k_z [Eq. (166)] is already the influence function k'_z of tilt, and a local depth d to the neutral fibre of

$$d = C(R - r) \quad (254)$$

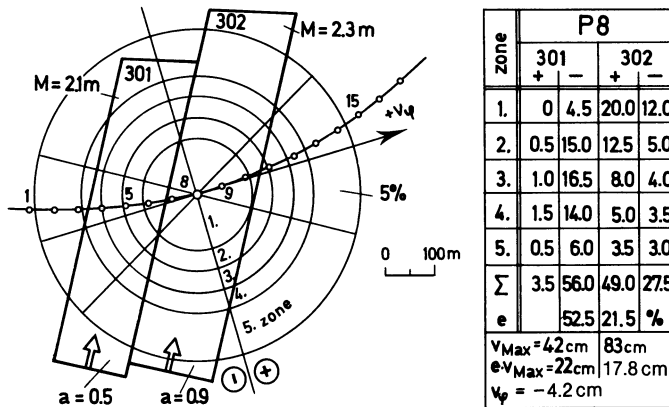


Fig. 221. Calculation of displacement of point 8. Extraction areas 301 (caving) and 302 (stowing). Integration grid after Beyer (Table 12)

factors and mined seam thicknesses, must be calculated separately. Point 8 is displaced by 4.2 cm towards point 7. Maximum possible displacement v_{Max} was taken as being $0.4 aM$.

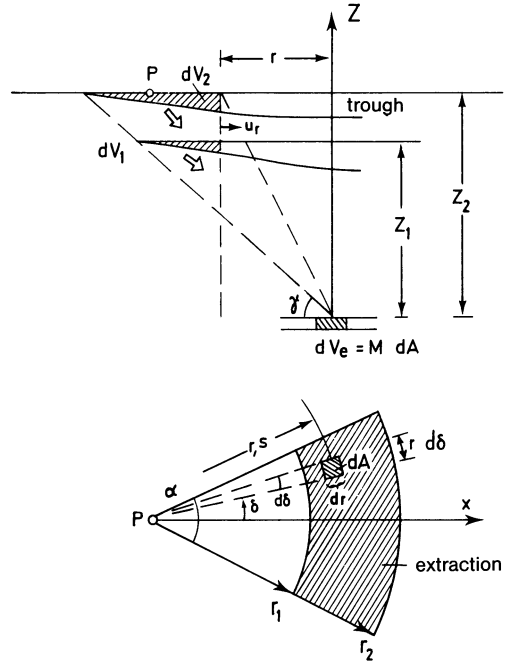
With Beyer's method it is noteworthy that the *depth of deformation* is a new strata characteristic. In place of point movement towards the focal point of extraction — a movement associated with the notion of a cohesionless mass of sand grains — there is substituted the descriptive model of the bending of a layer of ground. It is thus assumed that the strata bond at the bedding planes is relaxed by the bending forces acting in a longitudinal direction and that, with severe curvature, the loosened beds shift by several decimetres (Fig. 38). The thickness of the layers formed by these thrust fractures is unknown. From the diagram in Fig. 219, top, it can be seen that, with severe curvature of the ground to a radius of $\rho_z = 10$ km, and with extension or compression amounting to $\pm \varepsilon = 5$ mm/m, the bed thickness $2d$ must, by Eq. (251), come to 100 m. The extent of displacement and linear change accordingly depends not only on convergence aM in the mine excavation but also on the thickness $2d$ of the surface layer of ground deforming as a single entity, and on the line of curvature of the underlying solid rock. The influence of bed thickness and bedding-plane friction on the amount of displacement was also confirmed (Fig. 232) by observations on layered models (Sect. 9.6). What is still unestablished is how far, when a ground layer with predominantly plastic properties subsides as a trough, it in fact absorbs linear changes through a reduction of pore volume — i.e., through compaction — and through internal slip movements, without allowing extensive ground stress to develop (see Sect. 11.2).

9.3.3 Displacement by the Statistical Theory

Calculation of displacement by the statistical-integral theory used in Poland (Kochmanski, 1958) is based on the following considerations.

The removal of a small extraction element dV_e of ground plan dA produces at the strata horizons z_1 and z_2 the well-known elementary troughs, with volumes of

Fig. 222. Deriving displacement u , after the theory of Kochmanski



dV_1 and dV_2 outside a circular cylinder of radius r (Fig. 222, top). Subsidence in the outer zone in question is only possible, however, if a part of the outer rock body corresponding to the difference $dV_2 - dV_1$ in trough-margin volume (vertical compression) can penetrate into the interior of the circular cylinder. The volume of rock body moving inwards can alternatively be calculated from the mean elementary displacement u_r of strata points at the periphery of the circle between the two horizons concerned⁴. This leads to the relation

$$dV_2 - dV_1 = 2 \pi r u_r (z_2 - z_1) . \quad (258)$$

In the limiting case $z_1 \rightarrow z_2 = z$, horizontal strata displacement passes into elementary point displacement of a horizon z

$$u_r = \frac{dV}{dz} \frac{1}{2 \pi r} . \quad (259)$$

Since the volume of the outer trough zone [see Eq. (190)] is

$$\begin{aligned} dV &= \text{total volume less volume of inner zone} \\ dV &= aM dA [1 - \varphi(\rho, b)] \end{aligned} \quad (260)$$

the equation for the elementary horizontal displacement of a point in the direction of the extracted element will read

$$u_r = - \frac{aM}{2 \pi r} \frac{\partial \varphi(\rho, z)}{\partial z} dA . \quad (261)$$

⁴ In assuming constancy of strata volume, the reduction of pore volume in the zone of vertical compression is ignored.

Here $\varphi(\rho, b)$ or alternatively $\varphi(\rho, z)$ is an empirically found e function, with the arguments of the abstract radius ρ equal to r/r_0 , and the depth parameter b or vertical distance z similar to Eq. (188).

In deriving an *integration grid*, the component in the direction of the bisector of the angle

$$u_r \cos \delta = - \frac{aM}{2\pi r} \frac{\partial \varphi(\rho, z)}{\partial z} r d\delta dr \cos \delta \quad (262)$$

is determined from the elementary displacement of the point P as caused by the element $dA = r d\delta dr$ of an extracted annular ring sector (Fig. 222, bottom). Extraction of the whole annular ring sector leads ultimately to the final displacement

$$v_x = \iint_A u_r \cos \delta = - \frac{aM}{2} \int_{-\alpha/2}^{+\alpha/2} \cos \delta d\delta \int_{r_1}^{r_2} \frac{1}{\pi} \frac{\partial \varphi(s, z)}{\partial z} ds, \quad (263)$$

where s signifies the horizontal distance of the element under consideration dA from the calculation point P – a distance which lies between the radial distances r_1 and r_2 . The first integral expression has the value $2 \sin \alpha/2$. The second integral can be solved according to the rule

$$\frac{\partial \varphi(s, z)}{\partial z} = \frac{\partial \varphi}{\partial b} \frac{db}{dz} + \frac{\partial \varphi}{\partial r_0} \frac{dr_0}{dz} \quad (264)$$

on the assumption that the trough shape is determined by – apart from the factor a – the parameters b and r_0 . If the unit radius r_0 in the roof does not alter significantly ($dr_0/dz = 0$), then Eq. (263) can be simplified to

$$v_x = - aM \sin \frac{\alpha}{2} \frac{db}{dz} \int_{r_1}^{r_2} \frac{1}{\pi} \frac{\partial \varphi(s, b)}{\partial b} ds. \quad (265)$$

After the arguments have been expanded by r_0 (i. e., $s = \sigma \times r_0$, $ds = d\sigma \times r_0$, $r_i = \rho_i \times r_0$), and the integral

$$\int_0^{\rho} \frac{1}{\pi} \frac{\partial \varphi(\sigma, b)}{\partial b} d\sigma = \psi(\rho, b)$$

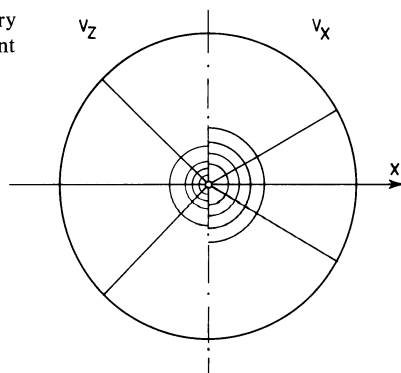
has been substituted by the values of the tabulated function ψ , the final equation for displacement by the statistical-integral theory will read

$$v_x = - aM \sin \frac{\alpha}{2} r_0 \frac{db}{dz} [\psi(\rho_2, b) - \psi(\rho_1, b)]. \quad (266)$$

With this Eq. (266) the critical extraction area can be divided into zones and sectors of equal displacement effect (Fig. 223).

For *example*, from the table $10^3 \times \psi(\rho)$ for the given parameters $b = 1.05$ and $r_0 = 50$ m at a critical-area radius of 600 m (i. e., $\rho_{\max} = 600/50 = 12.0$), may be found a functional value of 969 which, when divided by 5, gives the tabulated values for the five zonal radii ρ_i or r_i :

Fig. 223. Division into zones after Kochmanski's theory for the calculation of subsidence (*left*) and displacement (*right*)



$$\begin{array}{lll}
 10^3 \times \psi = 969; & \rho_i = 12.00; & r_5 = 12.0 \times 50 = 600 \text{ m} \\
 776 & 4.25; & r_4 = 4.25 \times 50 = 212 \text{ m} \\
 582 & 3.20; & r_3 = 3.2 \times 50 = 160 \text{ m} \\
 388 & 2.30; & r_2 = 2.3 \times 50 = 115 \text{ m} \\
 194 & 1.60; & r_1 = 1.6 \times 50 = 80 \text{ m} .
 \end{array}$$

For $aM = 1 \text{ m}$ and $db/dz = -C/H = -1.2/180 \text{ m}$ (with C from 1.5 to 0.6 on increasing depth), critical displacement on extraction of half a critical area ($\alpha = 200 \text{ gr}$) shows a value of

$$v_{\text{Max}} = -1 \times 1 \times 50 \frac{-1.2}{180} 0.969 = 0.323 \text{ m} .$$

The ratio between critical displacement and critical subsidence is in this case 0.31 (Kochmanski and Magdziorz, 1972).

In the integration grids for displacement and subsidence calculation portrayed in Fig. 223, one is struck by the great width of the fifth zone. This is undoubtedly connected with the asymptotic influence function and the limiting of the area of influence, reaching to almost 100% ϵ (the practical limit angle), but also, in places, with the absence of cover over the Carboniferous in Upper Silesia (see Saar District).

As already mentioned, the statistical-integral method can be extended also to *inclined measures*, if the oblique-lying extraction panel is converted into horizontal strips (Fig. 192). The contributions of these strips to linear change in the vicinity of a surface point in the section down the dip can then be calculated with the aid of nomograms (e.g., Fig. 193). In doing this it is assumed that vertical extension or compression can be equated to horizontal linear change at the place being investigated:

$$-\epsilon_z = \epsilon = \frac{dw}{dz} . \quad (267)$$

In the nomogram, subsidence is determined with two auxiliary lines of the extraction profile, which are drawn at a distance of $\Delta b = 0.05$ above and below the actual profile line in relation to the calculation point. The ϕ values read off from

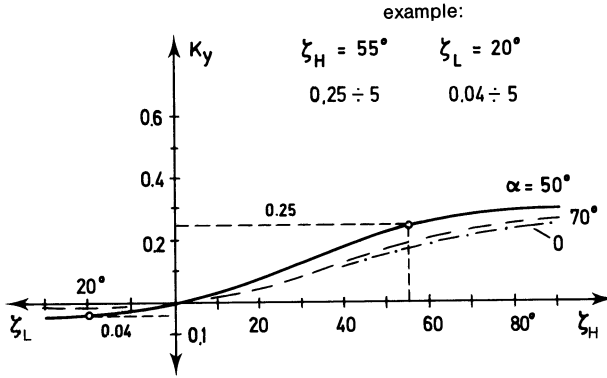


Fig. 224. Integral curve by Eq. (269) for division into zones in dip section. (After Schleier, 1956)

the upper and lower lines for the n individual partial sections are subtracted one from the other ($\Delta\varphi_i = \varphi_{i\text{upper}} - \varphi_{i\text{lower}}$) and introduced into the final formula (Ochab, 1959):

$$\varepsilon = aMC \sum_{i=1}^n \left(\Delta\varphi_i \frac{10}{H_i} \right) \quad [\text{mm/m}]. \quad (268)$$

The calculation itself is very time-consuming.

9.3.4 Displacement Grids for Inclined Measures

Just as with subsidence, displacement in inclined measures can also be calculated experimentally with an oval integration grid, derived by central projection from a circular zonal grid (see Fig. 188).

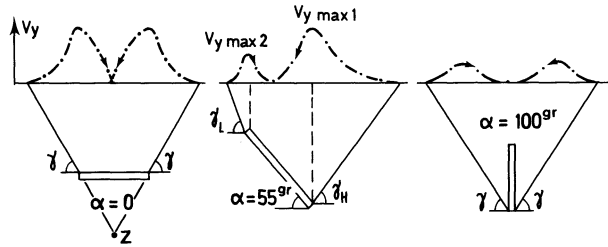
An originating grid is obtained by setting up, on similar lines to Eq. (171), a distribution function

$$k_y = \frac{dA}{H \cos \alpha} \cos(\zeta \mp \alpha) \cos^3 \zeta \, d\zeta \quad (269)$$

for the shares of displacement of the extraction elements dA in the direction of dip (Schleier, 1956) and by integrating them over half the critical area. In the dip section, the integral curve K_y of Eq. (269) is of the shape represented in Fig. 224 in the case of a seam dip of between $\alpha = 50^\circ$ and $\alpha = 70^\circ$. From this curve the zonal angles for the individual K_i values can be read off and then erected in the cross-section of the vertical axis $P - P'$, as Fig. 226 demonstrates, by way of indication, using an example with given limit angles. In the strike section through P' , the zonal angles read off for $\alpha = 0^\circ$ are used in constructing the zones. Zones in diagonal sections can be traced either according to the defining elements of a table or as the points of an ellipse. Sector lines in accordance with the sine function are laid through P' and not, as in a subsidence grid, through the centre Z .

Maximal displacement over the upper and lower ribsides of a panel is of varying magnitude in steep-lying measures (Fig. 225). Generally, the greater displacement is found over the lower ribside. This can be explained by the greater defor-

Fig. 225. Displacement curves in flat and steep stratification



mation depth of the strata at that point and by the strata movement directed perpendicularly to the mine excavation, which subjects the overburden to a thrust towards the footwall. On the extent of this maximal or critical displacement, little has been published. One possibility of calculating it from subsidence consists in multiplying the subsidence value according to Eq. (233), ascertained for the dip section over the ribside, by the tangent of the focal-point angle pointing to the upper or lower half of the critical area (Fig. 226). The focal-point angle μ of the two halves of the critical area is obtained with the zonal-angle nomogram (Fig. 186) from $\frac{1}{2}K_{z1}$ and $\frac{1}{2}K_{z2}$ for the relevant starting values (seam dip, limit angle). The values calculated in this way in Table 18 can be regarded as an indication of the relative magnitude of critical displacement in the two areas concerned.

Besides the methods discussed here, other distribution functions for displacement calculation have been proposed, such as those obtained, for example, by conversion of the Gaussian subsidence function (Sann, 1949) on the basis of

$$k_x = k_z \frac{r}{H} \tag{270}$$

or by empirical subdivision of the critical area on the basis of observed point displacements.

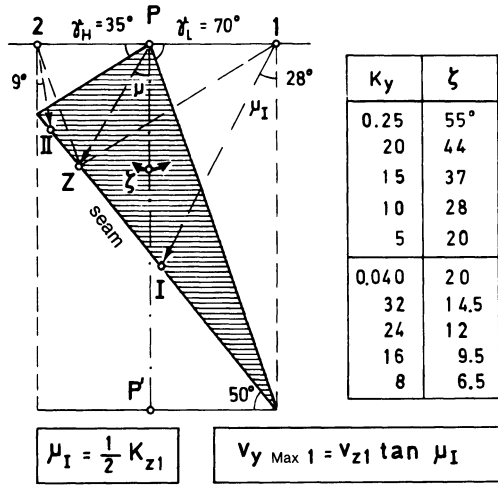


Fig. 226. Determination of maximal displacement in steep-lying measures. *Right:* zonal angles for displacement grid, after Schleier

Table 18. Critical displacement in steep-lying measures, calculated by the focal-point method

Dip α°	Critical displacement in the dip section over the	
	Lower ribside $v_{y\text{Max}1}$	Upper ribside $v_{y\text{Max}2}$
0	45% $v_{z\text{Max}}$	45% $v_{z\text{Max}}$
30	40	25
50	35	5 – 10
70°	30	2 – 5

9.4 Calculating Displacement from Tilt

A glance at the ground-movement curve in Fig. 128 shows that the displacement curve corresponds closely in shape to the tilt curve. Both present a nil value at the margin and in the centre of the trough and a maximum value over the ribside. Between these extremes both profile curves follow an approximately sinusoidal course. This similarity in the movement components v_x and v'_z is made use of in programmed ground-movement calculation to ascertain displacement from the tilt of the ground in the calculation direction concerned:

$$v_x = k v'_z . \quad (271)$$

With the proportionality factor k , derived from the critical-area radius R , which in the Ruhr has a value of

$$k = 0.033 R_{[m]} \quad (272)$$

the tilt number expressed in mm/m is converted into the amount of displacement in cm appropriate to the mining district concerned. For $R = 500$ m at a mining depth of 700 m, and with a tilt of 1 mm/m, a point displacement of 16.5 cm is obtained in this way.

The amount of tilt at any given place can be determined either from the subsidence of two neighbouring marker points in accordance with Eq. (113) and Fig. 214, or directly by means of a tilt integration grid, bypassing subsidence calculations. Tilt grids of this type have been designed for the influence functions of Eqs. (166) and (168).

Three methods of calculating displacement⁵ from tilt are known of:

- a) the computer programme of Ruhrkohle AG (influence function);
- b) the profile method of the Donetz District;
- c) the profile method for inclined measures.

In the computer *programme of Ruhrkohle AG* for flat-lying measures, tilt in the two directions $\partial v_z / \partial x$ and $\partial v_z / \partial y$ is calculated and rendered proportional to

⁵ See also Eq. (303): $\Delta v'_z$ prop. ε (factor d/l).

the required displacement (Maassen, 1961). Linking up with Eq. (194), differentiation of the influence function k_z gives the share of tilt

$$\frac{\partial v_z}{\partial x} = c \left[x \exp \left(-\frac{x^2}{2k^2} \right) dx \right] \exp \left(-\frac{y^2}{2k^2} \right) dy. \quad (273)$$

By analogy with Eq. (200), tilt in the x direction at the surface point P , resulting from the extraction of an infinitesimal trapezoidal area, is – by Simpson's rule –

$$v'_{z \text{ trapez.}} = \int_{y_1}^{y_2} \left[k^2 \left(1 - \exp \left(-\frac{x(y)^2}{2k^2} \right) \right) \exp \left(-\frac{y^2}{2k^2} \right) \right] dy. \quad (274)$$

Tilt in the y direction is obtained in a manner similar to Eq. (202):

$$v'_{z \text{ trapez.}} = \int_{y_1}^{y_2} \left[\frac{k}{2} \sqrt{2\pi} \psi \left(\frac{x(y)}{\sqrt{2}k} \right) y \exp \left(-\frac{y^2}{2k^2} \right) \right] dy. \quad (275)$$

Tilt values must be worked out for all trapezoidal extraction areas and then, after summation, be converted to displacement using the conversion factor 0.033 R [Eq. (272)]. With the displacement components v_x and v_y found in this way, displacement values in any desired direction can then be determined (Fig. 140).

Using an unaltered factor k in Eq. (271), however, it is not possible to obtain equally good results for the whole trough when converting tilt to displacement. Experience shows that point movements in the trough margin zone are in an almost horizontal direction, with the result that tilt is minimal and displacement arrived at by conversion is too small (Fig. 213). For the trough margin therefore, the factor k given in Eq. (272) must be increased. Furthermore, it should be taken into account when selecting the k factor that tilt values observed in the Ruhr are twice as great as the calculated values when caving is employed instead of pneumatic stowing.

As compared with calculating displacement from the difference in subsidence between two points in accordance with Eqs. (271) and (113), the method described has the advantage that the displacement of a point is calculated directly with an influence function (1 value) and not from the difference between figures (2 values = double the calculating time) which may or may not be accurate.

In the same way as displacement of the point P is derived from calculated tilt at the point P , the computer programme of Ruhrkohle AG follows up by deriving linear change from curvature. For this purpose the tilt function in Eq. (273) is differentiated once more, to obtain the influence function for curvature. Mining-induced curvature of the land surface, as obtained by calculation for the point P in a given direction, yields when multiplied by a factor d [see Eq. (252)] the required extension or compression: ε in mm/m = $362 R v''_z$ (with R in m, and v''_z in 1/m).

If the extraction area slopes at more than 20 gr, recourse is had to the focal-point method in calculating displacement. The requisite subsidence values and focal-point coordinates in accordance with Eqs. (233) and (237) are determined

with an influence function after Perz [Eq. (173)], as has been described with the aid of Fig. 198. Finally, linear changes are derived from the displacement values for four subsidiary points as calculated by Eq. (118) (Fig. 214).

Thus in the Ruhrkohle programme, subsidence, tilt, and curvature at the point P are calculated, in the case of flat-lying strata, with influence functions; displacement and linear change are set in proportion to tilt or curvature as the case may be. In steep-lying strata only subsidence is obtained with an influence function, and only for the point P and for four points nearby; from these subsidence values all other trough elements are then derived (see Appendix II).

Between tilt and displacement there is not only a similarity of curve, as in Fig. 128, but also a mechanical connection, as between the flexure of a bent beam and linear change on its upper side, in accordance with Eq. (251). Further mechanical connections between the quantities v_x and v'_z are illustrated in Figs. 74 and 230.

As proof that displacement v_x is proportional to tilt v'_z by Eq. (271), the quotient $1/\rho_z$ in Eq. (250) $v_x = dl/\rho_z$ is replaced by the angle at the centre ϕ , and this in turn by the tilt $v'_z = \phi$ (see point 1' in Fig. 219). From Eq. (250) we obtain then with half the beam thickness d a horizontal component v_x for the change in position of the point 1 to 1' as a result of the bending of the beam section 1–2. This component v_x is equal to d times ϕ , which equals d times v'_z – i.e., v_x proportional to v'_z . The increment Δv_x in the displacement of the two points 6 and 7 in Fig. 219 corresponds to the central angle $\Delta\phi$ of the section of arc 6–7, or to the difference $\Delta v'_z = v'_{z7} - v'_{z6}$ in tilt at these two points: i.e. Δv_x proportional to $\Delta\phi = \Delta v'_z$. If Δv_x and $\Delta\phi$ are divided by the distance l between the points, we return with $\Delta v_x/l = \varepsilon$ and with $\Delta\phi/l = 1/\rho_z = v''_z$ once again to Eq. (251) (ε proportional to v''_z), on which the Beyer method (Sect. 9.32) is based, or to Eq. (303) of the British method (Sect. 9.7), by which $\Delta v'_z$ is proportional to ε (factor d/l).

In other words if the surface layer is approximated to a beam which is bent like the trough profile (Fig. 219), then displacement v_x on the convex upper side, from point 5 (trough margin) up to be transition point 7, increases with the angle at the centre ϕ and the amount of tilt v'_z , and thereafter, in the concave zone 7–c, decreases again to zero at the trough centre c as the angle at the centre (at the point o) and the tilt values become smaller. Equations (250), (251), (271), and (303) thus all have the same mechanical significance.

In the method employed in the *Donetz District*, as in the computer programme of Ruhrkohle AG, use is made of the knowledge that displacement is approximately proportional to tilt. Accordingly, the degree of tilt derived as a first differential quotient from the subsidence function of Eq. (137) requires only to be multiplied, as in Eq. (271), by a factor of $k = 0.2$ for displacement to be obtained (all in metres):

$$v_x = 0.2 v_{z\max} \left[1 - \cos \left(2\pi \frac{x}{L} \right) \right]. \quad (276)$$

As in Fig. 164, the abscissa x of the calculation point is measured from the bottom of the trough, and the distance to the edge of the trough is designated as L .

The displacement formula for *inclined measures* (Kolbenkov, 1961) is noteworthy and is moreover in agreement with observations on the slab model. In such measures, adhesion and friction at the interface between the practically perpendicularly moving Carboniferous strata and the overburden are taken into account by means of an additional term in Eq. (276), thus:

$$v_y = v_{z\max} [k_1 f(y) + k_2 f'(y)] . \quad (277)$$

Here, the expressions employed have the following meanings:

$$k_1 = \tan \alpha - \frac{h}{H_m} ,$$

$$k_2 = 0.15 \text{ to } 0.25 ,$$

$$f(y) = 1 - \frac{y}{L} + \frac{1}{2\pi} \sin \left(2\pi \frac{y}{L} \right) ,$$

$$f'(y) = 1 - \cos \left(2\pi \frac{y}{L} \right) .$$

The first value in brackets (subsidence) implies that displacement of the overburden at its lower bedding plane is proportional to its subsidence [see Eq. (137)] and to the tangent of the seam dip. In addition, displacement of a surface point under the influence of the basement rock decreases, if the ratio of the thickness h of overburden to the average depth of extraction H_m increases, and if the point lies in the lower part of the trough. In the upper half of the trough, point displacement in an inwards direction is accentuated by friction from the basement rock (Fig. 227). The second term (tilt) in the brackets in Eq. (277) covers the

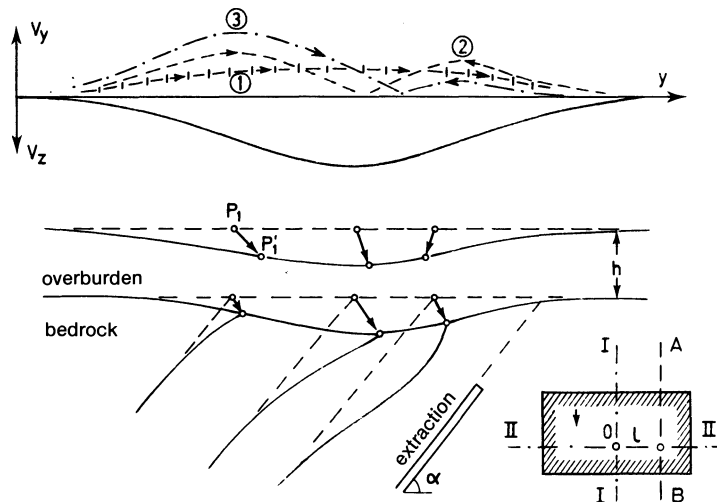


Fig. 227. Displacement in steep-lying measures, taking account of friction between basement rock and overburden (after Akimov and Korotkov, 1969). (1) Displacement at lower bedding plane of overburden; (2) displacement at ground surface without friction at bedding plane; (3) displacement resulting from (1) and (2)

share of displacement resulting from the slablike sagging of the overburden, as in Eq. (276).

For sections running parallel to the principal sections I – I down the dip and II – II along the strike at a distance l from the trough centre 0, an additional factor is introduced (see also Fig. 208). This factor has, for example, the value of the function $f(y)$ for $y = l$ at the point of intersection of the lines A – B and II – II (Fig. 227). Calculation of the components of movement in oblique-angled sections is effected in accordance with an analytical method of continuum mechanics (Akimov and Korotkov, 1969).

9.5 Methods Employing Theoretical Models

As with subsidence, the horizontal components of point movement can be derived on the basis of a stochastic, an elastic, or a plastic rock body.

9.5.1 The Stochastic Model

If the equation for point displacement $u(r)$ within the elementary trough applicable to the *stochastic medium* – namely Eq. (81) – is introduced into the basic equation (160), what is obtained is half the volumetric content of the shares-of-influence body for displacement:

$$K_{xy} = 2 \int_0^R r u(r) dr = \frac{aMB(h)}{4\pi\varphi^2(h)} \int_0^R r^2 \exp\left(-\frac{r^2}{4\varphi}\right) dr. \quad (278)$$

To be able to calculate the zonal radii of an integration grid, it is necessary to put

$$\begin{aligned} \int_0^R r^2 \exp\left(-\frac{r^2}{4\varphi}\right) dr &= 4\varphi\sqrt{\varphi} \int_0^Q \exp(-q^2) dq - Q \exp(-Q^2) \\ &= 4\varphi\sqrt{\varphi} \psi(Q) \end{aligned} \quad (279)$$

for the integral expression, in order that the values of the Gaussian error integral

$$\int_0^Q \exp(-q^2) dq$$

recorded in tables can be drawn upon for the solution (Bräuner, 1961). The initial equation (278) thereby assumes the form

$$K_{xy} = \frac{aMB}{\pi\sqrt{\varphi}} \psi(Q). \quad (280)$$

The values of the function ψ are in tabulated form for the arguments Q from 0 to 2.0 (in Bräuner, 1961, p. 13). Finally, with the structural functions B and φ of Eq. (76), the following zonal radii are found for a limit angle of 50 gr: $r_1 = 0.4 R$, $r_2 = 0.55 R$, $r_3 = 0.68 R$, $r_4 = 0.85 R$, and $r_5 = 1.00 R$ (see Fig. 180).

In Poland, in recent years the *stochastic theory* on the movement of strata points (Litwiniszyn, 1957) has also been extended to the calculation of displacement (Szpetkowski, 1972). As was discussed in Sect. 3.3.4, infinitesimal displacements and tilt movements are proportional to each other [Eq. (80)] – a fact which can be deduced from the assumed volume constancy of the deformed section of strata. Linear change should also be proportional to curvature and to vertical compression or extension. In both cases a rock-mechanics structural function [Eq. (76)] appears as proportionality factor.

Integration of the horizontal linear change ε , which as in Eq. (267) is derived from vertical extension/compression

$$\varepsilon = c \varepsilon_z = c \frac{\partial v_z}{\partial z} \quad (281)$$

(c being approx. 8–12) gives rise to the displacement function (profile curve) for the trough over a critical area

$$v_x = v_{\text{Max}} \exp \left[-0.5 \left(\frac{x}{\rho} \right)^2 \right] \quad (282)$$

with critical displacement amounting to

$$v_{\text{Max}} = 2 \rho H^{-1} a M. \quad (283)$$

For the extraction of a subcritical area (Fig. 203) the formulae in the x -axis read

$$v_x = v_{\text{Max}} A_y B_x \quad (284)$$

and in the y -axis

$$v_y = v_{\text{Max}} A_x B_y$$

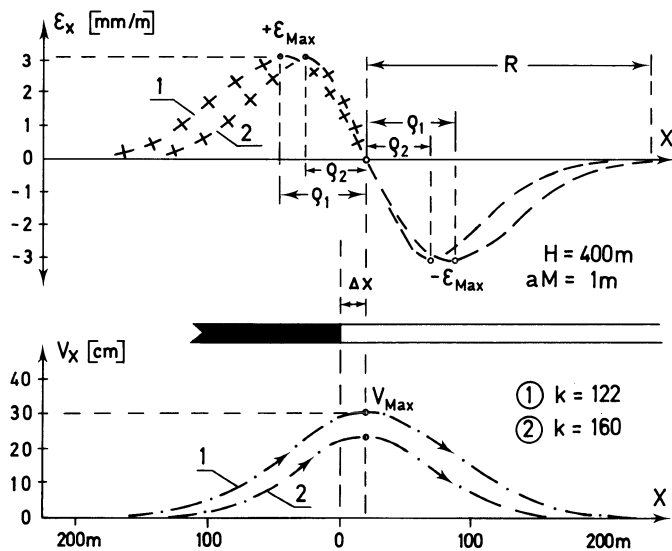


Fig. 228. Displacement on the basis of Eq. (282), and linear change derived from it. (After Szpetkowski, 1979)

with

$$B_x = e^{-0.5 v_2^2} - e^{-0.5 v_1^2}$$

and

$$B_y = e^{-0.5 v_4^2} - e^{-0.5 v_3^2}.$$

A_x , A_y , v , and ρ have the same meaning as in Eq. (220). By Eq. (221), the strata characteristic lies between 100 and 170. With a greater k value, calculated tilt and curvature certainly increase as a result of the steeper trough profile (Fig. 204), but displacement and the width of the trough both decrease (Fig. 228). Polish investigations show critical displacement as standing in a ratio of 0.30–0.475 to critical subsidence, depending on the mining district.

In *inclined stratification* the displacement curve is shifted by $\Delta x = H_m \times \tan \mu$ or, where the thickness h of the overburden is considerable, by $\Delta x = (H_m - h) \tan \mu'$ in the direction of the dip (Fig. 229). Up to a dip of 60° , μ is approximately equal to 0.7α or $\alpha \times 80/k$. Calculation of ground movements is carried out in a system of coordinates rotated by μ or μ' , in accordance with Fig. 228. In the rotated system the displacement formula then reads

$$v_x(\mu) = 2 \rho' H'^{-1} \frac{aM}{\cos(\alpha - \mu)} \left\{ \exp \left[-0.5 \left(\frac{x'}{\rho'} \right)^2 \right] - \exp \left[-0.5 \left(\frac{l' - x'}{\rho'} \right)^2 \right] \right\} \quad (285)$$

with

$$x' = x \cos \mu$$

$$l' = l \cos(\alpha - \mu)$$

$$\rho' = 400 k^{-1} \sqrt{H'}.$$

The required horizontal point-displacements of the ground surface in the dip section then become, over the upper ribside

$$v_x = -v_z(\mu) \sin \mu + v_x(\mu) \cos \mu$$

and over the lower ribside

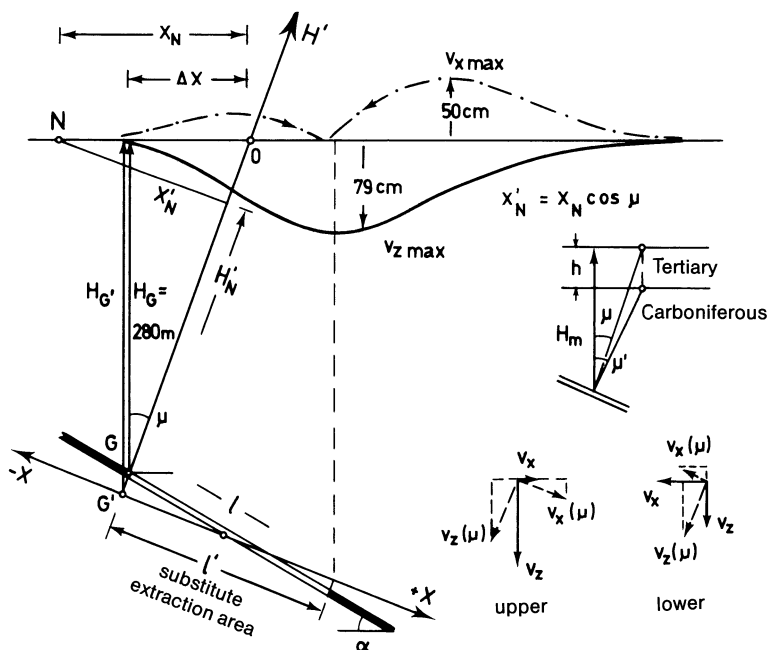
$$v_x = -v_z(\mu) \sin \mu - v_x(\mu) \cos \mu.$$

(286)

Subsidence values $v_z(\mu)$ are calculated using Table 16 based on the equation

$$v_z(\mu) = \frac{aM}{\cos(\alpha - \mu)} \left[\Theta \left(\frac{x'}{\rho'} \right) - \Theta \left(\frac{l' - x'}{\rho'} \right) \right]. \quad (287)$$

Exponential-function values can be taken from works of tabulation. The remaining quantities required for the calculations (e.g., H' , l' , x') are best determined diagrammatically. When using the above formulae, the width of extraction Δy at right angles to the profile axis must be considerably greater than the length of extraction l down the dip. In the case of seams dipping steeply, it is recommended that the extraction panel be divided into strips, as otherwise the average depth of extraction alters too much (Szpetkowski, 1972).



$$-\frac{\partial w}{\partial r} = \frac{\partial u}{\partial z}. \quad (290)$$

This equation implies that, from any given place on the surface, subsidence movements in a radial direction must increase by as much as displacement movements from that point decrease with greater depth, if the surface of the elastic medium is to remain in equilibrium. If from Eq. (224), with $k^2 = 20$ and $R = \sqrt{a^2 r^2 + H^2}$, the equation

$$\frac{\partial w}{\partial r} = \text{Const.} \left(-\frac{k^2 r}{R^3} - \frac{3a^2 r H^2}{R^5} \right), \quad (291)$$

is formed, then after several calculational steps the influence function for displacement is reached:

$$u = -\text{Const.} \frac{20 H r}{\sqrt{20 r^2 + H^2}^3}. \quad (292)$$

In conjunction with Eq. (160), and after indefinite integration

$$v = -20 H \int \frac{r^2}{\sqrt{20 r^2 + H^2}^3} dr \quad (293)$$

the zonal radii $r_1 = 0.33 R$, $r_2 = 0.49 R$, $r_3 = 0.63 R$, $r_4 = 0.80 R$, and $r_5 = 1.00 R$ of the integration grid are obtained for a limit angle of 60 gr, by employing approximation procedures.

As Table 12 shows, the displacement grid worked out according to the laws of an elastic continuum is largely identical with the grid derived from ground curvature (Beyer). What is particularly noteworthy in this elastic model is that, from the volumes of the shares-of-subsidence and -displacement bodies, a ratio of

$$\frac{V_{\text{Max}}}{V_{z\text{Max}}} = 0.44 \quad (294)$$

for a limit angle of 61 gr (55°) can be found – a ratio which corresponds closely to the components of movement observed in the Ruhr, and which decreases only slightly to 0.40 at a limit angle of 50 gr (45°).

If it is remembered how many approximation steps had to be taken with the elastic model mentioned, in order to produce an integration grid as a practical means of determining displacement, the advance that the *finite-elements method* represents in the calculation of elastic continua will be evident. For each individual element, the connection is established between the forces exerted at the corner points and the routes appertaining to those corner points over the stiffness of the element [Eqs. (83) and (84)] and through the selected function of the paths of point movement (coordinates matrix, Table 17). With the original measurements and characteristics of the stratified rock body, it is thereby possible to make a simple description of the equilibrium state for the forces and routes of the elastic system in Eq. (84). It is also unnecessary to know the critical value v_{Max} and introduce it into the calculation as a reference value, as in the integration

grid. This is because displacement, as a horizontal component of point movement, necessarily appears at the correct value if the elastic characteristics selected, or the given convergence on extraction, agree with those of the strata. The peak of displacement over the ribside, even by the finite-elements method, is then approximately half as great as the subsidence of the central trough point, as shown by Fig. 209.

The finite-elements method has, however, the disadvantage as compared with the integration grid that two networks, for the section along the strike and the section down the dip, must be prepared for each extraction, and that, in the event of varying strata conditions, the programmed network can be simplified only to a certain degree. The finite-elements method is therefore less suited at present to the daily calculation of panels of varying depth and size than to special investigations – for example, into the influence of strata structure and the dip of the beds.

9.5.3 The Plastic Model

For a plastic medium with the three equilibrium conditions of Eq. (67), one arrives via the movement vector ψ [Eq. (227)], derived on the basis of ∂y , to a displacement formula

$$v_x = -\frac{aM}{2} \left[x_1 \tanh \left(\omega \frac{x_1}{H} \right) - x_2 \tanh \left(\omega \frac{x_2}{H} \right) - \frac{H}{\omega} \ln \frac{\cosh \left(\omega \frac{x_1}{H} \right)}{\cosh \left(\omega \frac{x_2}{H} \right)} \right], \quad (295)$$

in which $x_1 = x - l/2$, $x_2 = x + l/2$, and l denotes the length of the extraction and x the running coordinate of the calculation point at the surface of the ground (Fig. 205). The strata characteristic ω is described by Eq. (228). Here also therefore, the link between subsidence and displacement is established via an equilibrium condition (Cernyj, 1966) – just as in Eq. (289) for the elastic medium. The formula for displacement in steep-lying stratification (Fig. 206) is arrived at by substituting in Eq. (295) the factors x_1 by x_1/H_1 and x_2 by x_2/H_2 , and also the hyperbolic expressions by $\omega(x_1 - \lambda_1)/H_1$ or $\omega(x_2 - \lambda_2)/H_2$, and H/ω by $1/\omega$. ($\lambda_1 = \lambda H_1$, $\lambda_2 = \lambda H_2$).

9.6 Displacement as Observed on Strata Models

The difficulties already mentioned in the measurement and analysis of point displacements at the surface of the ground led at an early stage to investigations on strata models constructed of plasticine, gelatine, gypsum and equivalent gypseous materials, rubber and sheet steel, or foamed plastics, and provided with

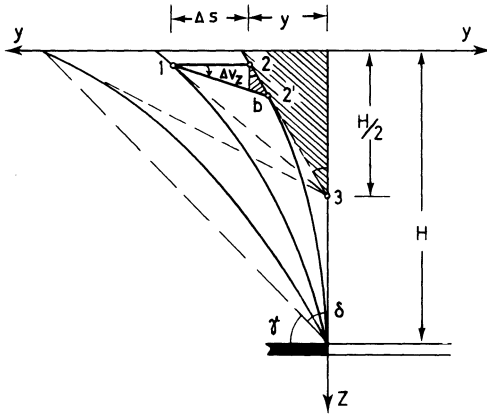


Fig. 230. Deriving displacement from tilt.
(After King and Whetton, 1957)

an extraction cavity in the form of a slit. The great advantages of such investigations are the ease with which alterations to the model and to the course of the trial can be made, as well as the ability to observe numerous measuring points, from the extraction slit to the surface. Geometrical and mechanical values affecting the movement process can thus be established experimentally and statistically. Nevertheless, it is not always possible to develop reproducible trial procedures or to maintain fidelity of scale, both in the strength properties of the material (compression, tension, shear) and in elastic characteristics (modulus of elasticity, Poisson's ratio), as also in dead-weight and friction forces and in model dimensions (e.g., seam thickness mined).

On the *gelatine model* of strata affected by underlying workings, the subsidence/displacement ratio was derived from experimentally established lines of equal three-dimensional movement v , which can be regarded as a family of parabolae radiating from the ribside (King and Whetton, 1957). In Fig. 230 the section Δs of the ground surface assumes the inclined position 1–2' after subsidence, having lengthened by b between the parabolic movement paths. The hatched similar triangles stand in the relationship

$$\frac{H/2}{y} \approx \frac{\Delta v_z}{b}, \quad (296)$$

which, given the tilt of $v'_z = \Delta v_z / \Delta s$ and the linear change $\varepsilon = b / \Delta s$, can be rewritten [cf. Eq. (281)] as

$$\varepsilon = \frac{2y}{H} v'_z. \quad (297)$$

If for tilt v'_z we substitute the value dv_z/dy derived from the subsidence curve found experimentally and then integrate Eq. (297) over the trough half-length, we obtain the displacement formulae

$$v_{\text{Max}} = 0.346 a M \tan \delta \quad (298)$$

and

$$v_x = aM \tan \delta \left[0.346 - \left(\frac{n \tanh 2n}{2} - \frac{\ln \cosh 2n}{4} \right) \right] \quad (299)$$

with

$$n = \frac{y}{H \tan \delta} \quad \delta = 100 \text{ gr} - \gamma$$

related to the transition point of the subsidence curve ($y = 0$). The hyperbolic functions in Eq. (299) are formed, when $\delta = 50 \text{ gr}$, from the sections $y \div H/2$ over the tangents to the parabolae, which characterize the direction of movement of the surface points and which intersect at point 3 above the ribside.

Conceived as a substitute for observations lacking in nature, there are the investigations carried out on a 2 m long and approx. 50 cm high and wide model consisting of several layers of *foamed plastic* (moltoprene; Hoffmann, 1963). The mine excavation is simulated by bars 2.8 mm thick which can be withdrawn. The movements of points, as observed with a theodolite, at the front of the model, which represents a longitudinal strata section through the working along the strike, reveal in the end-situation a direction in relation to extraction which is almost horizontal at the edge of the model, just under 50 gr above the ribside, and vertical over the centre of the working (Fig. 231).

In a first approximation, the circular function

$$\mu = 100 \text{ gr} - \left(i \frac{\pi}{4} \right) \quad (300)$$

reflects the movement directions μ related to the vertical, when the point coordinates are expressed as $i = r/R$. Where subsidence is known

$$v_z = \sin^2 \left(\frac{\pi}{4} i \right) \quad (301)$$

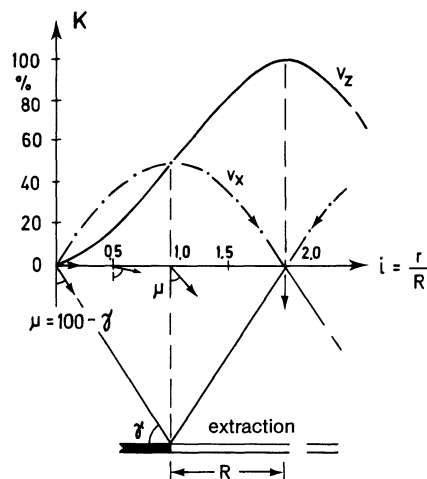


Fig. 231. Normal curves of subsidence and displacement on stripwise extraction of the critical area. (After Spettmann, 1963)

we find via Eq. (233) the displacement of a surface point

$$v_x = \frac{1}{2} \sin \left(\frac{\pi}{2} i \right) \quad (302)$$

as a function of the state of extraction i for $aM = 1$ (Spettmann, 1963). With a larger exponent in Eq. (301), the subsidence profile curve can be adjusted to the funnel shape observed in some places today. Thus, with an exponent of 3, subsidence over the ribside amounts to only 0.35 instead of 0.50 aM . This so-called normal displacement curve, which is associated with extraction of the critical area in strips on a front at least $2R$ long, is symmetrical to the $\pi/2$ axis. For the ratio v_{Max}/v_{zMax} , model trials yielded widely diverging values of 0.1 to 0.37, according to depth of working and strata friction. Practical application of the model equation (302) is effected with tabulated values K_x for extraction fronts in a straight line, as in Fig. 189. The model trials have thus demonstrated that the nadir angle μ of point movement in accordance with Eq. (300) can be of any size from 0 to 100 gr, whereas with the focal point method it is much too large at the trough margin with $\mu = 100 - \gamma$ (Fig. 231).

Other investigations with sheets of foamed plastics approximately 1 m^2 , intended to simulate, not a strata section, but an extensive overburden trough seen in plan, had as their object the gaining of new understanding of the peak of displacement. The model table developed for this purpose at the Berlin University of Technology (1968) consists essentially of a screw bed, adjustable in height,

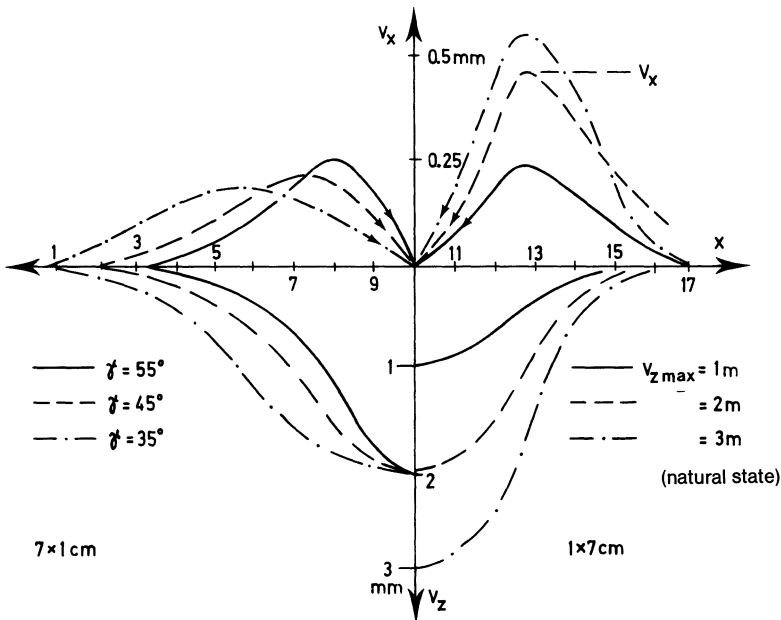


Fig. 232. Influence of trough shape on extent of displacement in a sheet model. *Left:* layered model consisting of 7 sheets (1 cm thickness = 10 m in natural state). *Right:* model consisting of 1 layer (7 cm thick) showing large displacements (TU-Berlin, 1968)

for setting up the trough, and a layer of Pará rubber overlying the screw bed and bearing the model strata of overburden in the given shape of the Carboniferous limit plane. Point displacements at the upper surface of a bed, occurring on a given amount of subsidence, were measured with a coordinatograph. By altering trial conditions such as thickness of bed, shape of trough, modulus of elasticity, and stratification, it was possible to establish the effect of individual influences on displacement. It was found that the extent of the *critical displacement* increased with

- a) the thickness of the uppermost bed (regarded as a continuum) of a group of strata;
- b) the amount of friction acting between the uppermost and the next following bed;
- c) a decrease in the modulus of elasticity of a group of strata;
- d) the amplitude of critical subsidence; and
- e) the size of the limit angle (Fig. 232).

It follows from point (a) that, if there is shear movement at the bedding planes (slip planes), displacement of surface points will increase very little with further increase in subsidence. The influence factors enumerated are not completely independent of each other, and a numerical evaluation, capable of being translated into practical application, cannot yet be relied upon. It can nevertheless be said that critical displacement, unlike critical subsidence, is determined by many geological and other factors, and consequently that the prediction of displacement must be complemented in each mining district by statistically evaluated observations. Ground displacement resulting from curvature of the surface layer and from changes in the field of stress are either reinforced or restrained by deformation and the transfer of friction forces in lower strata (see Figs. 38 and 227), as well as influenced by strata structure. An additional factor is the widely varying horizontal deformation of the ground, depending on its nature (pore volume) and moisture content, when the surface layer subsides and bends.

9.7 The Graphical Method of the U.K. National Coal Board

Figure 233 may serve as an example of the fact that the displacement curve can vary from coalfield to coalfield and also can deviate from the normal outline, depending on the degree of reworking in the block of strata concerned. The Figure shows displacements measured over a flat-lying panel at a depth of 315 m, rising to a maximum value of 75 cm some 30 m ahead of the lateral ribside. Whereas the peak of displacement generally occurs over the zone just within the panel perimeter (Figs. 131 and 228), in this case it lies over the forefield ahead of the face. Empirical procedures are better suited than theoretical calculation methods to such deviations from the normal in ground movement. This advantage is made use of in the British coal-mining industry for the prediction of horizontal linear change, and the calculation of displacement is dispensed with.

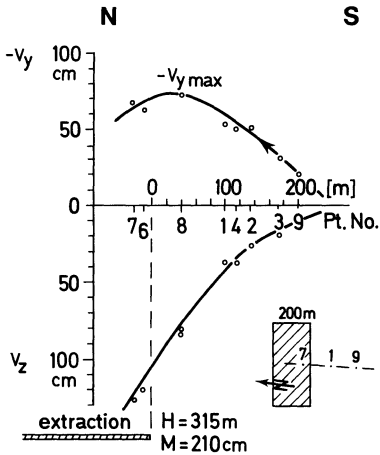


Fig. 233. Measured displacement and subsidence. (After Steudel, 1978)

Empirical values for linear change (tensile/compressive strain), gathered from all coalfields in the U.K., have been combined into *nomograms* in the same way as for subsidence (National Coal Board, 1966). Displacement, proving to be of little practical value, therefore does not require to be calculated as the starting point for horizontal trough elements. An example of NCB practice would be to construct, from the lines of equal linear change in nomogram 2 of Fig. 234, the profile of linear change for an extraction panel for width l_d and length l_s , starting from the ribside or from the point of transition (from tensile to compressive strain). To do this, it is first necessary to read off from diagram 1 of Fig. 234 the ratio of greatest linear change $\epsilon_{\max}$ of the profile in question to the width of extraction l_d and to the greatest subsidence $v_{z,\max}$ (both values related to the depth of working H). For example, for a panel 250 m wide, at a depth of 500 m ($l_d/H = 0.5$) a factor of 0.8 is obtained for tensile strain, and of 1.35 for compressive strain. The maximum of subsidence in the profile is determined with dia-

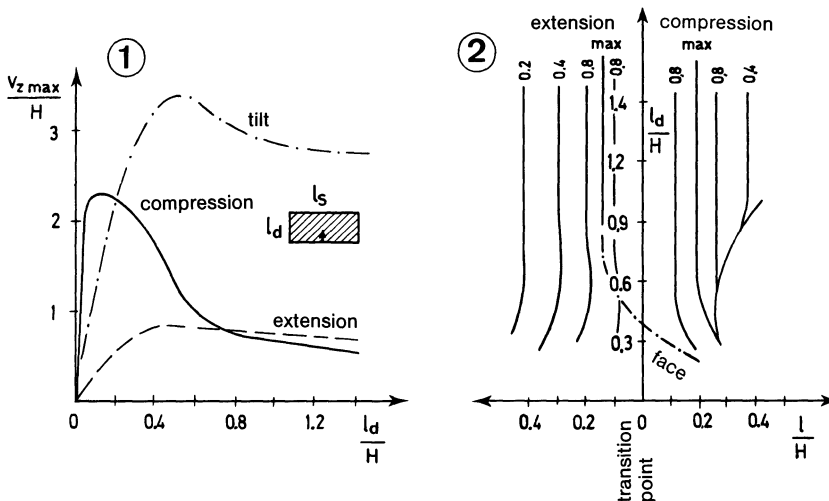


Fig. 234. Diagrams for calculation of linear change. (After National Coal Board, 1965)

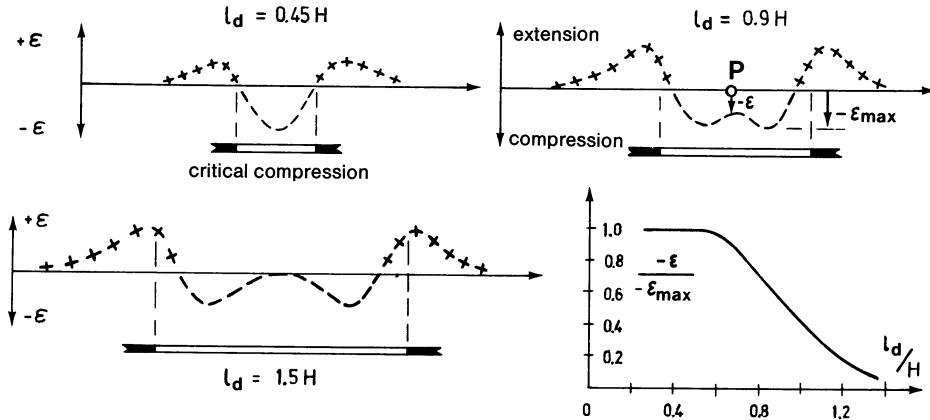


Fig. 235. Linear change over a subcritical and a critical area (*left*). Diagram for calculating the smallest and the largest amount of compressive strain $-\epsilon$ at the central point P. (After National Coal Board, 1965)

grams 1 and 2 of Fig. 162, taking account of the length of extraction l_s – e. g., as being 0.50 m, or 500 mm. The maxima of linear change, again relating maximum subsidence to depth (here 500 mm $\div$ 500 m) amount on this basis to:

$$+\epsilon_{\max} = 0.8 \frac{500 \text{ mm}}{500 \text{ m}} = 0.8 \text{ mm/m} \quad \text{and}$$

$$-\epsilon_{\max} = 1.35 \frac{500 \text{ mm}}{500 \text{ m}} = 1.35 \text{ mm/m}.$$

Individual profile points of the linear change curve can then be found with nomogram 2 of Fig. 234. Additionally, indentations in the curve of compressive strain, occurring where extraction is more than 0.6 H wide, can be found from the diagram in Fig. 235.

It should, however, be more accurate to calculate linear change from the subsidence profile curve than from these diagrams, and indeed it is known that linear change increases as the radius of curvature ρ decreases [see also Eq. (250)]:

$$\epsilon \text{ prop. } \frac{1}{\rho} = \frac{\phi}{l} = \frac{\Delta v'_z}{l}. \quad (303)$$

As can be seen from Fig. 236, the angle at the centre ϕ , enclosed by perpendiculars dropped to the centre of the lines 1–2 and 2–3, can be calculated from the difference in tilt of these two lines. The length of line l (in m) should be approximately 1/20 of the depth of working – i. e., 40 m at a depth of 800 m – to enable short, highly curved sections to be separately registered. For tilts of, for example, 0.00117, or 1.17 mm/m, and 0.00300, or 3.0 mm/m, in two neighbouring stretches, the difference in tilt is 0.00183, or +1.83 mm/m. With a distance of 30 m between centres, this gives a ratio ϕ/l of 0.00006 ($\rho_z = 17 \text{ km}$), for which the diagram in Fig. 236 enables a tensile strain of 0.0022, or 2.2 mm/m, to be read off, in accordance with the equation

10. The Time Factor

10.1 Significance of the Time Factor

The pre-calculation methods dealt with in the preceding sections are based on the final state of ground movement over a single extraction area. In coal-mining this movement process can be regarded as complete 6 months to 5 years after the halting of operations, but in room-and-pillar working in, say, potash or other ore mining it may not be complete until after 100 years. Consequently, in mine planning and the protection of structures, account must also be taken of the interim development of ground movement resulting from the constant enlargement of the extraction area and its long-term influence. As is known, the local values for subsidence, tilt, and curvature of the ground, as well as of displacement and tensile or compressive strain, alter during the period of working, and a structure which, for example, may end up under compression can in the intervening period be exposed to tensile stress. Figures 237 to 240 show how greatly the trough elements alter during mining activity and how the time-related movement and deformation at a point in the trough can be graphically represented.

The *time factor* z , which expresses the time-dependent development of subsidence over a mine working in terms of 100 parts (%) of its final value

$$V_{z\text{dyn}} = V_{z\text{end}} Z \quad (305)$$

thus has a considerable practical significance in relation to determining

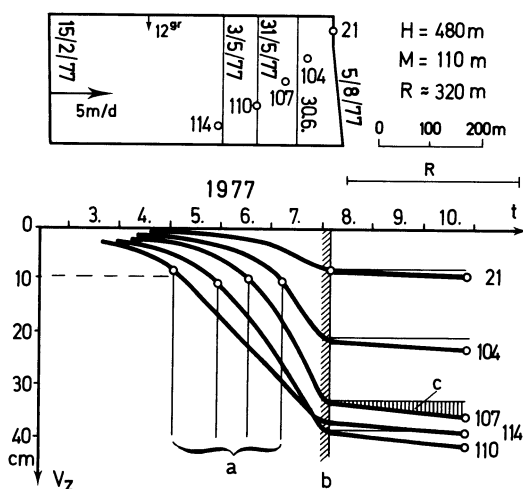


Fig. 237. Subsidence over time of the surface points 21...114. *Top:* sketch plan of the panel. (a) Passage of the working face; (b) end of extraction; (c) residual subsidence. (After Gilles, 1978)

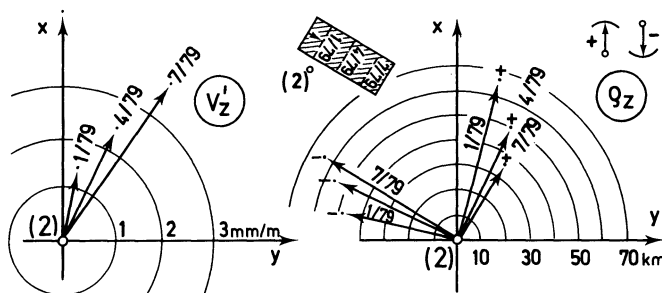


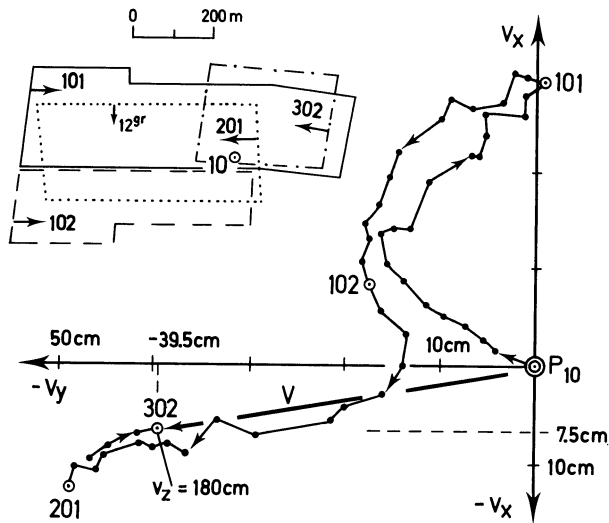
Fig. 238. Change over time in the tilt (*left*) and the radii of curvature (*right*) at the point 2

- the point in time when the effects of mining first become dangerous, in order to prepare timely measures against the expectation of operational difficulties;
- when ground movement will be at an end, with a view to beginning thorough-going repairs to the damage caused;
- where the greatest stress to a structure will occur during mining, in order to be in a position to measure its effects;
- interim amounts of subsidence, in order to draw up a time plan for raising tracks or bridges or for regulatory works on watercourses, canals, dikes, and drainage works;
- the overlapping of ground movements over several mine workings, in order to develop an operational plan which will produce the least tilt, curvature, and linear change in the ground and thus the least mining damage; or
- the interim movements of vertical extension, compression, and tilting of the shaft, in order to judge of possible damage to the shaft lining and consequent prejudice to hoisting operations.

The underground opening created in a deposit does not lead immediately to a subsidence trough of equal volume at the surface. The subsidence trough forms gradually, extending in the direction of the advancing extraction front and continuing to deepen even after the working has been halted. This *delayed subsidence* of the overlying strata has its origin, on the one hand in the stability of the underground workings up to the moment of cave-in, in the gradualness of the collapse of roof strata into the mine excavation, in the slowly increasing compaction of stowage material and fragmented roof rock (in caving methods), and in room-and-pillar mining, in the time- and load-dependent convergence of the pillars (Figs. 5 and 19); and on the other hand in the time-dependent interplay between the imposition and relief of load, vertical expansion and contraction, bending of strata, fracture propagation, strata separation, and sliding (plastic) movement in the overlying and underlying strata (Figs. 42, 115 and 116).

It is true that in coalfields in which seams are mined over a large area without supporting pillars, ground subsidence in the mm to cm range has been measured as soon as the undermined roof of the working reaches a minimum size and sags (see Fig. 250, field I). In such cases the strata overlying the workings all subside simultaneously as a single group. A reduction of load takes place over the mine excavation, however, in a direction perpendicular to the undermined strata, which consequently expand and open out vertically, with the result that the subsidence movement decreases and slows down as it moves higher (Sect. 2.1).

Fig. 239. Change over time in the position of the point P_{10} , brought about by the extraction of panels 101, 102, 201, and 302. *Small circles:* end of operations in the panel concerned. Final values: $v_y = -39.5$ cm; $v_x = -7.5$ cm; $v = 40$ cm (after Gilles, 1978). Depth of mining: 500 m. Duration: 16 months. Rate of face advance: 5–7 m/d



A period of greater or lesser length thus elapses before the convergence movement in the extraction area is propagated in full through all the strata up to the surface of the ground in the form of a wide trough (Fig. 55). If the extraction area increases further in size, the stresses and strains in the rock body alter continuously, with the result that local subsidence at the surface may increase, while local tilt, curvature, and linear changes may decrease in magnitude and even change their sign, depending on the position of the working front. Only when this front has finally halted, and the mine excavation has been completely closed up and stowage material completely compacted by strata pressure, can a new equilibrium of forces be established in the rock body, the undermined strata come to rest, and the trough reaches its final shape.

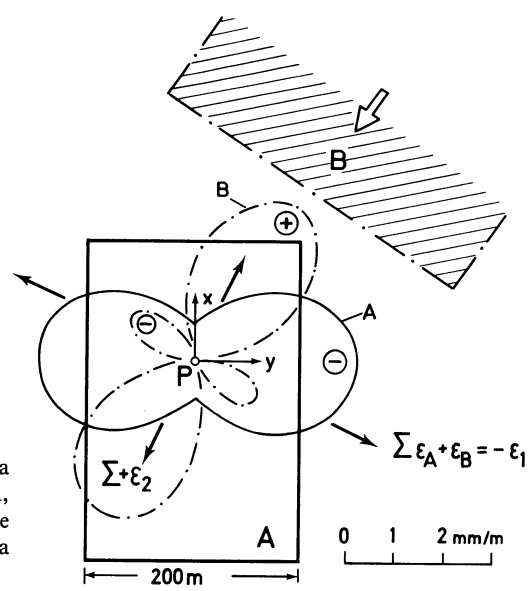


Fig. 240. The deformation figure of a ground element at P , over an old panel A , alters in direction, magnitude, and nature (tension/compression) on the approach of a second working B

10.2 Analysis of Ground Movement over Time

The first comprehensive observations of transitional forms of ground movement over time stem from the coal-mining industry. They show how movement at the surface travels forward like the bow wave of a ship with the advance of extraction. In Fig. 241 can be seen not only the intermediate trough shapes related to mining positions 1 to 7 but also the movements of individual points connected with the mining position at the time. These intermediate, “dynamic” *subsidence troughs* are distinguished, in longitudinal section, from the troughs found at the final situation by their lack of symmetry and their flat shape. A surface measuring line along the strike – i.e. at right angles to the extraction front – runs moreover under the zone of transition from tensile to compressive strain above the extraction front and of change in the direction of tilt and ground curvature (Fig. 124). In a transverse direction, on the other hand, the shape of the trough remains locally unchanged: all that alters as it deepens is the magnitude of movement and deformation values (Fig. 123). Parallel to the extraction front, therefore, there is no change from extension to compression, nor from convex curvature over the forefield to concave curvature. The tilt of a surface element directed towards the migrating focal point of extraction continues to increase even after the face has passed. The tilt of a line along the strike, on the other hand, decreases after the passage of the face.

In analysing ground movement over time, a distinction must be drawn between observations at a single measuring point and those taken at a measuring line along the strike or down the dip; and likewise between movements over the stationary rib where the face was opened out, over the advancing face working, and over the extraction front when finally halted.

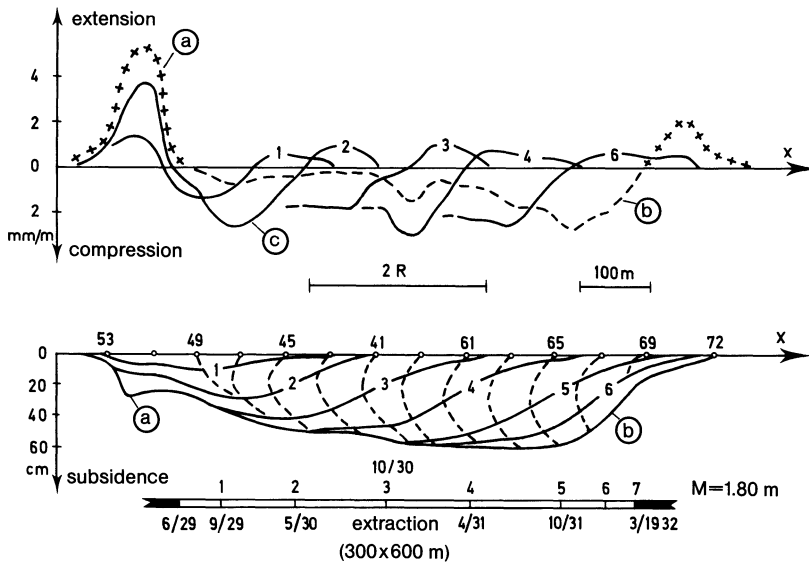


Fig. 241. Ground movement, in terms of time, over a working advancing from left to right at a depth of 175 m (after Janus and Fläschenträger, 1956). Critical subsidence: 90 cm. Solid stowing. (a) Crack zone; (b) final position; (c) critical peak of compression at face position 2

10.2.1 The Curve of a Surface Point's Subsidence over Time

At a single measuring point, what is obtained is the *curve of point movement* over time; at a measuring line, on the other hand, what is obtained is a sectional diagram of trough deformation. A surface point P_1 located over or behind the working face exhibits a steep increase of subsidence in the unit of time sooner than does an observation point P_2 lying ahead of the advancing face and subsequently undermined by it (Fig. 242). A point P_1 over the stationary rib where the face was opened out is increasingly affected by extraction exceeding the area at which rock strata sag fully, whereas parts of the working lying at the margin of the influence area have only a minimal and much later effect on it – a fact which is evident from the flat course followed by the continuation of the curve. The point P_2 is at first caused to subside only minimally by the approaching face, and the increase in subsidence following the passage of the face is in fact steeper than in the curve for P_1 . The steepness of the curve in this principal subsidence phase is determined, at a given depth of working, by the advance of extraction in m/day, by the amount of convergence aM in the working, and by the width l_d of the panel. The faster that extraction proceeds, the faster the surface point subsides. Similarly, the greater the seam thickness mined, the subsidence factor, and the influence factor (= width of working), the greater – as in Eq. (164) – is ground subsidence over time. The rate of point subsidence (in mm/10 days) is consequently almost twice as rapid with caving as with stowing¹.

Over the halted extraction face, from the moment of halting extraction, measurements are made of the delayed subsidence of point P_3 caused by the sta-

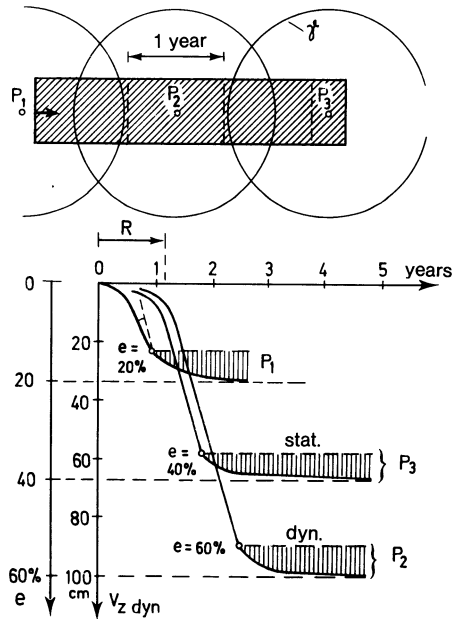


Fig. 242. Subsidence over time of the points P_1 , P_2 and P_3 . *Hatched areas:* static or dynamic delayed subsidence

¹ The influence of a , M , and e on the steepness of the time-subsidence curve is indiscernible if the relative amounts of subsidence ($v_{z\text{end}} = 100\%$) are compared with each other (Fig. 248). Other terms for delayed subsidence: residual or post-mining subsidence (see Appendix III).

tionary extraction area itself, without the added – active – influence of any new extraction. During this phase of delayed post-mining subsidence (e-factor constant), the roof behind the stationary face gradually exerts pressure until the compacted stowage material, with the exception of the bridged-over marginal zone, adequately bears the weight of the strata (Fig. 37). This static delayed subsidence of the point P_3 over the stationary working, occurring after the halting of mining activity, lasts longer than the dynamic delayed subsidence of point P_2 on the continuation of extraction beyond the circumference of the critical area, in circumstances in which the roof is further undermined and the marginal zone of the extraction panel is dynamically subsided as the unsupported span of roof lengthens.

In Fig. 243 the difference between static and dynamic delayed subsidence is made clear numerically. The extraction area represented in the lower part of the Figure is divided into a zone A exhibiting full (“critical”) subsidence of the overlying strata and a zone B showing reduced subsidence. In case (1) extraction is halted on 1 January 1976, at which point in time the subsidence at the surface point P caused by the extraction areas A and B amounts to $50 + 5 = 55$ cm. Static delayed subsidence within the next half-year amounts of 22 cm, of which 15 cm is attributable to area A, and 7 cm to area B. Before the ground is at rest again, the point P will subside by a further $85 - 77 = 8$ cm.

In case (2), extraction is continued without interruption, and by 1 January 1976 the point P shows a subsidence of 55 cm, as in case (1). In the following half-year, area A acts on point P with 20 cm subsidence, and area B with 14 cm. The effect produced by area B has thus doubled as compared with case (1). This is because there is no longer a marginal zone, and with a doubled length of extraction, vertical pressure and convergence in field A have increased. Of the final subsidence effect of 90 cm, by 1 January 1976 only 1 cm remains to come as delayed subsidence.

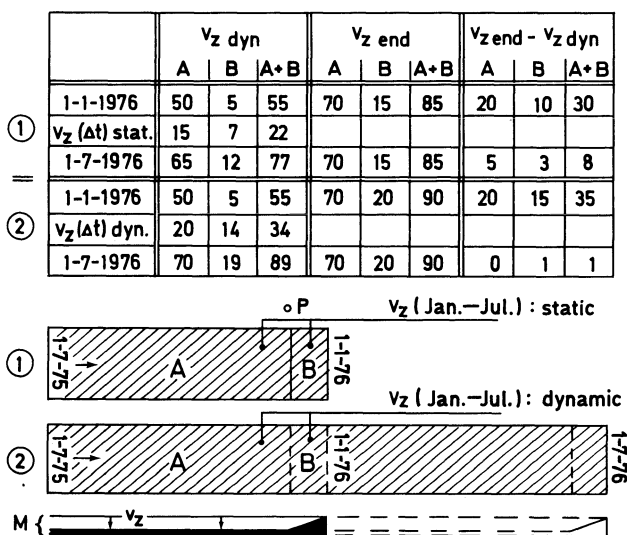


Fig. 243. Static and dynamic delayed subsidence

As can be seen from Fig. 242, the highest *rate of subsidence* – i. e. the largest increase in subsidence in a unit of time – varies somewhat for the individual surface points of a measuring line along the strike, over the advancing face. It is greater at the centre of the trough than at the margin: which is understandable in view of the more intense effects produced on the central point during mining. The increasing steepness of the curves of subsidence against time towards the centre of a working is also discernible in the transverse profile in Fig. 237. (The curve of subsidence over a period for the points 114 and 21 lying over the marginal zones of the panel are flatter, after the passage of the face, than those for the centrally located points 110, 107, and 104.) The rate of subsidence of a surface point is consequently dependent not only on the rate of face advance but also on the position of the point in relation to the working. In each unit of time the point in question receives a fresh “influence impulse”, varying according to the size of the working, the rate of face advance, and the lie of the extraction front. This produces a corresponding variation in the rate of subsidence, accelerating it or slowing it down. The steady increase and later decrease in these “impulses” is expressed in the shape of the curves of subsidence against time. If extraction operations are interrupted, the development of subsidence is delayed; but where the workings are at some depth, the earlier shape of the curve is quickly re-established on the resumption of operations (Fig. 56).

With shallow workings (depth H less than 100 m) it is even possible to make out the rhythm of cyclic longwall mining from the subsidence of a surface point, if this is continuously registered with very precise apparatus. During the face-cutting operation the surface point subsides relatively strongly, in a slight up-and-down movement, by between 0.1 and over 1.0 mm/h. Ahead of and behind the advancing face, intact strata sag at once while loosened blocks of strata go down spasmodically. This intensive subsidence phase is thus determined by gravitational settlement as the “active” element; the rheological, time-dependent element of strata deformation participates to only a small extent. In the filling and preparation shifts which follow (with the working face stationary), subsidence continues only slowly, increasing linearly; what is concerned here is purely time-dependent, delayed, and stationary subsidence of the rock mass over the previously created excavation along the working front, combined with consolidation of fresh stowing material, the slow slipping of blocks of strata down separation surfaces, and a redistribution of stresses in the rock mass. There then follows, with the start of a new cutting shift, a steep fall once again in the time-subsidence curve as active subsidence is superimposed on the rheological component (Orchard and Allen, 1974). During work-free week-ends, intermediate, time-dependent subsidence slows down. Over deep workings, however, the two elements of subsidence are no longer distinguishable at the surface; the thickness of the rock mass smoothes out the separate stages of subsidence into a uniform curve of long-term subsidence.

10.2.1.1 Examples from Longwall Coal-mining

The normal curve of the time-related subsidence of a surface point represented in Fig. 244 is based on values measured in many British coalfields. The horizontal

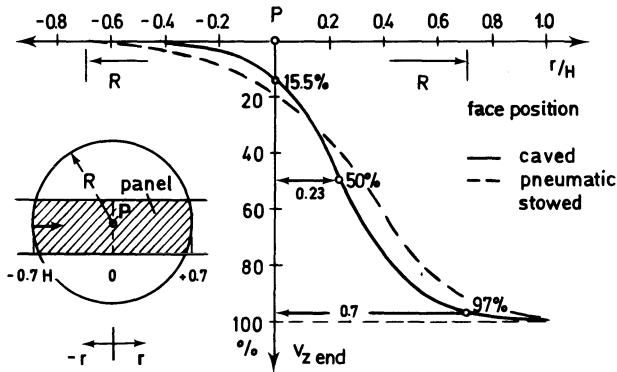


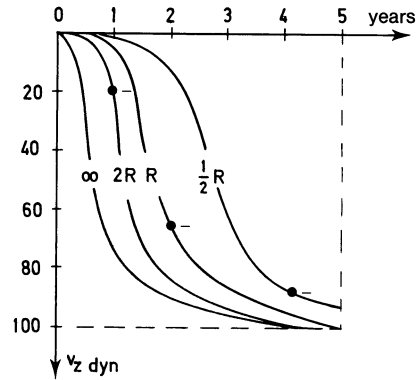
Fig. 244. The time-subsidence curve in U.K. coal-mining at depths of between 300 and 600 m (after National Coal Board, 1965). Total subsidence $v_{z\text{end}}$ — e.g. 80 cm (stowed) or 140 cm (caved) for $M = 250$ cm and $e = 60\%$

axis of the graph reflects the horizontal distance $\pm r$ of the working face from the surface point P, expressed as a proportion of the depth of working H , and the vertical axis reflects subsidence as a percentage of final (total) subsidence. In this way the graph can be applied to other depths and amplitudes. The radius R of the critical area, by which the progress in extraction is judged, has in this example a value of $r/H = 0.7$, in accordance with Eq. (161). It will be noticed from the *time-subsidence curve* of Fig. 244 that, once the working face has advanced by a distance of more than $0.7 H$ or $1 R$ beyond the observation point, only a very small residual amount of subsidence remains. At the moment when the face passes directly under it, subsidence at the observation point has attained 15.5% of the final subsidence occurring when the face has traversed the whole critical area. The half-subsidence value is found when the face has passed under the point and moved on by a distance of $0.23 H$, or approximately 92 m when $H = 400$ m depth. The type of stowing has a noticeable influence on the shape of time-subsidence curve: with pneumatic stowing the curve is flatter than with caving or pillar-and-stall working. Delayed subsidence can be set off by coal pillars of a minimum breadth of $0.1 H$ or by roadside packs, if either collapses after many years. It has further been found in the U. K. that, at an equal rate of face advance (m/d), the deeper the working, the longer the delayed subsidence persists.

In other coalfields also, delayed subsidence is small if the block of strata has been broken up by repeated working. When this occurs in the Ruhr, the curve illustrated in Fig. 182f applies. On the passage of the working face under the point P, usually only 10% of final (total) subsidence has occurred. In this first phase (I), subsidence of the point P is determined by deformation of the block of strata resting on the seam edge. The undermined surface point next moves into the zone in which strata overlying the working are subsiding rapidly and steeply, with the result that the surface point can drop by 20 mm/d shortly after the passage of the face. By the time the critical length $2R$ has been extracted, 90% – 95% of final subsidence at the point P has been attained. Thus, with a rate of advance of 3 – 5 m/d in fractured strata, the main part of subsidence has been completed in phase II, in the first 6 months approximately.

This main subsidence, which in total extraction (longwall mining) includes phase I, initial subsidence, and phase II, accelerated subsidence — i.e., approximately 90% of the total subsidence of a point — is referred to in the literature as

Fig. 245. Subsidence over time of a surface point at different rates of face advance – namely, 0.5 R/year, 1 R/year, 2 R/year and infinitely rapidly. (After Perz, 1948). Strata unbroken, visco-elastic. Points in the curves: e factor constant, the beginning of delayed subsidence increasing with the rate of face advance



“active” subsidence, as distinguished from the purely rheological, “post-mining” or “residual” subsidence of the closing phase III, here called “delayed subsidence”. During active subsidence the influence factor e relating to the point P grows in intensity, and the extraction area in size. During post-mining subsidence the e factor remains constant, either because mining has ceased (case 1) or because the working face has left the critical area (case 2 – see Appendix III). In the post-mining phase therefore, no mining is taking place within the critical area relating to the surface point P.

The steepness of the time-subsidence curve in this active phase II, at a given convergence aM and width of panel, depends mainly on the rate of face advance but also on the depth of working and the degree of reworking of the strata. In the subsidence curves for various rates of advance calculated in accordance with Eq. (319) and shown in Fig. 245, the rate of subsidence is near enough proportional to the rate of face advance. Figure 246 shows the maximum rates of subsidence to be expected in fractured strata. In comparison with unbroken strata, reacting still in a predominantly visco-elastic manner, the rate of subsidence in fractured strata is greater, because movement develops more rapidly and is concentrated as to 80% on phase II, with only slight initial and delayed subsidence. The resultant curve is a steep one.

The influence of working depth clearly seen in Fig. 246 – at a constant rate of face advance the maximum rate of subsidence decreases with increasing depth – is exemplified in Fig. 247 with two workings at H_1 and H_2 advancing at an

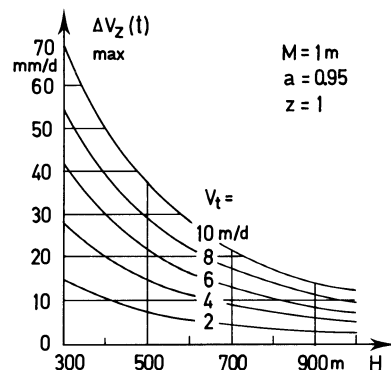


Fig. 246. Maximum rate of subsidence of a surface point, as a function of rate of face advance v_t and depth H of mining. (After Sauer, 1975). Strata fractured. The maximum rate of subsidence increases with v_t as shown in Fig. 245 but the amount of delayed subsidence is nearly equal for all rates of face advance

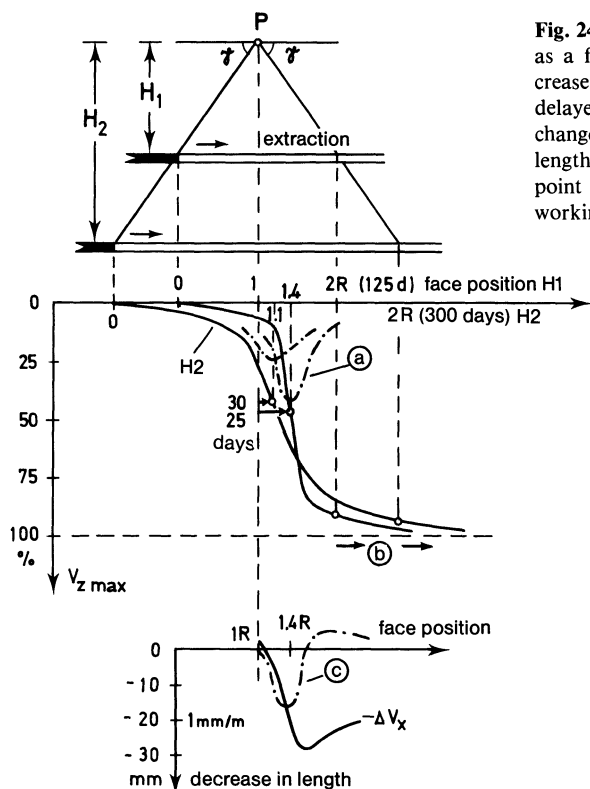


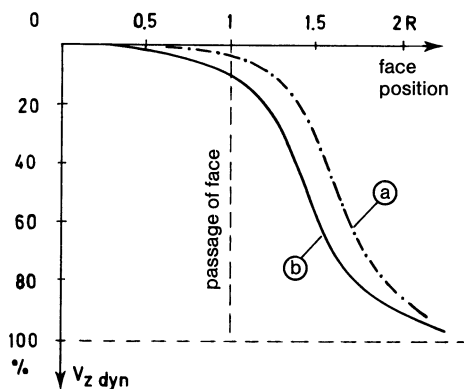
Fig. 247. Subsidence over time of a point P, as a function of depth of mining H. **a** Increase in subsidence over 10 days; **b** dynamic delayed subsidence; **c** increase in linear change over 10 days. *Bottom:* decrease in length of a 20 m long measuring line at the point P, measured in relation to the upper working. (After Drent, 1957)

equal pace. At the greater depth H_2 the working requires a longer time (300 days as against 125 days) to traverse the critical area of the point P, with the result that the respective time-subsidence curve is broader and has a flatter slope. Surface points thus subside more slowly over the deeper working H_2 , and it is not until 30 days after the face has passed, when extraction has reached a length of $1.1 - 1.2 R$, that the surface point P attains the maximum rate of subsidence (working H_1 : 25 days = $1.4 R$).

To make workings at varying depths comparable as regards their subsidence influence in terms of time, the rate of face advance is related to the critical-area radius R . A relative rate of advance expressed in this way as $0.1 R/\text{month}$ then represents an absolute advance of, for example, 0.7 m/d at a depth of 200 m ($R = 140 \text{ m}$), or of $0.7 \times 4 = 2.8 \text{ m/d}$ at a depth of 800 m ($R = 560 \text{ m}$).

In the Netherlands coal-mining industry, the subsidences of surface points overlying both caving and stowing operations were observed and their relative values compared (Fig. 248). This showed that stowing delayed the settlement of roof beds, because the small-sized stowage material supported the strata behind the face sooner than the coarse rubble from the collapsed roof bed could, since the large pore volume of the latter meant that it did not become load-bearing until after convergence of over 50%. This indication of the course of subsidence in near-surface mining will undoubtedly apply also to deeper workings (Drent, 1957). Stowing procedures thus reduce the amount of intermediate tilt, curva-

Fig. 248. Time-subsidence curves for stowing methods (a) and caving methods (b) when working at a depth of 50 m (after Drent, 1957). $50\% v_{z \text{ dyn}} (\text{caved}) = 1.45 R$; see Fig. 244: $0.23 r/H = 1.3 R$ (depth > 300 m)



ture, and linear change over the working face and thereby also the costs of mining damage.

Another point of interest is the comparison of dynamic ground subsidence $v_{z \text{ dyn}}$ at the point P, and mining convergence c_{dyn} at a point P' vertically under it in the mine roof (Fig. 249). It will be noticed that the roof point P' attains its final amount of subsidence after the passage of the face sooner than does the surface point P, and that a smaller amount of subsidence at the surface corresponds to the flatter convergence curve for the near-surface working. The curve of subsidence increase of the surface point continues to rise until the stowage material lying $0.3 - 0.4 R$ behind the face has attained a sufficient compaction and load-bearing capacity and the subsidence movement of the roof point P' has come to an end (Wardell and Webster, 1957).

In Fig. 250 the subsidence over a period of time of the *roof bed* in an increasingly large working is portrayed in 16 sections numbered I to XVI, in which it is assumed that final convergence in the section concerned is reached when the working face has moved on by five further sections. Subsidence of the roof bed

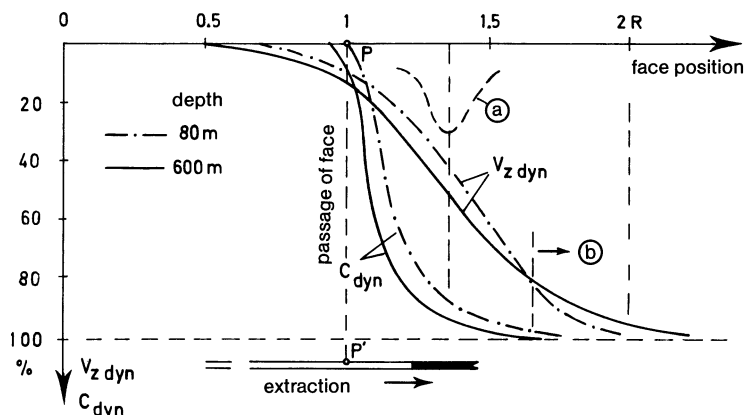


Fig. 249. Subsidence over time at a surface point P, and convergence over time in the extraction area, at a point P' vertically under P. **a** Increase in subsidence; **b** flattening-out of time-subsidence curves after compaction of stowage material. (After Wardell and Webster, 1957)

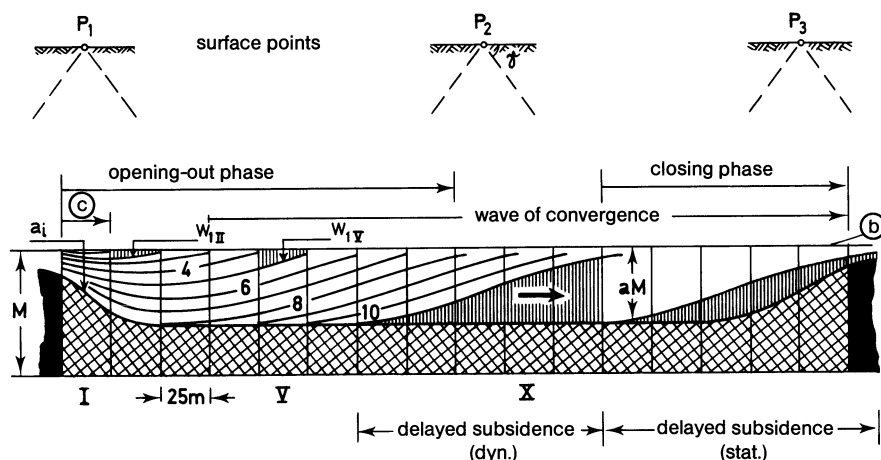


Fig. 250. Subsidence of the roof bed over time, for an extraction face continually advancing from left to right. a_i Subsidence factor in the section concerned; w subsidence of the roof bed; b original position of the roof bed; c scope for subsidence in the extraction panel. (Heaving of the mine floor is ignored.)

thus increases from w_1 after mining of the section concerned to a final value of w_5^2 . The main roof settles over a caved working in a similar manner. It will be noticed that the amount of subsidence w_{1II} (hatched) in the section II just mined is significantly smaller than the first subsidence step w_{1V} in section V. Only from face position VIII onwards does there form, as in Fig. 37, a constant roof-bed subsidence line, or main-roof line-of-sag, moving with the working face. This convergence wave can be compared with the “bow wave” at the surface represented in Fig. 241. With Fig. 250 it also becomes clear why, as in Fig. 237, the surface points 114 and 21 lying over the margin of the panel subside more slowly than the points 110 and 107 over its central zone. Under the central point P_2 (Fig. 250) the convergence wave has fully formed, and in the five sections behind the advancing face the stowage material, or the broken rock from the descending roof, is quickly compacted by the volume of the hatched portion until it follows the smoothed-out, horizontal line of the roof bed. In the extraction sections I to VIII, on the other hand, vertical strata pressure increases only gradually during the initial phase, with the result that the surface point P_1 subsides more slowly and less far than point P_2 . With the ending of extraction under point P_3 , the convergence wave also halts, and the residual subsidence of the roof bed in sections XIII to XVI during the closing phase is slower and smaller than in the centre or on the left-hand side of the panel (cf. Fig. 242).

The zones of vertical compression and extension in the roof move forward with the working face (Fig. 115). The rate of convergence in the extraction area is chiefly influenced by the amount of “give” in the stowage material or caved roof beds, by the increase in strata pressure, and by the seam thickness mined M

² The influence of the extraction element on a surface point increases with time in accordance with an exponential function (Kochmanski and Magdziors, 1972).

(Sects. 1.3 and 3.2). Under a constant load (force), final convergence sets in sooner in a working of low seam thickness than in one having a thick seam, where compaction of the stowage material (distance moved) requires more work and time (work = force $\times$ distance moved).

10.2.1.2 Influence of the Rock's Mechanical Properties on Subsidence over Time in Room-and-Pillar Working

In methods of mining for minerals employing permanent pillars (Table 2), the roof of the mine opening is supported by 10–20 m wide room pillars and entry pillars, and additionally, at the margin of larger workings, by barrier pillars of up to 50 m width (Figs. 1 and 2). The load-carrying capacity T of a pillar is often calculated by the formula

$$T = \sigma' \sqrt{\frac{b}{h}} \quad (306)$$

from its cube crushing strength σ' (30–40 MN/m² in the case of saline rock, 100–200 MN/m² for silicate rock), as well as from its breadth b and height h . The smaller the reserve of load-carrying capacity³ T/Q – where Q denotes the average load on the pillar in MPa, as in Eq. (10) – the greater the vertical compression on the pillar, and consequently the greater the settlement of the roof (Börger, 1971).

In *ore-mining*, the pillars, after an initial vertical compression, are usually so stable that the mined-out chambers remain open even after abandonment of the mine, or become only loosely filled with broken-off fragments of roof bed (Fig. 49). On the other hand, permanent pillars of coal, of sedimentary ores, or of weak limestones exposed to mine waters and damp mine air weather in the course of time and collapse gradually, or even suddenly (Figs. 52 and 321).

In *salt-mining*, the chambers, which in flat-lying deposits may today be up to 200 m long, 5–20 m wide and, depending on the thickness of the deposit, 3–20 m high, close up again partially or even wholly, because pillars of saline rock deform plastically under the strata load. As the roof of the mine working settles, it is followed down by the overlying strata as a single group, with the result that shortly after the start of pillar convergence a depression forms at the surface of the ground.

In steep-lying deposits (Fig. 4), pressure normal to the stratification bearing on the pillars and on the roof of the chambers is less than the vertical pressure in a flat-lying deposit, in accordance with Eq. (7) and Table 4. Pillars can therefore be narrower and chambers wider than in flat-lying deposits. Because of the large dimensions of the trough over steep-lying deposits, the rate of subsidence is reduced.

As set out in Sects. 1.2, 1.3, and 2.2, the *amount of subsidence over time* in strata overlying room-and-pillar workings with permanent pillars depends essentially on

³ Denoted in Fig. 177 by the safety coefficient n . For other formulae see Dreyer (1967).

- the deformation behaviour of the pillar rock;
- the amount of load on the pillar (in N/cm^2);
- the shape of the pillar (width against height, and length);
- the “grip” on the pillar in roof and floor;
- the degree of back-filling of the chambers;
- the width (span) of the chambers; and
- in salt-mining, the expected life of the pillars.

In addition, subsidence over time is influenced by the size of the mine working, by the rate at which pillars become loaded (i.e., the rate of face advance), by the size and position of unmined portions of the deposit (residual pillars), and by the distribution of room pillars and chain pillars within the mine working. Indeed, for an equal sacrifice of mineral V_L , the number, size, and arrangement of pillars – in other words, the amount of support for the undermined roof strata – can differ greatly, as the variety of structures in the examples of workings in Figs. 2, 6 and 253 shows.

Note on a): deformation behaviour. Silicate and calcareous rocks deform elastically under load. The fracture process is signalled by shear and separation

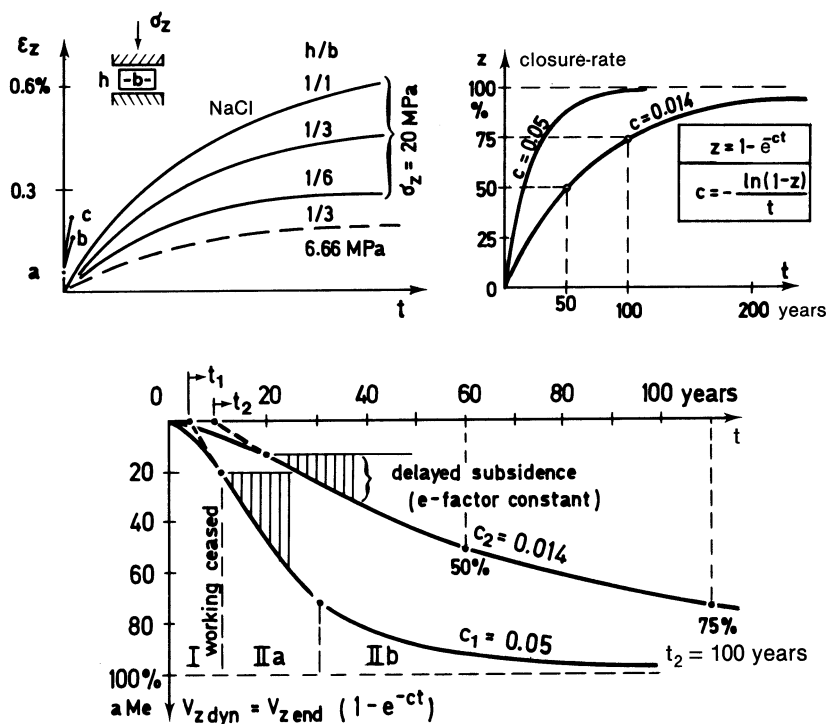


Fig. 251. *Left:* time-dependent deformation of rock salt, and also of basalt (a), limestone (b), and sandstone (c), under a permanent load of 20 MPa. *Right:* curves of the time factor z in salt-mining; c strata parameter. *Bottom:* subsidence over time of points at the ground surface, over chamber workings with yielding pillars. *Phase I:* active subsidence during mining. *Phase IIa:* linearly increasing delayed subsidence. *Phase IIb:* closing stages of delayed subsidence. t_1, t_2 = time as counted from the middle of the period of working

cracks (Fig. 27). In saline rocks, the viscous and plastic components of deformation predominate, with very little elastic reaction: if loading is increased gradually, these rocks flow fracture-free along intracrystalline translation surfaces. Fracture planes, which form on too rapid loading or on blasting, accelerate and increase deformation but may “heal” through a “briquetting” effect and recrystallization. Coal and carnallite, on the other hand, fracture suddenly and show a brittle behaviour on a rapid increase in load. The more jointed, moist, and coarse-grained a rock is, the more rapidly and severely it deforms under load. Important roles in the extent of deformation are also played by the interlocking of the mineral substances, the amount of stressing and stress-relief undergone previously, and also, in the case of salt, by temperature.

Most saline rocks behave, like rock salt, visco-elastically under a mild load and visco-plastically under a severe load. Nevertheless, under an equal load of, for example, 20 MPa, deformation in hard-salt at 0.2%⁴ and in sylvinites at 0.35% of the initial height is less than in rock salt at 0.5%. By comparison, at a pressure of 20 MPa basalt is vertically compressed by only 0.03%, limestone by 0.1%, and sandstone by 0.2% (Fig. 251, left). Transverse expansion ratio for rock salt stands at $m = 4$ or, under high loading, at $m = 2$. Young's modulus E at the start of loading comes to approximately 6,500 MN/m², and tensile strength at 2.5 MN/m² is very low.

Note on b): load on the pillar. The average load on the pillar σ_p , in Pa, is related to the depth of mining and the ratio between the area of the mine working and the ground area of all pillars [Figs. 2 and 18, Eq. (10)]. The outer pillars of the working carry a smaller load than the inner ones (Fig. 14). The larger the working and therefore the greater the number of pillars bearing the maximum load at its centre, the more and the faster the roof sinks; for with a larger pillar load, elastic vertical compression of silicate rocks increases and visco-plastic flow in saline rocks accelerates (Fig. 19). A faster rate of face advance leads to a more rapid increase in load in the working chamber and a greater initial settlement of the roof.

The following stages of loading on pillar and chamber develop in room-and-pillar working:

Immediately after the excavation of the chambers the undermined roof begins to sag, and in this process the exposed sides of the pillars, being equivalent to rib-sides, are loaded considerably more severely than the pillar centres, with a part of the weight of undermined strata in addition to the general pressure of overlying strata γH [Eq. (14)]. This is indicated by the dashed line σ_1 in Fig. 21. As the pillar sides become exposed, however, the lateral pressure – in salt $\sigma_2 = \gamma H$, and in ore $\sigma_2 \approx 0.25 \gamma H$ – is removed from them. The result is that there is at first a very rapid elastic and plastic deformation of the pillar sides outwards and into the chambers. This is accompanied by a drop in horizontal compressive stress σ_2 from a maximum of γH , or alternatively of $0.25 \gamma H$, at the pillar centre to nil at the pillar side (Fig. 21). At the pillar edge the rock becomes fractured at the point where the difference between the principal stresses $\sigma_1 - \sigma_2$ exceeds double the

⁴ These are average values, after Dreyer (1967, p. 33) and depend on crystal structure and previous tectonic stress (e.g., through depth pressure or folding). By “hard-salt” is meant a mixture of sylvine (or sylvite, KCl), rock salt (NaCl), and kieserite (MgSO₄).

value of its shear strength [see Eq. (12)]. The fractured margin zone of the pillar loses in load-bearing capacity as a result of the transverse deformation and fissuring, but it still provides considerable support to the pillar core at its open sides exposed to the chamber (mantle pressure). In this process the initial high vertical pressure σ_1 on the pillar edge becomes reduced, and the peak of pressure is shifted to the pillar core. In the course of time the creep and fracture zones in the pillar increase in depth, and the distance between the edges of the working itself, which bear the main load of the undermined strata, increases as mining progresses (Fig. 14). As a result, the pillar core becomes increasingly loaded as the working is enlarged, the support provided by the edges of the working at its centre decreases [Eq. (24)], and the outer layer of the pillar seeks to evade the vertical pressure by transverse deformation or fracture. In the end-stage of the mine working's development, the vertical pressure σ_1^* at the pillar core finally exceeds the specific area-load σ_{Ap} of Eq. (10).

As regards load distribution in the working area, room-and-pillar working differs from longwall working, just as ore-mining differs from salt-mining. In mining on a long front, strata pressure behind the face working drops from the pressure of overlying strata γH to practically nil, and rises again in the centre to the peak value of γH only on the attainment of the critical area (Fig. 10). In room-and-pillar working, all pillars – even the least loaded outer pillars of the working – carry a load, from first exposure up to extraction of the critical area, of between one and two times the overlying-strata pressure (waste of mineral 50%). Whereas in ore-mining the pillar loading reached by the time that work is halted may remain unchanged for hundreds of years, the loading on salt pillars (in Pa) decreases with the passage of time – in fact, on the complete closing-up of the mining excavations, back to no more than the overlying-strata pressure γH (Fig. 14). The reason for this is the sustained visco-plastic flow behaviour of saline rock under a continuous load. The halting of working does not halt the flow behaviour of salt pillars, with the result that their ground area increases through transverse expansion, and the constant residual load of the undermined strata (in N) is distributed over an ever increasing pillar area. The chambers moreover, as they are narrowed “like cushions” (Fig. 17), become filled with slabs of salt flaking off from the sides of pillars and with lumps breaking off from roof and floor beds, constituting a form of “stowing material”, which becomes increasingly load-bearing as the mine cavities close up. In this way pressure differences gradually even out across the whole working area. The abutment pressure on the edges of the working, and on the pillars in its central zone, recedes – in the end to zero, when the chambers have completely closed up again. Throughout the mining horizon an even, overlying-strata pressure of γH will then prevail as before extraction began.

Note on c): shape of a pillar. Apart from the size of the load on it, the shape of a pillar influences its deformation. Under a continuous and constant load, a pillar of equal height h and breadth b ($h/b = 1/1$) deforms far more and – especially in the case of salt – more rapidly than a pillar of breadth three times its height ($h/b = 1/3$), as indicated for rock salt in the left-hand diagram of Fig. 251. The reason is that, in a flattish pillar, transverse expansion in its upper and lower

zones is impeded by the bond with the roof and the floor, and a horizontal compressive stress σ_2 builds up, showing an increase from the pillar face towards its interior (Fig. 21). This horizontal pressure supports the core of the pillar like a stiff casing or “mantle”, increasing its load-bearing capacity and reducing the amount by which it is vertically compressed. For example, because of this mantle, the core of the hard-salt pillar ($h/b = 1/2.5$) in Fig. 17 is vertically compressed by only 10 mm, or 1.7 mm/m, despite the high vertical pressure of 130 MPa. Even an outer layer (mantle) which has been fractured and brought into a state of flow by overstressing still exerts a lateral supporting pressure on the pillar core – a pressure which is nevertheless, depending on the degree of fracturing and the cohesion of the rock fragments, less than in the previous elastic state. The broader a pillar is, the less its load-bearing capacity is reduced by a fractured or plastic outer layer. Pillars of rectangular, elongated outline in plan, such that the mantle pressure along their longitudinal axis is very high, are therefore used as supporting pillars for haulage ways requiring to be kept open for long periods, and as roof supports at the margins of workings.

From trials with models it is known that even small mantle stresses (σ_3 in Fig. 27d) increase significantly the crushing strength⁵ of a cylindrical rock sample, since by Eq. (11) the shear stress which is decisive for failure is reduced along with any horizontal compressive stress σ_3 . The greater the height of a pillar, the less will be the supporting mantle pressure halfway up that height, because the restraining influence of roof and floor on transverse deformation decreases with distance. Consequently, with each height/breadth ratio of the pillar there goes not only a definite, uniaxial crushing strength σ_B , as emerges from Figs. 20 and 27c, but also a definite value for vertical compression at an equal pillar load σ_{A_p} (in Pa).

Notes on d) to f): grip, backfilling, and width. Transverse deformation of the pillar is sometimes promoted by beds of, for example, clay at the interface with roof and floor, which are liable to slip, thereby lowering the mantle pressure on the pillar core and reducing the compressive strength of the pillar by around 40%. Or alternatively, a mine floor which has arched upwards and mine roof which has sagged may have the effect of impeding the expansion of pillars into chambers which have narrowed like a vice (Fig. 17). Filling the chambers tightly with solid material – e.g., mill residues – may also create a mantle pressure on the sides of pillars. In brining operations or in flooded mines, the hydrostatic pressure of, say, 8 MPa at a depth of 800 m will support the sides of pillars and roofs of chambers.

Settlement of a roof over a working, which becomes increasingly undulating in outline with increasing breadth of chambers, as illustrated in Fig. 41, evens out in higher horizons into a smooth, trough-shaped line (Fig. 116). The influence of chamber width on the amplitude of subsidence is given by Eq. (15). In very wide chambers the roof beds may cave in and the chambers become partially filled; floor beds may also arch upwards.

⁵ A distinction is drawn between the uniaxial crushing strength of a cube with a side length of 10 cm ($h/b = 1/1$), a prism ($h/b = \text{any given ratio}$) or cylinder, and the triaxial crushing strength of a cylinder or cube ($\sigma_1 \neq \sigma_2 \neq \sigma_3$), where the mantle compressive strength is exceptionally $\sigma_2 = \sigma_3$.

Note on g): expected life of the pillar. Whereas in longwall coal-mining, or in room-and-pillar ore-mining, it is essentially only the rate of face advance which determines the development of ground subsidence over time (because delayed subsidence – Fig. 242 – is minimal), in salt-mining the expected life of the pillars is a significant additional factor: the principal subsidence occurs only after abandonment of the mine working and may last, in the form of delayed subsidence, for many decades (Fig. 251, bottom, and Appendix III).

Loaded saline and silicate rocks react very differently under the influence of time. The curves of vertical compression in Fig. 251, left, show the dependence of vertical compression on the h/b ratio. They also show delayed subsidence over a very long period in the case of the saline rocks. Pillars of hard silicate rock, on the other hand, which as in ore-mining are created with such dimensions that the load-bearing core is stressed only within the elastic limit⁶, undergo very rapid convergence. In the case of comparatively slender pillars in fact, the rate of convergence is directly proportional to the loading on them [Eq. (29)]. The end-state of deformation is somewhat delayed by the fact that a new equilibrium of forces must be established between the outer fractured layer and the pillar core: which takes some time. As soon as a constant, end-state loading on pillars of silicate rock has been reached – which happens on abandonment of the working, or on attainment of a critical area at the centre of the working (Fig. 14) – the convergence movement in pillars ceases. The roof sags no further over the chambers, and strata and ground movement in this part of the working comes to an end.

Salt pillars, on the other hand, behave in a predominantly visco-plastic manner under continuous load, with convergence in a vertical direction spread over a long period of time. Elastic deformation is minimal (Fig. 15). Under a constant load, salt masses deform at first rapidly and then more slowly. The rate of deformation – the so-called creep rate ϵ/t , represented by the slope of the deformation curves in Fig. 251, left – depends as much on the specific area load (in Pa) as on the h/b ratio of the salt pillars. The convergence of salt pillars continues even when the load on them has reached its final value. The question is whether this convergence ceases after a time and the salt pillars carry the subsided roof strata permanently in this height (see Fig. 252c), or whether the convergence movement continues until the complete closing-up of the mine excavations. The following discussion of investigations on pillar models in the laboratory may clarify the rheological behaviour of saline rock.

Where the continuous load on a salt pillar is small, the rate of vertical convergence and horizontal divergence decreases in the course of time. If the creep rate, after the pillar has stood for some time, returns to zero, the pillar will have reached its permanent, final height. This is explained by consolidation of the crystal lattice to the point where it resists further translation movement – so-called *strain hardening*. Salt deformation such as this, which is designated as “primary flow” in Fig. 15, is illustrated by the visco-elastic Kelvin model of

⁶ The pillars are dimensioned in such a way that they are loaded up to only one-half or one-third (safety factor 2 or 3) of the failure load σ_B established in the laboratory on equivalent samples, which is dependent on the h/b ratio [Fig. 20 and Eq. (306)]. The specific area load of pillars, in Pa, is given by Eq. (10) or by a nomogram in accordance with Fig. 18.

Fig. 61. There, a continuous and constant load produces deformation which tends asymptotically towards an end-value of $\varepsilon = \sigma/E$, because the tensile or compressive force of the spring, corresponding to the elasticity constant E , grows in proportion to the deformation ε , until it is equal to the applied load. The delay in deformation is produced by the Newtonian damper consisting of a piston in liquid. Thus a $10 \times 10 \times 10$ cm cube of pure rock salt (uniaxial crushing strength $2620 \text{ N/cm}^2 = 26.2 \text{ MPa}$), under a constant load of $1000 \text{ N/cm}^2 = 10 \text{ MPa}$, exhibited an asymptotic end-value in deformation after 150 days with 1.7% vertical compression (Dreyer, 1967).

In an identical cube of salt under a greater constant load of $1400 \text{ N/cm}^2 = 14 \text{ MPa}$, a constant creep rate of 0.4% per 10-day period set in after 60 days. In other words, from a particular level of loading onwards, the marginal value of which depends in part on the h/b ratio of the sample, no further “consolidation” takes place in the cube of salt: the salt in fact “flows” fracture-free. This secondary, plastic flow, accompanied by an at first degressive and later constant creep rate, can be represented by a visco-plastic Bingham model – a parallel arrangement of friction bodies and pistons-in-liquid preceded by springs (Fig. 15). The initially small and elastic deformation of the model when under a constant load is transformed, when the load overcomes the friction resistance of the St. Venant body – the flow boundary⁷ – into a degressive, viscous movement, and then into an endless, uniform, plastic movement. The rate of deformation – the creep rate – is determined by the level of load and the viscosity of the Newtonian body. In contrast to the Kelvin model, the plastic deformation reached at a given point in time persists in the Bingham model (see Fig. 61).

An even higher loading of, for example, $1650 \text{ N/cm}^2 = 16.5 \text{ MPa}$ leads to accelerating creep rates and, in the end, to failure of the salt cube (tertiary stage of flow, Fig. 15). A salt pillar can therefore be regarded as load-bearing when the creep rate is declining, and as stable when the creep rate becomes constant. By contrast, where the pillars are of silicate rock, a constant rate of deformation is a signal that there is already a risk of failure.

Although carnallite ($\text{KCl} + \text{MgCl}_2$) deforms visco-plastically when subjected to a slowly increasing load, in the same way as other varieties of saline rock but to a greater degree than rock salt, it is the exception among saline rocks and deforms elastically, and only to a small degree, when loaded rapidly. That means, where the increase in loading is rapid, elastic deformation energy in carnallite becomes stored, particularly if, for example, it is in the core zone of a broad pillar and is restrained by mantle pressure in the outer zone from deforming and relaxing horizontally. Furthermore, since the uniaxial cube-crushing strength of carnallite, at approximately 5 MN/m^2 , is comparatively low, unfavourable rock-mechanical conditions can lead, if the shear strength of a carnallite pillar is exceeded, to the stored energy of deformation's becoming released percussively as brittle fracture (rock burst). Neighbouring pillars are then suddenly loaded with extra strata pressure and similarly fail: which can lead to

⁷ With the Maxwell model (Fig. 71), flow behaviour, which is similarly irreversible, begins immediately on the application of load (viscous flow with no flow boundary, in contrast to plastic flow with a flow boundary).

the collapse of an entire working. The overlying strata and the ground surface subside over the collapsed working by a metre or more after only a brief lapse of time.

It can be seen from the above that secondary, long-term flow behaviour in salt rock, although not dangerous to the mine operation, is associated with considerable subsidence and mine damage. In ore-mining and coal-mining, it is the intensity of weathering that determines when old mine workings converge or collapse. In salt-mining, *long-term flow behaviour* can be expected:

- where the loading on pillars is high;
- where a salt rock is particularly prone to flow behaviour or has been shattered by shot-firing;
- where the pillar cross-section is a slender one ($h/b > 1/3$);
- where the bonding of the pillar with roof and floor is capable of slipping; and
- where chambers are high and have not been back-filled.

In other words, long-term flow in salt-mining can be expected in a large-area working operated with a low sacrifice of mineral, at great depth, and in conditions of high strata temperature. Only if the salt continues to flow uninterrupted in the secondary stage, however, will mine chambers close up completely over a period of tens or hundreds of years. Since flow in pillars reduces the specific area-load bearing on them, makes the h/b ratio smaller and therefore more favourable, and leads to caved waste providing support in the narrowed mine excavation, long-term flow in salt can later revert to consolidating flow. The result is that, where the depth of working is not great, the convergence movement in pillars and the subsidence in overlying strata can come to an end before the mine excavations have closed up completely.

On the basis of measurements of vertical deformation in pillars, the creep rate is at its maximum, with 10–20 mm per 100 days, during the 2- or 3-year period when the working is being developed (Fig.19). After that it decreases, but owing to the shortness of the periods of observation so far, it is generally impossible to say whether the creep regime will recede completely in a few centuries or continue at a fixed rate – i.e., whether the degressive creep rate is to be ascribed to consolidation in the salt or, where secondary flow is continuous, only to a reduction in the specific area load. Unlike the experience with short-period compression trials in the laboratory, where the continuous load is constant, primary flow in a salt pillar in situ can display, in cases where the load is provisionally increasing, and even after the initial phase, a weakening upward trend; conversely, secondary flow can slow down if the specific area load on the pillar decreases. Because of this alteration in loading and in pillar shape, to which, as has been shown, salt reacts very rapidly with an increased or decreased creep rate, it is difficult to draw conclusions from short-term convergence measurements⁸ about the long-term creep regime in salt.

⁸ Whereas Dreyer (1967) and others accept the consolidation of salt unequivocally established in pressure trials in the laboratory as being equally applicable to the pillar core, Baar (1977) disputes salt consolidation under mining conditions. According to Baar, the pillar core is also affected by the de-stressing and re-stressing influences of creep in salt.

Primary flow, with creep rates which decrease with the age of the pillar according to an exponential function, is to be expected only at shallow mining depths (under 400 m in the case of rock salt) or in very wide and low pillars of a hard saline rock. Although measured subsidence movements are small (Fig. 253), so far – if a few special cases are ignored (Figs. 177 and 257) – a complete cessation of subsidence has not been observed in the German salt-mining industry. A cessation of flow in pillar salt therefore does not appear to be substantiated by in-situ measurements. Proof of this is made even more difficult by the fact that rock salt and potash have been mined in any appreciable quantity underground only in the past 100 years (in Germany since 1861) and that almost all the older shaft installations are now under water. In Canada, on the other hand, workings 2.3 m high at a depth of 900 m were closed up by flow in the surrounding salt in as little as 15 years (Baar, 1977). From the few numerical details of subsidence published in Germany it can be deduced that, in the years from 1910 to 1950, the long-term mean of annual subsidence movements accompanying the slow rates of face advance obtaining in that period was in general under 10 mm a year, and only in isolated cases several centimetres a year. For instance, the largest subsidence measured in 17 years amounted to 1.25 m (Hilbig, 1953). Annual subsidence movements of up to 5% (2 dm) of the final value have very recently been measured over a rock-salt working 800 m deep ($M = 15$ m, $a = 0.3$).

10.2.1.3 Determination of the Time Factor in Salt-mining

In the literature on salt-mining with permanent pillars (Figs. 1 and 2) there are no data on the value of the time factor z as expressed in Eq. (305). Its determination is difficult, because

in salt-mining we are dealing with delayed subsidence spread over a lengthy period, and above none of the workings under observation has an end to ground movement yet been seen – in other words, the final subsidence value v_{zend} of Eq. (305) has not been measured;

the mining of large, uninterrupted areas is prevented by the presence of wide chain pillars and barrier pillars, or by protective pillars between caverns in the case of solution mining by borehole; the result is that at greater depths no critical-area extraction can be expected, such as would facilitate the inferred derivation of the factors z , a , and e from subsidence values at the trough centre (where $e = 1$ and $a = v_{\text{zend}}/M$);

the rate of subsidence is so slow that the time factor cannot be related to the area extracted quarterly, as in longwall mining, but to a year's extraction – i.e., to an area of approximately 0.5 km^2 ;

the time factor in a working alters if convergence in an old working is accelerated by new workings alongside;

in the course of century-long subsidence movements, several workings act simultaneously upon a measuring point at the surface, and the influence of each separate working is thus not directly measurable;

the proportions of pillars and chambers vary from working to working, and so the time factors found for one working are not applicable to others;

the rate of convergence depends not on the mine geometry alone but also on the depth (strata pressure and temperature) and the method of mining (blasting or solution mining); the validity of a time factor as determined is therefore, unlike the situation in coal-mining, limited to the mining depth and mining method to which it is related – i.e., to the intensity of blasting and the shattering of rock caused thereby;

complete or partial closing-up of chambers is brought about not only by a more or less even flow in the salt but also by roof layers collapsing at a time which cannot be determined, floor layers breaking upwards, and pillar sides slabbing off.

To establish the end-value of subsidence $v_{z\text{end}}$, it is necessary to know when the *final stage of convergence* in the workings has been reached and to distinguish between (see Fig. 252a–c) complete closing-up of stowage-free chambers, complete closing-up of back-filled chambers, and incomplete closing-up of stowage-free or partially back-filled chambers.

In the case of complete closing-up of stowage-free chambers, the subsidence end-value $v_{z\text{end}} = c_r$ for the roof layer and for the surface point over the centre of a critical area ($v_{z\text{end}} = v_{z\text{Max}}$ – see Fig. 16, bottom) is calculated in accordance with Eq. (9) from the thickness of seam mined M (= pillar and chamber height h) and the waste of mineral V_L [see Eq. (11)]. In Eq. (9) the factor in brackets $(1 - V_L)$ which equals $1 - A_p/A_p + A_r$ corresponds to the extraction ratio V_w and the subsidence factor a in longwall caving. In the case of back-filled chambers, the thickness mined M has to be reduced by the height of the compressed stowage material (factor $a' < 1$) if pillar convergence progresses to the point when the roof rests closely on the stowage material. If the roof becomes stationary while still freely suspended, before coming to rest on the stowage material or the caved slabs of strata, the subsidence factor a'' is equal to c_r''/M , the ratio between the average convergence c_r'' of the roof and the chamber and pillar height M . Over the centre of an extracted critical area, subsidence at the surface is then equal to average settlement of the roof over the working:

$$v_{z\text{end}} = v_{z\text{Max}} = a''M = c_r'' \quad (\text{freely suspended chamber roof}) .$$

This applies also to the two other cases of complete closing-up of chambers:

$$v_{z\text{end}} = v_{z\text{Max}} = a'M = c_r' \quad (\text{with stowage})$$

$$v_{z\text{end}} = v_{z\text{Max}} = aM = c_r \quad (\text{stowage-free}) .$$

(Formulae for c_r , c_r' and c_r'' see Fig. 252.)

Factors a_{dyn} for dynamic subsidence relating to each point in time concerned can also be derived from dynamic subsidence movements $v_{z\text{dyn}}$ as measured at the trough centre over a critical area: $a_{\text{dyn}} = v_{z\text{dyn}}/M$.

Since, however, only subcritical areas are found as a rule, the influence factor e is introduced as an unmeasurable quantity into all formulae mentioned:

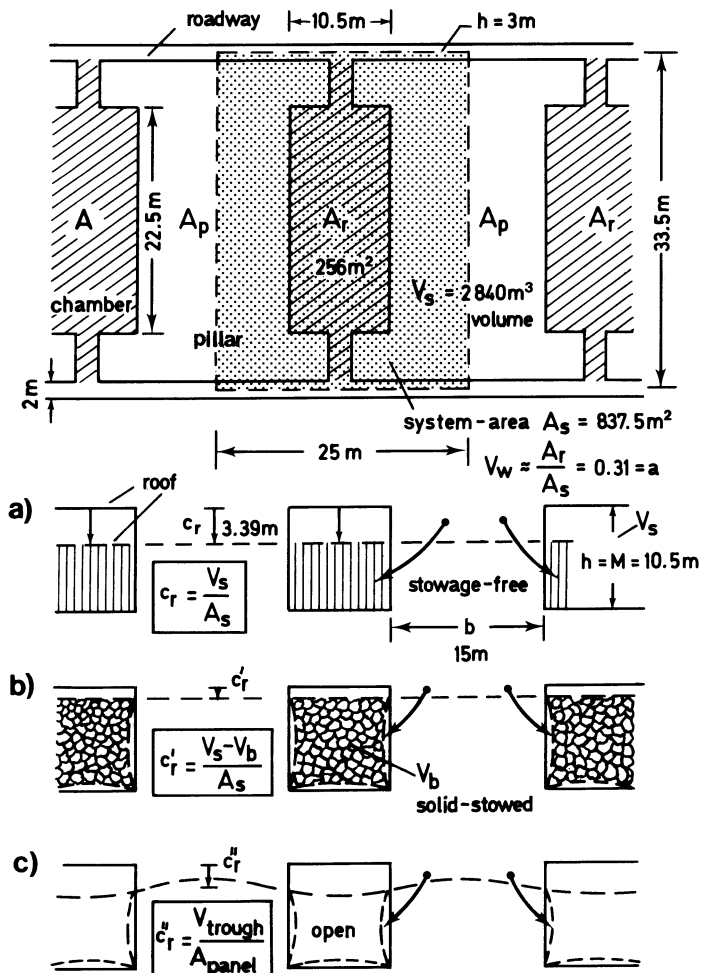


Fig. 252. *Top:* Plan of typical layout for room-and-pillar working. $A_s = A_r + A_p$ = area of system comprising room (A_r) or chamber and pillar (A_p). $V_s = A_r M$ = volume of excavation. V_w = extraction ratio. *Bottom:* section through room-and-pillar working with roof subsidence c_r . **a** Complete closing-up of stowage-free chambers; **b** complete closing-up of back-filled chambers; V_b volume of back-filling; **c** incomplete closing-up of stowage-free chambers

$v_{z\text{end}} = a M e = c_r e$, etc. The value of the e factor must therefore be determined with an integration grid. From the few investigations that have been published, the influence functions of Sann and Bals, in which the inner zones are given extra value, appear to reflect best the trough shape over a chamber working in the end-stage (see Fig. 205). Dynamic subsidence movements, which so far have only been measured, do not enable a suitable influence function to be derived, because dynamic shapes of the trough are unsymmetrical and are not similar to the final shape (see Fig. 256). It follows from this that a dynamic trough profile for delayed subsidence over a completed working can be calculated only approximately from its end-shape by means of a constant reduction factor z as in

Eq. (305); for over a chamber working which has reached its final size it is the trough centre which subsides first, and disproportionately rapidly, while the marginal zone catches up the delayed subsidence only later. Trough profiles in the intervening period are therefore not only unsymmetrical but also more acute-angled than in their final stage.

The *time factors* $z(t)$ of a working, from the time of its abandonment up to the end of the convergence movement, can be determined as follows (Erasmus, 1967). For the first time sector of 30 to 50 years it is assumed that convergence in the salt pillar increases linearly with the time t (see Fig. 251, bottom, stage I and IIa). For each extraction panel i affecting the measuring point at the surface, the proportionate share v_{zi} of the measured subsidence $v_{z\text{dyn}}$ at a given point is calculated, taking account of the influence time t (number of years since abandonment of the working), in accordance with the relation

$$v_{zi} = \frac{t_i v_{zi\text{end}}}{\sum t_i v_{zi\text{end}}} v_{z\text{dyn}} \quad (307)$$

The end-value $v_{zi\text{end}}$ of subsidence at the relevant surface point caused by each working is calculated – for example – graphically with an integration grid after Sann or Bals (see Fig. 177). For this the average final convergence c_r , c'_r or c''_r of the roof over the working concerned must be known or must be estimated from Eq. (9). The sum of all calculated shares of subsidence will be equal to the measured point subsidence: $\sum v_{zi} = v_{z\text{dyn}}$. Then by taking the ratio of the subsidence that has occurred to the final subsidence

$$z(t) = \frac{v_{zi}}{v_{zi\text{end}}} \quad (308)$$

the time factor z for the duration of influence t of the extraction area concerned i can be found.

Example. Subsidence at the point P measured on 1 January 1980: $v_{z\text{dyn}} = 25$ cm
Final subsidence at point P as ascertained with an integration grid and attributable to (see Appendix IV):

Working A ($t = 5$ years):	$v_{zA\text{end}} = 12$ cm;	$t_A \times v_{zA\text{end}} = 60$ cm
Working B ($t = 10$ years):	$v_{zB\text{end}} = 30$ cm;	$t_B \times v_{zB\text{end}} = 300$ cm
Working C ($t = 7$ years):	$v_{zC\text{end}} = 18$ cm;	$t_C \times v_{zC\text{end}} = 126$ cm
		$\sum v_{zi\text{end}} = 60$ cm; $\sum t_i \times v_{zi\text{end}} = 486$ cm

Share of these workings in the subsidence of the point P up to 1.1.80 as calculated by Eq. (307):

$$v_{zA} = \frac{5 \times 12}{486} 25 = 3.1 \text{ cm}$$

$$v_{zB} = \frac{10 \times 30}{486} 25 = 15.4 \text{ cm}$$

$$v_{zC} = \frac{7 \times 18}{486} \quad 25 = 6.5 \text{ cm}$$

$$\sum v_{zi} = 25.0 \text{ cm} = v_{z \text{ dyn}}$$

Time factors for the relevant duration of influence t_i of the workings as calculated by Eq. (308):

$$\text{Working A: } z(5 \text{ years}) = \frac{3.1}{12} = 25.8\%; \quad z(1 \text{ year}) = 5.1\%$$

$$\text{Working B: } z(10 \text{ years}) = \frac{15.4}{30} = 51.3\%; \quad z(1 \text{ year}) = 5.1\%$$

$$\text{Working C: } z(7 \text{ years}) = \frac{6.5}{18} = 36.1\%; \quad z(1 \text{ year}) = 5.1\% .$$

The calculation of time factors can be refined if it is assumed that the rate of convergence increases linearly with the depth H , the seam height M , and the subsidence factor a . These quantities H , M , and a can be introduced as additional weighting factors into the numerator and denominator of Eq. (307):

$$v_{zi} = \frac{H_i M_i a_i t_i v_{zi \text{ end}}}{\sum H_i M_i a_i t_i v_{zi \text{ end}}} v_{z \text{ end}} \quad (307a)$$

In this way specific time factors can be obtained for each individual working as a function of the working depth and the relative dimensions of pillars and chambers. The time factors for a given working as calculated from the subsidence of a number of points are averaged.

To be able to make an assertion about the further shape of the time-factor curve up to when the movement dies out, the factors for a brief period of time, calculated from measured subsidence in accordance with Eq. (308), can be extrapolated into the future by means of a geometrical progression or e function [e. g., Eq. (323)], or recourse may be had to pillar-and-chamber models constructed of salt or plasticine. Photographs are taken of the narrowing of chambers in the model – a process which increases with time under a constant load – up to the presumed end-stage c_r , c'_r , or c''_r of roof settlement. The required shape of the time-factor curve is then obtained by measuring with a planimeter the areas of convergence-over-time and final convergence. The time scale is obtained by comparison with the time factors derived by Eq. (307) or (307a) from in-situ measurements. Figure 251, right, shows a time-factor curve established in this way for salt-mining, assuming complete closing-up of stowage free chambers. On this basis the chambers are half-filled within 50 years of the beginning of mining, and will have narrowed to approximately 75% of their initial volume within 100 years. The closing-up of their remaining volume by the flow mechanism will be spread out over a very long period of time, because the pressure δ_{A_p} on the pillars gradually decreases to γH as those pillars flow and widen (thus increasing their ground area).

Approximated time factors for individual workings can be found in another and simpler way by placing a grid with 100 m or 200 m lines over the subsidence area as located on a map of the mine, and then determining for each grid point

the quotients $z = v_{z\text{dyn}(1980)} / v_{z\text{end}}$, representing the amounts of subsidence measured (for example, in 1980) as a proportion of final subsidence calculated with an integration grid. The z -values of the grid points lying centrally over an extraction area offer a good approximation of the time factor for the t -years old working, but it must be remembered that neighbouring workings have also had a greater or lesser influence on these grid points. The empirical time factors obtained in this way are then progressively improved by comparing the dynamic subsidence values for the year 1980 calculated with them, on the one hand, with the measured values on the other (see Appendix IV).

Finally, the whole time-factor curve can be constructed on the basis of the Eq. (323) given on the right-hand side of Fig. 251. For this purpose, the exponent $c < 1 = -\ln(1 - z)/t$ must first be calculated with the time factor z found and the given time t , the age of the working in the year 1980. Beginning with the developed half panel ($t_1 = 0$), the long-term subsidence in Fig. 251, bottom, is:

$$v_{z\text{dyn}} = v_{z\text{end}}(1 - e^{-ct_1}). \quad (307b)$$

“Active” subsidence during working within the time t_1 (phase I, continuous line) can be calculated by the formula

$$v_{z\text{dyn}}(\text{I}) = v_{z\text{end}}(1 - e^{-ct_1/2}) t^2/t_1^2 \quad (307c)$$

with $t = 0$ for the start of working ($t = t_1 + t_1/2$) and $e = 2.718$.

With today's intensive blasting, using millisecond fuses, of salt reached by drilling, neighbouring strata become shattered and consequently subside three times more quickly in the initial phase than is shown in Fig. 251, which refers to blasting with a slower firing sequence. Thus, for example, in the time-factor curves derived according to Eq. (323) for a new, 0.8 km by 1.8 km chamber working in rock salt ($H = 800$ m, $M = 18$ m, $a = 0.3$), the exponent c had a value of 0.06 – meaning that, in as little as 10 years after the start of subsidence, 50% of the calculated final subsidence of 4.8 m had been reached (time of half-value $t(50\%) = 10$ years). This compares with a value of $c = 0.014$ for smaller, older workings [see Fig. 251, right: $t(50\%) = 50$ years] and, in the case of a solution-mining operation ($M = 10$ m, $a = 0.3$), as little as $c = 0.007$ [$t(50\%) = 100$ years]. With pillars of carnallite, coal, or sedimentary ore, failure and sudden collapse of the remaining openings must be expected in the final stage of convergence (Fig. 27).

The individual workings and panels not only act together on points in the ground but also exercise an influence on each other at the working horizon, thus affecting the development of pillar convergence over time and the closing-up of the chambers. Abutment pressure in a working extends 100 – 250 m into the fore-field ahead of the face, depending on depth, size of working, waste of mineral, and bending resistance of the roof (Fig. 14). Insofar as this additional pressure is not absorbed by wide barrier-pillars at the edge of the working, an extra load is placed on the outer pillars of the neighbouring new working. The creep rate in these pillars is thereby increased, and this in turn leads to both a more rapid and a greater increase in pressure at the centre of the working. Conversely, the new working and its abutment pressure act in return on the marginal pillars of the old

working (see Fig. 104). Since the creep rate in the salt, and therewith the convergence in the pillars, alter with the loading on the pillars, every time factor extrapolated into the future, on the basis of the previous development of convergence in a working, must later be corrected to allow for the amount by which neighbouring workings increase in area.

The connection, as taken over an area, between settlement of the roof bed and subsidence at the surface of the ground is made clear in Fig. 253. The surface over a pillar-and-chamber working at a depth of 800–950 m has subsided by up to 40 mm between 1954 and 1966. Roof settlement over this period came to 60 mm at its maximum. It is interesting that such a small subsidence movement of the roof bed is propagated up to the surface and creates a trough there of a depth which is up to two-thirds of the maximum of roof settlement. Despite the relatively slow rate of face advance in salt-mining, the centres of subsidence at the surface and underground do not lie directly one over the other but are 1 km apart. The trough at the surface thus follows settlement of the roof in the direction of the advancing extraction front but with a delay in terms of time.

In another example, taken from potash mining, the previous average rate of subsidence of 50 mm a year (Fig. 19) increased tenfold when the pillars were given smaller dimensions. The safety factor $n = T/Q$ or σ_B/σ_{Ap} had become less than 2 [Eq. (306)]. Annual subsidence of the point P in Fig. 177 accordingly increased to 560 mm, and it was only by subsequent hydraulic stowing of two workings (b) that the rate of subsidence could be brought down to 200 mm a year. Despite subsidence of the surface which was unusually large and rapid for potash mining, there was in the working area itself, apart from considerable

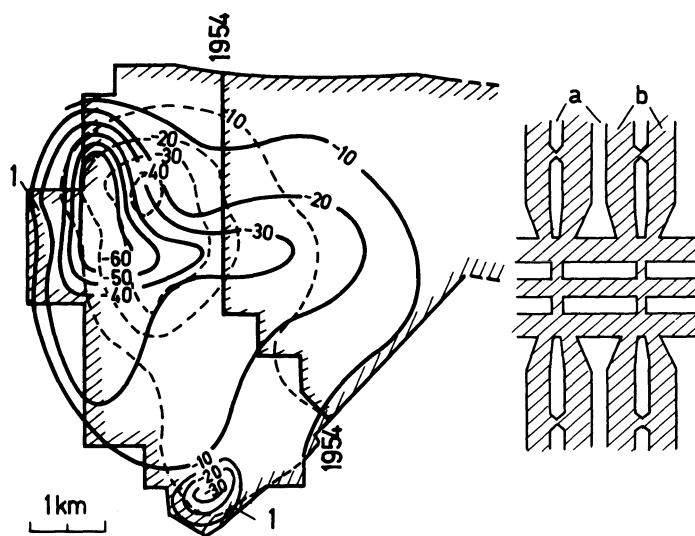


Fig. 253. Settlement of the roof (*continuous lines*) in salt-mining, expressed in mm, and subsidence of the ground surface (*dotted lines*) from 1954 to 1966 (after Uhlenbecker, 1969). *Right:* ground plan of the working with pillars (*a*) and chambers (*b*). Waste of mineral 33%–56%. Depth of mining 900 m. *Hatched line:* perimeter of the working. (*1*) Room-and-pillar working since 1966, with considerable roof settlement

slabbing-off from pillar sides, no collapse of pillars or roofs. The overlying strata deformed fracture-free over the converging pillars.

10.2.2 The Dynamic Trough Profile

If subsidence movements measured simultaneously along a measuring line are projected perpendicularly upwards from that measuring line at an exaggerated scale, and the points thus obtained are joined up by a line, what is obtained is the dynamic trough profile for the point of time and the stage of mining concerned (Fig. 241). Trough profiles such as these enable the vertical deformation of the ground surface to be perceived: displacement over a period can be deduced from the extent of dynamic tilt, and tensile or compressive strains over a period from the extent of dynamic curvature. Dynamic trough profiles are consequently more informative for reaching an opinion on mining damage than are the time-subsidence curves⁹ of individual measuring points.

The shape of the dynamic trough profile depends on the rate of subsidence of points in the profile – i.e., for a given height of convergence aM and depth of working H , it depends on the rate of face advance, the degree of reworking of the strata, and the size of the working.

Figure 254, top, shows the dynamic shape of the trough in the direction of steadily advancing extraction for various rates of face advance. The curves have

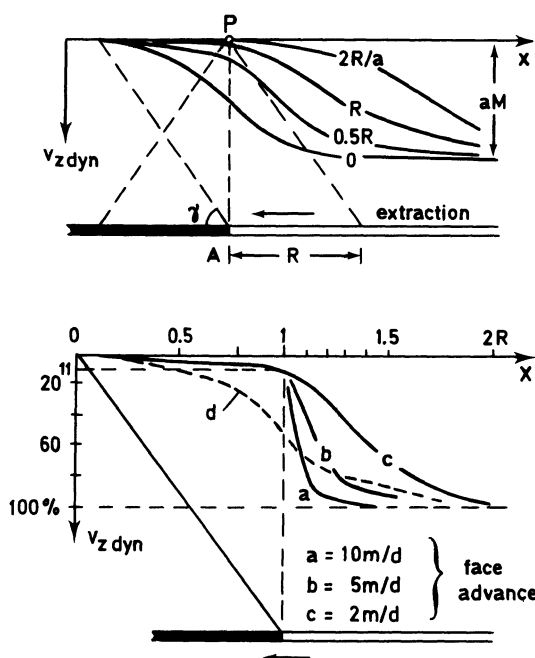


Fig. 254. Dynamic trough profiles at different rates of face advance (R /year or m /day), as compared with the normal subsidence curve 0 or d (final stage). *Top*: compact visco-elastic rock mass with delayed subsidence (after Perz, 1948). *Bottom*: fractured strata (after Pfläging, 1974). Curve d for unbroken strata (Bals)

⁹ The time-subsidence curve is identical with the dynamic trough profile in the direction of strike if the extraction front consistently traverses the critical area of the observation point from the left-hand to the right-hand edge.

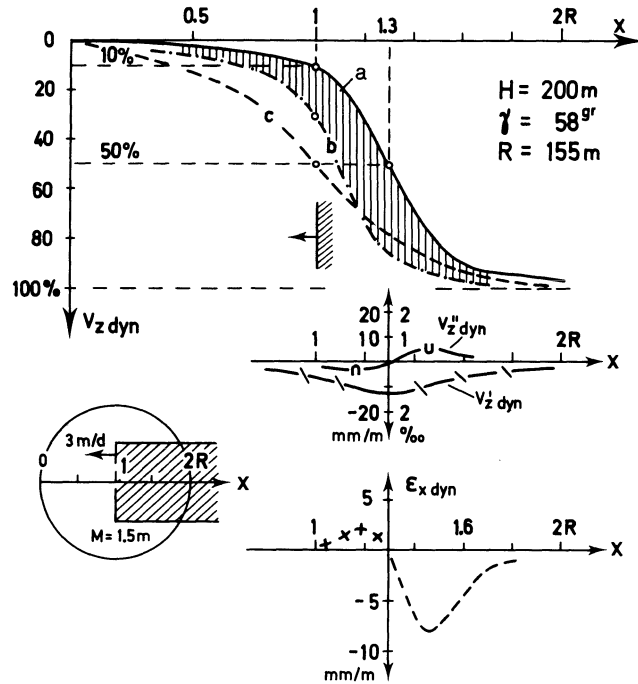


Fig. 255. Dynamic (a) and static (b) profile curve in fractured strata as compared with the normal subsidence curve (c) after Bals. *Bottom:* dynamic waves of tilt, curvature, and linear change. (After Pfläging, 1974). *Hatched:* delay or retardation of subsidence and extraction area

been constructed from subsidence movements over time at several points, as calculated by Eq. (319). It can be seen that the faster the rate of extraction, the flatter the trough profile becomes. For strata reacting visco-elastically, a rapid rate of face advance implies a brief period of influence on the surface point, mild bending of the strata, and little compression of the stowing material at the point in time of a given stage of extraction.

Dynamic trough profiles have a very different appearance in fractured and loosened strata (Fig. 254, bottom). In such conditions, the faster the extraction advances, the steeper the trough sides become: a development which is connected with the blockwise or silo-like subsidence of clastic strata (Figs. 113, right, and 182c). With a high rate of advance of 5 or 10 m/d, the fragmented roof beds settle more rapidly just behind the extraction front than in an elastic rock mass, in which the roof cantilevers over the margin zone (Fig. 113, left). In a clastic rock mass, strata settlement behind the extraction front is accelerated with increasing rates of advance, and the dynamic trough profile becomes steeper (curve a).

In Fig. 255 the dynamic trough profile shows only 10% of final subsidence over the extraction front ($1R$) in fractured strata (see Fig. 182f). The half-subsidence value is found behind the extraction front at $1.3R$, and 90% of final subsidence is reached at as little as $1.6R$. Coinciding with the location of the point of half final subsidence are also the points of transition from convex to concave

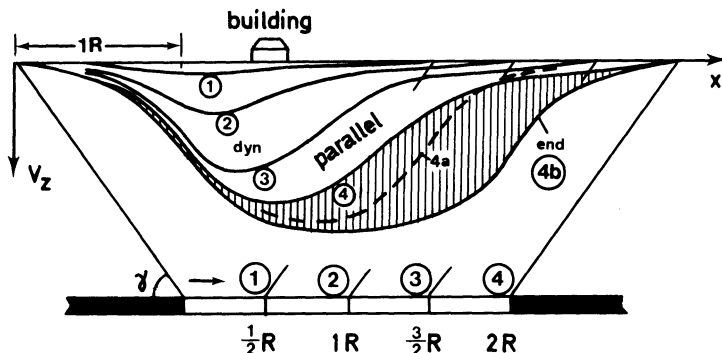


Fig. 256. Dynamic trough profiles 1–4 of total extraction advancing continuously up to the critical area in a compact visco-elastic rock mass and with delayed subsidence (*hatched*). (4a) Dynamic shape in a fractured strata; (4b) final shape

curvature and of maximum tilt value (here -12 mm/m). The flank of the dynamic trough is strongly curved, with maximum values of 0.5‰ m^{-1} (radius of curvature 2000 m). Even the static profile curve of the final stage (b) is steep in the central part but is less strongly curved. Over the face edge it has a subsidence value of 30% of the maximum, as compared with 50% in the normal curve of Bals.

Figure 256 shows the whole subsidence profile in the direction of an extraction proceeding from left to right, and also the interim shapes of the trough, for an unfractured, predominantly visco-elastic rock mass. In all the intermediate shapes, the lowest point of the trough lies nearer to the left-hand edge of the working (where the face was opened out) than to its centre. Dynamic trough profiles in the longitudinal direction of extraction are thus unsymmetrical. The trough side which travels with the face acquires a constant shape from mining stage (3) onwards, as soon as final convergence aM has been attained at the centre of the working. The wave of convergence in the working (Fig. 250) corresponds to the bow wave in the spreading trough. The large and long-drawn-out delayed subsidence in the unfractured rock mass of this example has the consequence that the trough side in the final stage (4 end) is steeper than the dynamic trough side. In contrast to Fig. 255 and the broken line 4a for fractured and loosened strata, tilt, curvature, and tensile strain increase over the extraction front after abandonment of the working. Only compressive strain at the trough centre decreases.

The unsymmetrical, dynamic trough profile which travels with the extraction also develops in potash mining (Fig. 257, top). The roof over the 2 m thick caving workings at a depth of 350 m subsides rapidly up to the surface. Because of the absence of supporting pillars, the rate of subsidence at 20 mm per 10 days is almost as rapid as in coal-mining. Subsidence over stowed panels (*hatched*) hangs distinctly back, and the humps or waves at the surface are 10 cm high. The broken lines show the increase in subsidence from measurement to measurement in the direction of extraction.

In chamber or room workings, subsidence is delayed by yielding salt-rock pillars left to support the overburden (Fig. 257, bottom). During the development

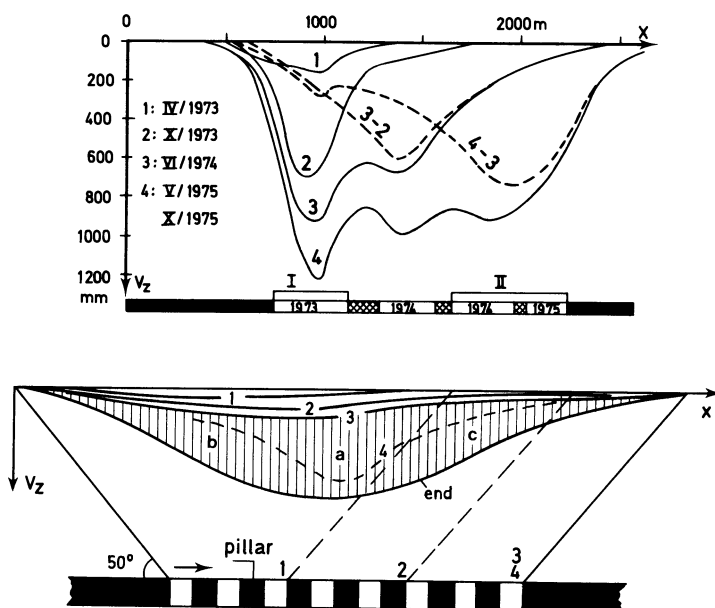


Fig. 257. Dynamic trough profiles of potash mining. *Top:* subsidence at the surface on total extraction of a potash seam (after Permjakow and Nesterow, 1977). *Bottom:* Trough profiles for partial extraction with yielding permanent pillars (delayed subsidence hatched)

of the panel before reaching stage 3 the dynamic trough profiles 1 – 3 are very flat. After suspension of mine work the rock mass subsides more rapidly above the centre (a) of the mined-out area than above the margin zones (curve 4). Many decades pass before the subsidence profile reaches its final shape above the margin zones (b) and (c) also.

In Fig. 258 the various curves of subsidence over time are brought together:

The curve (a) of normal subsidence $v_z(x)$ coincides from point 0 to point 10 with the half-trough profile in the x -axis over the critical area in the final stage (100% = aM). The curve is identical with the curve of the relevant final subsidence of a surface point ($z = 1$) on the progressive extraction of the critical area from one side to the other (Fig. 175).

Curve (b) of the influence factor $e(r)$ for a subcritical area shows the increase of influence on a surface point (here No. 3) as a function of the stage r of the extraction front for $z = 1$ (Fig. 266).

Curves (c) are of the dynamic, unsymmetrical trough profiles $v_{z\text{dyn}}(x)$ in longitudinal section from 1 to 7 (x -axis) over continually advancing extraction (Fig. 256).

Curve (d) is for subsidence $v_{z\text{dyn}}(r)$ over time for a surface point (here No. 3) as a function of stage r of the extraction front, related to final subsidence of 100% (Fig. 244).

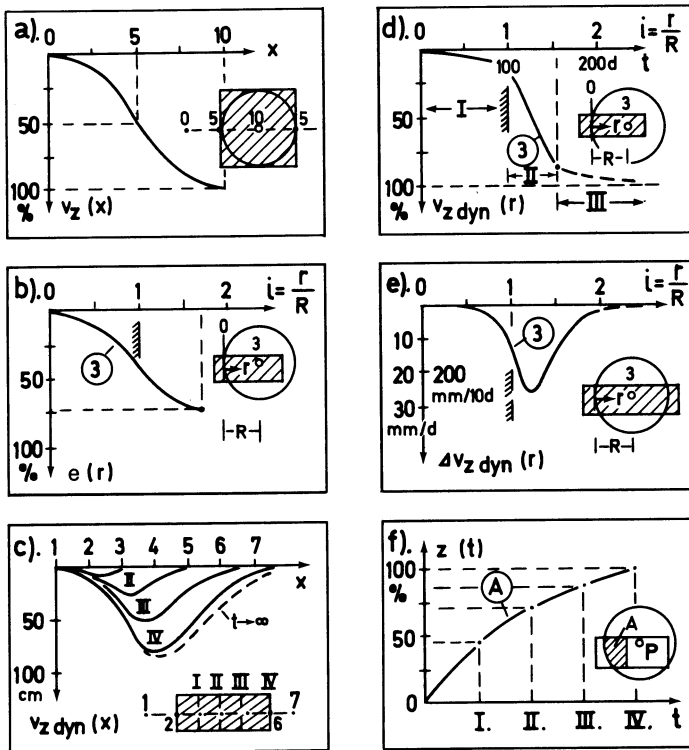


Fig. 258. The various curves of subsidence over time

Curve (e) is of the rate of subsidence $\Delta v_{z \text{ dyn}}(r)$ of a surface point (here No. 3) as a function of the stage of extraction (Fig. 247), related to the increase in subsidence in 1 day or in 10 days (mm/10d). The maximum point of the curve coincides with the transition point of curve (d).

Curve (f) is of the time factor $z(t)$ for the increasing influence exercised by the area of seam extracted within a period of 3 months (quarterly area A) on the surface point P, as a function of the time t or of the quarterly stages of extraction I – IV (Fig. 265, curve 5).

10.2.3 Dynamic Displacement and Linear Change

As is known, surface points become displaced towards the focal point of the extraction area influencing them. Since this focal point moves in the direction of the advancing extraction, the horizontal direction and magnitude of the displacement vector v_{dyn} are constantly changing (Fig. 239). The dynamic displacement of a point which, like point 64 in Fig. 259, lies over the extraction area at the final stage increases at first in the direction of the approaching extraction front ($-v_{x \text{ dyn}}$ up to 14 cm) and decreases again after the passage of the face (mining stage 10). The surface point can move in the opposite direction ($+v_{x \text{ dyn}}$) if the

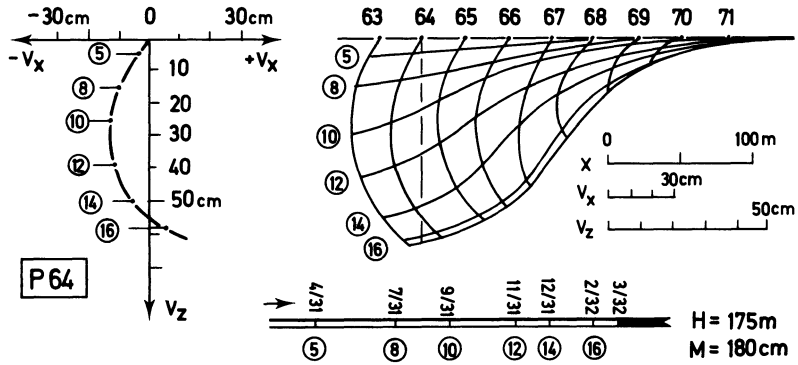


Fig. 259. Representation of ground movement in the direction of advancing extraction: (*left*) at a single point, No. 64; and (*right*) along a measuring line. (After Janus and Fläschenträger, 1956)

focal point of the influencing extraction area under it moves through to the other side of that extraction area. In the case of surface points which, like point 70, remain outside the horizontal extent of the extraction area, even at the final stage, dynamic displacement becomes constantly greater and never smaller. With points which, like point 2 in Fig. 238 or point 7 in Fig. 122, lie alongside the extraction panel (forefield III in Fig. 142), the change in the direction of dynamic displacement is particularly great.

The connection between dynamic subsidence and dynamic displacement, clearly brought out in the trough profile in Fig. 259, is examined in more detail in Fig. 260. In the left-hand diagram the two components of the movement of a surface point P are compared. It will be noticed that the curve of dynamic displacement appears similar to the curve of the rate of subsidence (Figs. 247 and 258 e).

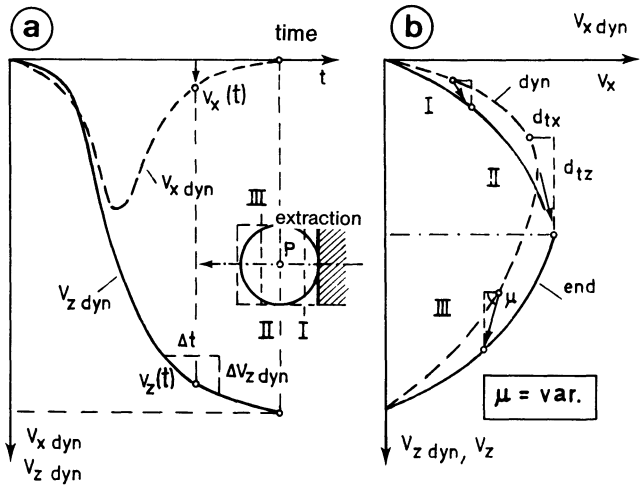


Fig. 260. Left: contrast between dynamic subsidence and displacement at the point P. Right: comparison of dynamic movement of the point P in the vertical plane with the final positions attaching to the stages of mining I to III concerned

Dynamic displacement reaches its maximum value at the point in time of the most rapid rate of subsidence – namely, shortly after the face has passed under the surface point. The magnitude of dynamic displacement $v_{x\text{dyn}}$ is approximately proportional to the rate of subsidence – i.e., to the difference in dynamic subsidence $\Delta v_{z\text{dyn}}$ within a period of time Δt :

$$v_{x\text{dyn}} = c \frac{\Delta v_{z\text{dyn}}}{\Delta t}. \quad (309)$$

In the right-hand diagram the path of dynamic movement of the surface point P within the profile is represented as a broken line. Extraction constantly traverses the critical area from right to left. The continuous line joins up the final values of the movement ($z = 1$) associated with an extraction area enlarged progressively from 0 to I, II, III, and IV. This curve is symmetrical to the mining stage II under the point P and is constructed from the values read from the normal curves of subsidence (Fig. 175) and displacement (Fig. 264a). The distance between the point locations on each curve associated with a given stage in mining corresponds to the time-lag in point movement. This time-lag is at its greatest during the passage of the face under a point. As can be seen, the ratio between the delay in subsidence d_{tz} and the delay in displacement d_{tx} varies along the path of movement:

$$\frac{v_z - v_{z\text{dyn}}}{v_x - v_{x\text{dyn}}} = \frac{d_{tz}}{d_{tx}} = \text{var}. \quad (310)$$

From this it follows that the time factors z for the two components are not identical; otherwise the angle μ in Fig. 260, right, would remain constant.

The curve of dynamic displacement $v_{x\text{dyn}}$ over the extraction front is displaced in relation to the static curve ($z = 1$) by the distance d towards the extracted area (Fig. 261). The point of greatest displacement lies at the transition point W of the dynamic trough profile, where tilt is also at its greatest. In a trough profile that is symmetrical about the transition point, the point W marks furthermore half the maximum of subsidence. If the static profile is of length $2R$ to the point of greatest subsidence, then the dynamic trough profile is longer by $2d$, and the same applies to the static and dynamic displacement curves. The dynamic radius of influence R_{dyn} related to the length of profile is therefore greater than the static radius of influence R from Eq. (161).

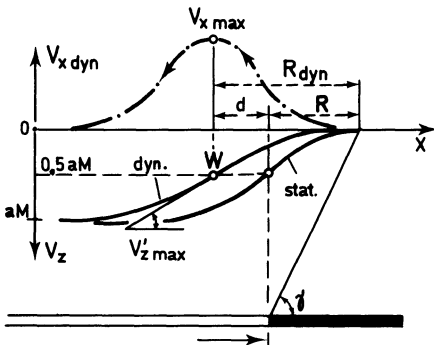


Fig. 261. Dynamic and static subsidence and dynamic displacement, over the extraction front of a supercritical area. (After Skinderowicz, 1979)

Just as with dynamic displacements, dynamic tensile and compressive strains derived from them vary in direction and amount (Fig. 240).

The development of linear change (extension, compression) over a period can be made out from the shape of the provisional trough profiles (Fig. 256). A building structure over working face 1 is at first exposed to tensile stress at mining position $R/2$, because the site on which it stands is humped upwards over that position. By mining position R the structure is in the vicinity of the transition point at which extension passes into compression. By mining position $1.5R$ to $2R$ the building site warps downwards in a trough, and the structure is exposed to compressive stress. When the ground at this place comes to rest again, this stress recedes somewhat.

Dynamic extension and compression over a field of extraction which is increasing in size are shown in Fig. 262 for an unfractured, visco-elastically reacting rock mass. For comparison, the final magnitudes of linear change ($z = 1$) applying to the mining positions $R/2$, $1R$ and $2R$ in question are also shown. A high compression which represents a danger to structures – the so-called critical peak of compression – appears at the end-stage ($z = 1$) over the extraction area of size $1R$. Dynamic linear change is in all three stages of extraction smaller than the final value, and it is only over the centre of the critical area $2R$ that dynamic compression grows less until the final stage. The place of transition from extension to compression – the neutral zone – lies, in the case of the dynamic curve, well behind the extraction front, over the extracted area. To the left of the stationary face this point is displaced, in subcritical extraction $R/2$ and $1R$, outwards into the forefield.

In a rock mass that has been worked through repeatedly and is consequently well fractured, there is little delayed subsidence, and the dynamic trough profile has almost the same steep shape as the static profile over the extraction front at the end-stage (Fig. 255). Dynamic linear changes consequently approach their final values, which, owing to the acute-angled trough shape, are very large in comparison with Fig. 241, attaining 2.5 mm/m extension and 7.5 mm/m compression. The neutral zone in thoroughly fractured strata, at a shallow mining depth of 200 m and a face advance of 3 m/d , lies $0.3R$ behind the extraction front.

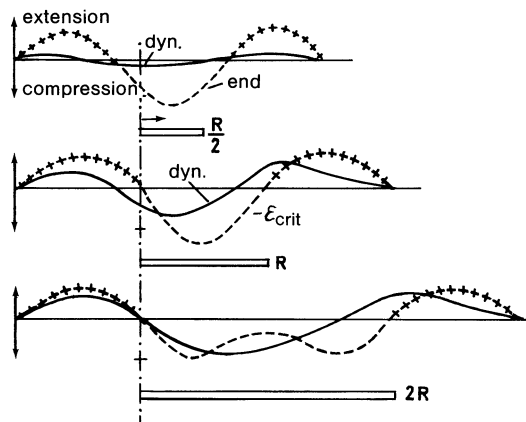


Fig. 262. Dynamic extension and compression (continuous line) as compared with the final situation (broken line) in a compact rock mass displaying delayed subsidence. (After Perz and Niemczyk, 1949)

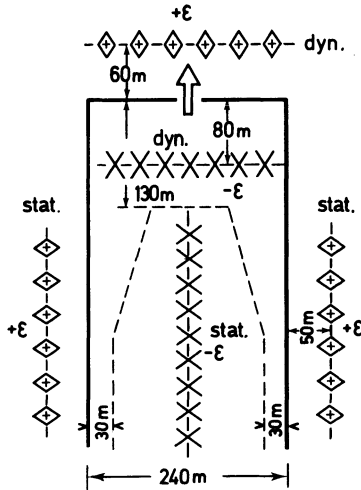


Fig. 263. Dynamic and stationary zones of extension and compression ahead of and behind the advancing extraction front, as seen in plan. (After Lippmann, 1974)

In caving operations of 2–3 m seam thickness at depths of 800 m, it has been observed in the Ruhr, with face advances of 4–5 m/d, that at the surface of the ground a zone of maximum extension travels 60 m ahead of and parallel to the extraction front, and that another parallel zone of maximum compression travels 80 m behind it (Fig. 263). Some 130 m behind the extraction front and at right angles to it, there arises a stationary peak of compression. In the stationary extension zone alongside the extracted area, the peak of extension is 50 m distant from the ribsides.

In the British coal-mining industry the peak of extension moves towards the ribside as the extracted area increases in size (Fig. 235).

With increasing rapidity of extraction dynamic extension and compression become greater, since the trough sides become steeper with increasing rates of face advance in clastic strata (Fig. 254, bottom). Height M of face, length l_d of face, and caving working all increase dynamic extension and compression over the extraction front, but dynamic linear changes become smaller at greater depths of mining.

Figure 264 assembles the various curves of dynamic and final displacement:

Curve (a) of normal displacement $v(x)$ represents the 2R-long branch of the graph of displacements ($z = 1$) in the x -direction measured over the critical area on the line 0–5–10. The same curve is obtained if the displacements which develop at the surface point 10 in the final stage ($z = 1$) on progressively extracting the critical area are drawn as a function of the stage of mining.

Curve (b) is of the influence factor $e(r)$ for displacement of the point 3 as a function of the stage of mining r (subcritical area). In contrast to the influence of subsidence, negative values also occur. The maximum negative value is attained when the extraction front is at point 3 (mining stage a). The zero position corresponds to an extraction area symmetrical to point 3 (mining stage b).

The curves at (c) are of dynamic displacement $v_{x\text{dyn}}(x)$ of the surface points 1 to 7 in the x -axis over the advancing extraction, for the three mining positions I, II,

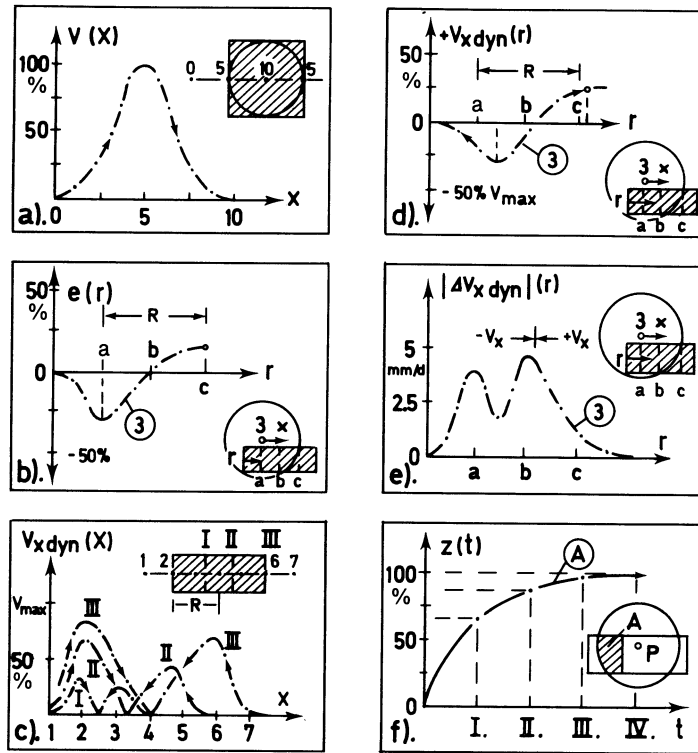


Fig. 264. The various curves of displacement compared

and III. In the branch of the curve which travels with the extraction front, the maximum value increases as the size of the extraction area increases.

The curve at (d), related to one point, is of displacement over time $v_{xdyn}(r)$ as a function of the mining stage r . As with the influence curve (b), both negative and positive values occur. Because of the delayed ground movement, however, the extreme values have a different position from those in curve (b).

Curve (e) is of the rate of absolute displacement $|\Delta v_{zdyn}|(r)$ of a point as a function of the mining stage r . The daily change in the magnitude of displacement is greatest at mining stages a (passage of the face) and b (symmetrical extraction area).

Curve (f) is of the time factor $z(t)$ for the influence, increasing with time, of an extraction area A on the displacement of a surface point P . Area A corresponds to a quarterly face advance from 0 to I.

10.3 The Calculation of Ground Movement over Time

The time factor z is the most uncertain calculation factor in the subsidence formula [Eq. (164)]:

$$v_{zdyn} = aMez.$$

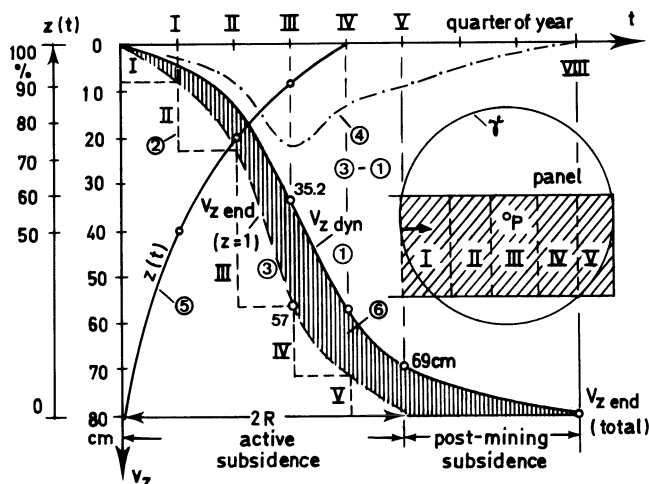


Fig. 265. Deriving quarterly (three-monthly) time factors (5) from dynamic subsidence movements (1) of the point P. (2) Increase in final subsidence from quarter to quarter; (3) summated curve of final (static) subsidence for P; (4) curve of retardation; (5) curve of quarterly time factors for one quarter's extraction area; (6) delay of subsidence (retardation)

It is derived from the measured subsidence over time $v_{z\text{dyn}}$ of a surface point P under which a working is proceeding. Depending on the calculation process, this is done either for an infinitesimal strip of extraction, or for a finite strip of extraction, or for the complete area of seam mined up to the point in time t . Into the determination of the time factor therefore will go the errors of several measurements as well as unknown anomalies in the development of movement. Furthermore, delayed subsidence over old workings and margin influences from neighbouring workings can also falsify the results.¹⁰

For the point P in Fig. 265, dynamic subsidence movements have, for example, been measured quarterly (curve 1), during which process the extraction area within the critical area has enlarged from I to V and subsidence has risen to 69 cm. Later subsidence amounts to 11 cm. If the final subsidence values for the point P are calculated for each stage of mining from I to V with $z = 1$, and with influence factors e_i determined with an integration grid for the extraction areas concerned, it becomes possible to ascertain the retardation d_t in the subsidence process (Table 19) by subtraction:

$$d_t = v_{z\text{end}}(t) - v_{z\text{dyn}} \quad (311)$$

The curve of retardation 4 portrayed in Fig. 265 reaches its maximum value of 21.8 cm during the undermining of the point P (extraction area III). Had the mining operation been ended after extraction of area III, the surface point P, already subsided by 35.2 cm, would in the course of time have subsided by a further 21.8 cm down to an end value of 57 cm. From a comparison of Figs. 265 and 247 it can be seen that the retardation of subsidence is at its greatest when the

¹⁰ Delayed subsidence: residual subsidence occurring in the post-mining phase (subsequent); delay or retardation of subsidence: amount of subsidence which has not yet occurred at the surface (out-standing, reserve). See also Appendix III.

Table 19. Subsidence as measured quarterly over a continuously advancing mine working; also final subsidence ($z = 1$), retardation, and time factors as calculated by Eq. (316) for each quarterly stage of mining

Quarterly stage of mining	Measured subsidence over period $v_{z \text{ dyn}}$	Calculated final subsidence ($z = 1$) $v_{z \text{ end}}(t) = e_t \times a \times M$		Retardation d_t	Time factor z_t (quarterly)
		separate (2)	cumulative (3)		
	(1)	(2)	(3)	(4)	(5)
I	4 cm	8 cm	8 cm	4 cm	0.50
II	13 cm	14 cm	22 cm	9 cm	0.75
III	35.2 cm	35 cm	57 cm	21.8 cm	0.90
IV	54 cm	15 cm	72 cm	18 cm	1.00
V	69 cm	8 cm	80 cm	11 cm	1.00
Delayed subsidence	11 cm	0			

rate of subsidence is maximal. The greater the movement potential still unconverted to subsidence in the undermined rock mass, the more rapidly will the surface point subside.

As another example, subsidence over a period of time measured at the point P is shown as a function of the influence factor e (Fig. 266). Dynamic subsidence values are divided by the critical subsidence value aM to make them comparable with other measurements. The quotient

$$\frac{v_{z \text{ dyn}}}{aM} = ez \quad (312)$$

signifies, in accordance with Eq. (164), the dynamic influence factor e_{dyn} or e_t — that is to say, the effect experienced at the surface point by the time stage or mining stage t , an effect which after a certain time increases to the final value e ($z = 1$) obtained with the integration grid for the mining stage concerned. The graph of subsidence runs in a curve from 0 through 1 and 2 to 3. The area I represents the delay of subsidence which has not yet occurred at the surface point by

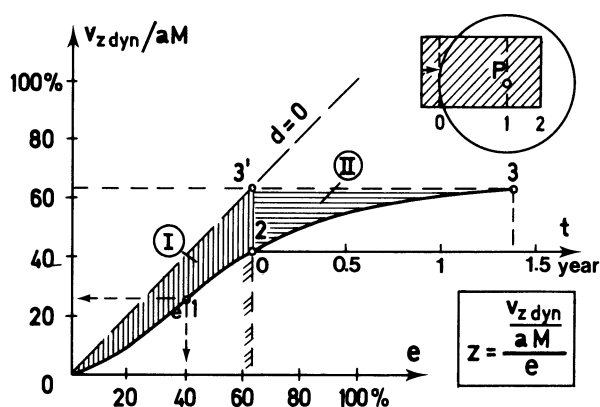


Fig. 266. Dynamic subsidence of the point P as a function of the influence factor e

the relevant mining stage. By mining stage 2 this retardation has a maximum value of $2 - 3' = 20\% v_{z\text{dyn}}/aM$. In the course of 1.4 years after abandonment of extraction at mining stage 2 this post-mining retardation decreases to 0 through delayed subsidence, as shown by area II. Without retardation ($d = 0$), and consequently also without delayed subsidence, the shape of the graph of subsidence would be a straight line from 0 to $3' -$ i.e., $v_{z\text{dyn}}$ proportional to factor e .

All methods employed today for calculating time-related subsidence or displacement are based on the principle of superposition and equivalence. It is assumed that extraction elements affect the surface equally over a period and that their effects are cumulative over time.

Three methods can be distinguished which differ in their basic approach:

Delayed subsidence is so slight that the subsidence and displacement values calculated with $z = 1$ for the various mining stages can be equated with the development of dynamic movement over the enlarging extraction area;

the whole working is divided up into areas of equal mining duration of, for example, three months or into very narrow, infinitesimal strips, and the influences calculated for the separate unit areas or strips, taking account of the time factor z_t , are added together or integrated up to the relevant mining stage to give the dynamic total influence;

the time-related effect for the whole extraction area up to the point in time t is calculated with a time factor which depends on the influence factor e of the mining stage at the time t .

In the mining of salt deposits with permanent pillars, the principal subsidence begins only after abandonment of the working and can continue for several centuries (see Fig. 251, bottom). From experience it has been found that large-scale delayed subsidence like this, over an extraction area which is no longer growing in size (e constant), is best calculated by Eq. (322). The rock-mechanics exponent c of this equation lies between 0.005 and 0.09, depending on size of working, depth, chamber height, and subsidence factor.

10.3.1 The Strip Method with $z = 1$

When it is planned to work a deposit under a thickly populated area or under industrial installations, ground movements to be expected at quarterly or half-yearly stages in the mining should be calculated, so as to be able to judge of possible mining damage from a knowledge of the amplitude, direction and nature of the dynamic ground movements. If earlier observations have shown that delayed subsidence is slight, it is generally sufficient for this purpose to calculate with a time factor of $z = 1$. With this, the simplest method, the area to be mined is divided up into narrow strips parallel to the extraction front, and the final value of subsidence and displacement is ascertained for each strip – for example, with an integration grid. The final values of the strips which together make up the mining stages at different times give the dynamic ground movement as a function of the enlarging extraction area. From the time-related subsidence and displacement movements obtained in this way can be derived the elements of dynamic

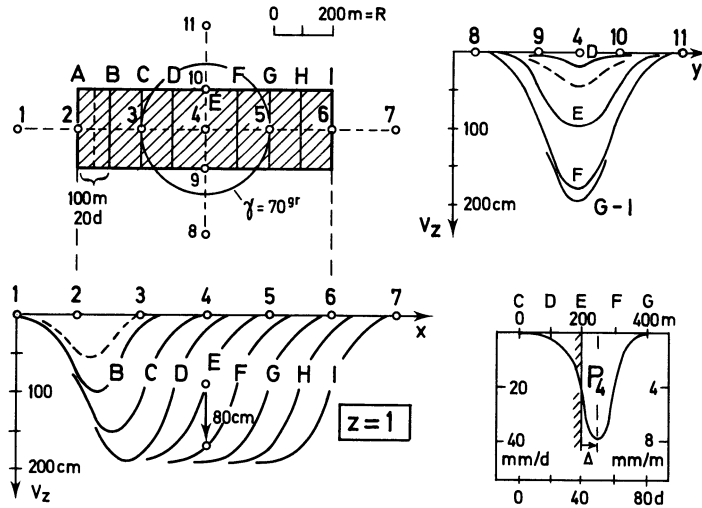


Fig. 267. Example for the strip method with $z = 1$ (after the calculation values of Sauer). Ground plan of the extraction area, with profile lines, trough profiles lengthwise and crosswise, and rate of subsidence for point 4

ground deformation; and with these values it is then possible to form an opinion of when tensile stress in a structure will pass into compressive stress, and of what peak value of tilt or curvature will occur.

Figure 267 is an example of this method of calculation taken from the Ruhr District, where delayed subsidence in repeatedly worked strata is slight. The calculation lines 1–7 and 8–11 lie symmetrically to the extraction area, which is over $2R$ long and at a depth of 400 m. It is divided up into strips 100 m wide which, at a face advance of 5 m/d, can be mined in 20 days. Given $z = 1$, $M = 2$ m and $a = 0.95$ (caving working), the final subsidence values of all strips are calculated for the points 1–11 in accordance with the method described in Sect. 8.36 and added together up to the respective mining stages B to I.

From the trough profiles traced in the Figure the “bow wave” which travels with the extraction front can be clearly seen, as also the progressive deepening of the stationary trough profile in a transverse direction (line 8–11) up to the point where the edge of the critical area is reached at mining stage G. In addition, the rate of subsidence at the central point 4 is calculated both in mm per day and in mm per metre of face advance. From being undermined at stage E up to mining stage F 100 m farther on, the point subsides by 800 mm in the course of 20 days. This corresponds to a rate of subsidence of 40 mm/d, or to an average increase in subsidence of 8 mm per 1 m of face advance. From the passage of the face under the point up to the moment of most rapid subsidence a time of $\Delta = 10$ days (= 50 m face advance) elapses. Within a period of 80 days from when the influence begins at mining stage C the subsidence process is complete.

If all that is required to be known is the duration of influence of a working – for example, in order to judge when repair works may begin, or when protective measures for an industrial or traffic installation may end – the nomogram in common use in the U.K. and shown in Fig. 268 may be employed. For the depth

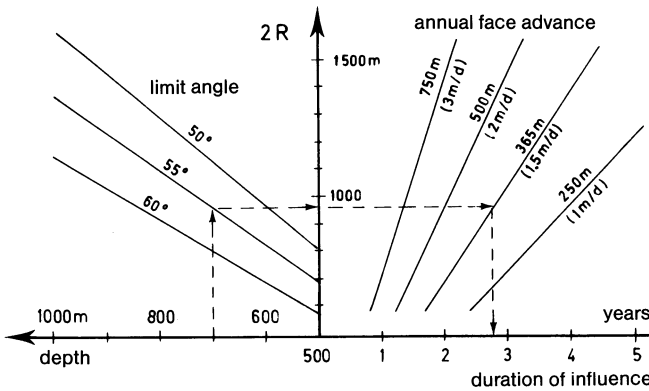


Fig. 268. Nomogram for estimating the critical area $2R$ and the duration of ground movement at a ground point ($z = 1$)

of working concerned, the nomogram enables the diameter of the critical area and the duration of mining influence to be read off for a given limit angle and rate of face advance. The time factor employed here is $z = 1$.

10.3.2 The Time Factor z_t for the Unit Area

In cases where delayed subsidence cannot be ignored, calculation should be done with a time factor of $z \leq 1$, which allows for the fact that the full influencing capacity of a strip of extraction develops only gradually with the passage of time. For this purpose the extraction area is divided up into strips parallel to the face of a uniform size such that each is mined within, say, 3 months. For these quarterly areas the time factors z_t for the 1st, 2nd, 3rd, 4th, 5th, etc. quarter after extraction of the unit area concerned are obtained from observations of the dynamic subsidence movements of a surface point. It is then assumed that these time factors are equally applicable to all other unit areas.

For example, a unit (quarterly) area I such as this in Fig. 269 will act on a surface point P immediately after its extraction at the end of a single unit of time (the first quarter) with 50% of its full influence capacity ($z = 0.50$), and at the end of a double unit of time (the second quarter) with 75% of that capacity ($z = 0.75$), if extraction at a constant rate of face advance has increased by a further unit area, after three units of time (third quarter) with 90% ($z = 0.90$), and so on (Table 19). The same time factors z_t as for area I apply to further unit areas of extraction II, III, etc., when account is taken of the duration of their influence. At the end of the third quarter (by 1 October 1979) the surface point P in Fig. 269 has therefore been caused to subside, by the action of the first unit area, by the amount of $0.90 aMe_I$, by the second unit area by $0.75 aMe_{II}$, and by the third unit area by $0.50 aMe_{III}$ – all together therefore after the third quarter by an amount of

$$v_{z3} = (0.90 e_I + 0.75 e_{II} + 0.50 e_{III}) aM.$$

The structural law for the calculation of interim ground movements is clear from the following equations set up for the interim subsidence movements of a surface

to 47% for the extraction area IV/78 and to 80% for the area III/78. In the area II/78, convergence since 1 July 1978 has increased by 40%, in area I/78 by 11%, and in area IV/77 by 4%. These increases must be added to the amounts of convergence already experienced up to 1 July 1978. Like Eq. (313), this calculation diagram can be devised without difficulty.

10.3.3 The Method with Differential Extraction Strips

The methods described so far can be further refined if, instead of wide strips and unit areas, infinitely narrow strips parallel to the extraction front are taken and their subsidence or displacement influence over time considered.

With this method, the differential subsidence influences dv_z of narrow extraction strips dx on a point P are reduced by using the time factor z , having regard to the varying duration of influence t for each strip, to the time-related values $dv_{z \text{ dyn}}$, which are then added together for the extraction area as a whole to give the required *interim subsidence value* $v_{z \text{ dyn}}$ of the point P. Thus, for example, at the mining stage i , when the strip-shaped extraction element lying $i - x$ behind the extraction front (Fig. 271, left) has exerted an influence, given an annual face advance of l (in m), for a period of

$$t = (i - x)/l \quad (\text{years}) \quad (317)$$

an extraction strip dx at the position x , with a final subsidence influence dv_z on the point P at the mining stage i , will cause an interim subsidence at the point P of

$$dv_{z \text{ dyn}} = dv_z z. \quad (318)$$

The time factor $z(t)$ in Fig. 271 must therefore be drawn towards the left from mining stage i onwards, to enable the time-related influence for each rearward

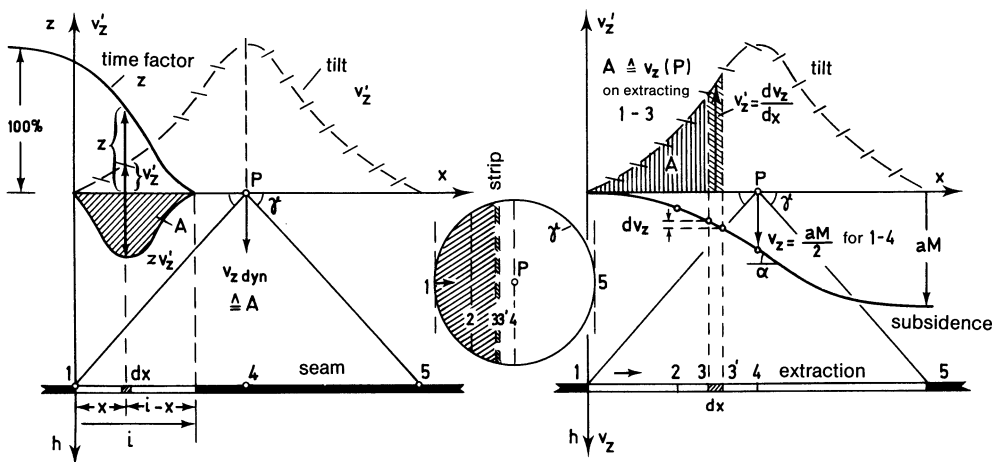


Fig. 271. Calculating dynamic subsidence from tilt and the time-factor curve (left) on extracting the critical area in strips. Share of the extraction strip 3-3' in subsidence at P. (After Perz, 1948)

extraction element to be read off it. Thus the interim subsidence of the point P, on extraction of the whole area, amounts at mining stage i to

$$v_{z \text{ dyn}} = \int_{x=0}^{x=i} z \, dv_z = \int_0^i z v'_z(x) \, dx = A. \quad (319)$$

In this equation, the differential subsidence influence dv_z of the narrow subsidence strip on the point P is replaced by the tilt $v'_z(x) = dv_z/dx$ of the normal subsidence curve at the position x of the extraction strip. As is known, the ultimate subsidence v_z of a surface point P, on stripwise extraction of the critical area from 1 to 5, increases in the form of a normal subsidence curve from 0 at mining stage 1 up to $v_{z5} = aM$ at mining stage 5 (Fig. 271, right). The increase dv_z in subsidence at the point P which occurs when the extraction front moves forward by an infinitely small amount of dx from 3 to 3' thus corresponds to the tilt $v'_z(x)$ of the normal subsidence curve above the position $x = 3$. Accordingly, an equal degree of subsidence influence $dv_z = v'_z(x_3)$ accrues to the narrow extraction strip dx at the position 3. Finally, the product of the amounts of tilt $v'_z(x)$ and the associated time factors $z(x)$ gives rise to a curve which describes a hatched area A representing the required interim point subsidence $v_{z \text{ dyn}}$ at the mining stage i .

With subcritical areas, the integration in Eq. (319) must also be carried out in the y -axis [see Eq. (195)].

The interim displacements $v_{x \text{ dyn}}$ can also be derived, by the same method, from the displacement effect dv_x of the extraction elements — i.e., from integration of the linear-change curve $\pm \varepsilon = dv_x/dx$ over the critical area for the point P, within the limits of the extraction front (e.g., a and b in Fig. 272):

$$v_{x \text{ dyn}} = \int_{x=a}^{x=b} z \, dv_x = \int_a^b z \varepsilon(x) \, dx. \quad (320)$$

In Fig. 272, the curve of linear change is at its minimum at the point P, because a narrow extraction strip vertically under it at position 2 has no displacement in-

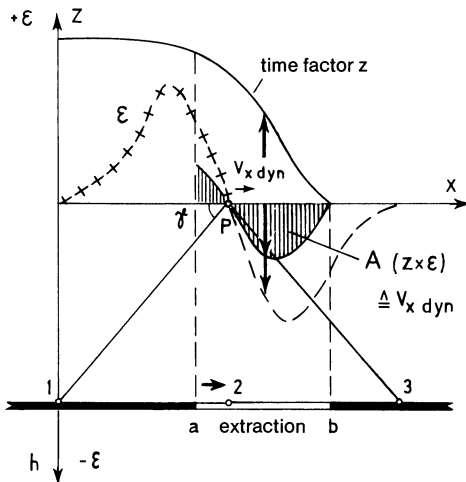


Fig. 272. Calculating dynamic displacement of the point P from linear change and the time-factor curve. (After Perz, 1948)

fluence $dv_x = \varepsilon$ on P (Perz, 1948). The linear change values which, as first derivatives of the displacement curve dv_x/dx , represent the point displacement associated with each extraction element and, as time-related values, are directly proportional to the interim subsidence, make themselves felt at the point P, given continuous extraction, only in proportion to the time factor z accruing to them. The same applies to the tilt values in Eq. (319).

The trough-profile curves and dynamic linear changes shown in Figs. 254, 256, and 262 have been calculated using Eqs. (319) and (320).

In Poland, the time-subsidence curves of surface points have also been described with equations, by analogy with the profile-curve method. Assuming that a working is mined in a time of $t = 0$ at an infinitely great rate of face advance $v = \infty$, the rate of subsidence over an extraction area of constant size is proportional to the difference between the constant final subsidence $v_{z\text{end}}$ and the dynamic subsidence $v_{z\text{dyn}}$ at a particular point in time t – i.e., proportional to the delay of subsidence (Salustowicz, 1958):

$$\frac{dv_{z\text{dyn}}}{dt} = c(v_{z\text{end}} - v_{z\text{dyn}}) . \quad (321)$$

The time coefficient $c \approx 0.3 - 2.0$ depends on the mechanical properties of the overlying strata. For an assumed start of subsidence at $t = 0$ immediately after sudden extraction of the working, the solution to the differential equation reads as follows:

$$v_{z\text{dyn}} = v_{z\text{end}}(1 - e^{-ct}) . \quad (322)$$

The exponential member of $e = 2.718$ in Eq. (322) – the delay factor d – approaches the value zero with increasing time t . Consequently, subsidence over time of a surface point over a stationary working tends to the final value aMe as delay in subsidence decreases (Fig. 273). Similarly, over a stationary working the rate of subsidence of an observed point decreases to zero with the passage of time. If Eqs. (322) and (305) are compared, the time factor z corresponds to the expression in brackets (Fig. 274) in Eq. (322):

$$z = 1 - e^{-ct} . \quad (323)$$

In salt-mining, the long-lasting delayed subsidence movements over chamber workings mined in 3 – 5 years can be satisfactorily calculated with Eq. (322) (see Fig. 251). According to this equation, subsidence of a surface point begins immediately with the greatest rapidity (Fig. 273). From coal-mining, however, it is known that subsidence over a working that is increasing in size accelerates only gradually, and only until approximately 50% of final subsidence at the surface point has taken place (Fig. 247). For a working that is continually developing therefore, Eq. (321) is modified to the following form (Knothe, 1950):

$$\frac{dv_{z\text{dyn}}}{dt} = c[v_{z\text{end}}(t) - v_{z\text{dyn}}(t)] . \quad (324)$$

Here the expression $v_{z\text{end}}(t)$ signifies the particular final subsidence aMe_t associated with the mining stage reached by the point in time t . The difference

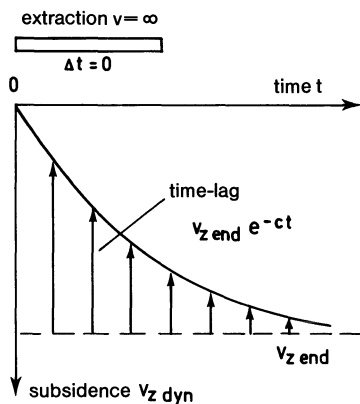


Fig. 273. Dynamic subsidence of a point in the ground after abrupt extraction of the area, on the basis of Eq. (322). The excavation is opened over its whole length at once

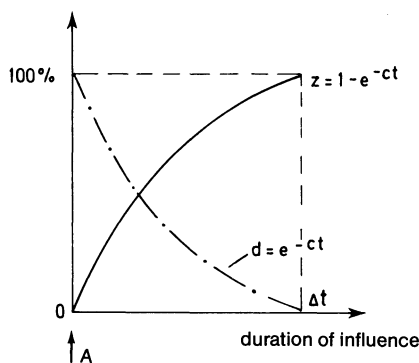


Fig. 274. The connection between the time factor z and the delay factor d . Extraction of the area A at the point in time 0

between the expressions in the brackets represents the delay in subsidence in the sense of Eq. (311). Equation (324) implies therefore that the rate of subsidence of a surface point over the advancing extraction is proportional to the delay in subsidence – a fact which can be seen from the similarity of the curves of increase and delay in subsidence in Figs. 247 and 265. The increase in subsidence is greatest at the moment when dynamic subsidence lags most behind the related amount of final subsidence $v_{z\text{end}}(t)$, which in Fig. 265 is shown as e_t -curve (3) for $z = 1$.

On the basis of Eq. (322), subsidence at the surface point P at the moment in time t , given a continually advancing face, amounts to

$$v_{z\text{dyn}} = v_{z\text{end}}(t) - \int_0^t dv_z(t_i) e^{-c(t-t_i)} dt. \quad (325)$$

The integral value to be subtracted from the final subsidence value $v_{z\text{end}}(t)$, which since t is a constant, and $t_i = 0$ to t , and

$$e^{-c(t-t_i)} = e^{-ct} e^{ct_i}$$

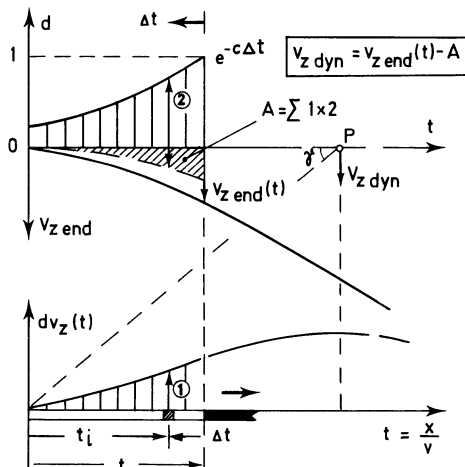
is frequently written also in the form

$$e^{-ct} \int_0^t dv_z(t) e^{ct} dt \quad (326)$$

gives the amount of delay in subsidence for the point in time and mining stage t .

The final subsidence value $v_{z\text{end}}(t)$ associated with the extraction area under consideration can be calculated with, for example, an integration grid. The amount of delay in subsidence can be obtained in the manner shown in Fig. 271 from the sum (area A) of the separate products resulting from multiplying the subsidence differential $dv_z(t)$ by the delay factor d (Fig. 275). As an influence function for the subsidence differentials dv_z of the extraction elements [Eq.

Fig. 275. Calculating dynamic subsidence for the ground point P by Eq. (325), using the influence function (*bottom*), the delay factor d , and the curve of final subsidence



(158)], it is possible to use a bell-shaped curve based on Eq. (182) (Knothe) with $x = vt$ (v = face advance) – simplified for ease of calculation to an isosceles triangle – or the first derivative dv_z/dx of the normal subsidence or e -factor curve. The further an extraction element, having at the position t_i a subsidence influence of $dv_z(t_i)$ on the surface point P, is distant from the working face, the greater its convergence and influence capacity become, with the result that the delay in subsidence decreases with an increase in the time span $\Delta t = t - t_i$. The amount of delay for the extraction element with $e^{-c_0} = 1$ is at its greatest immediately after extraction ($t_i = t$; $\Delta t = 0$). Equation (325) (Knothe) can thus be regarded as a mathematical description for the graphical method of time-subsidence calculation used in Fig. 271 (Perz), if the time factor z by Eq. (323) is introduced (Trojanowski, 1972).

Starting from Eq. (325), a method of calculation has been developed for rectangular extraction areas of any desired size, making use of tabulated values and nomograms (Dzgniuk, Pielok, Śroka, 1980).

10.3.4 The Influence-related Time Factor z_e

In the methods discussed in Sects. 10.3.2 and 10.3.3 it was assumed that the time factors selected are valid for all extraction strips, for every rate of face advance, and for every position of the surface point. In reality, the rate of convergence and of subsidence over the stationary rib where the face was opened out is slower than over the extraction front, and faster at the centre of the area extracted than at the margin. Movement in strata affected by underlying workings is also accelerated by more rapid rates of face advance.

The obvious course therefore is to relate the time factor, as in Fig. 266, to the whole extraction area – i.e., to make its value z_t dependent on the value of the influence factor e of the extraction area at the point of time t . The time factor z_e is found by taking the mean of the dynamic subsidence values measured at

several surface points at a given stage of mining and of the associated influence factors as calculated, thus:

$$z_e = \frac{\frac{v_{z \text{ dyn}}}{aM}}{e} . \quad (327)$$

This time factor is valid only for the rate of face advance and the depth range of the workings observed. At greater rates of face advance the time factor z_e will also be greater – time factors are approximately proportional to face advance and inversely proportional to mining depth. Special time factors are required for the period of delayed subsidence (e constant).

With this method, the position of the surface point in relation to the area extracted and the working front is not taken into account. A small extraction area lying centrally to the surface point can in fact have the same influence factor as a larger area at the margin, since the latter causes slower movement at the more distant surface point than does a central working.

For a similar graphical method of calculation based on the delay factor d_t , there are nomograms available to determine the time-related influence of the extraction area (Czubik, 1971).

10.3.5 The Calculation of Static Delayed Subsidence over a Chamber-and-Pillar Working

The calculation procedures dealt with so far have been concerned with dynamic subsidence over an enlarging extraction area with longwall-type working, where delayed post-mining subsidence is slight and of short duration (Appendix III). With chamber-and-pillar working in salt-mining, on the other hand, ground subsidence is slight during the development over many years of a working measuring 1 – 1.5 km² in size. What is required to be known is the considerable and long-lasting delayed subsidence after the conclusion of working (Fig. 251, bottom: phase II). Its prediction is complicated by local variations in the intensity of subsidence in strata overlying the workings and by the supporting effect of residual pillars and barrier pillars.

Delayed subsidence over a fully developed chamber working with yielding permanent pillars varies from place to place, as the hatched area in Fig. 257 shows. This is because the roof over the centre of the working, where pressure is high, subsides at first more rapidly than at the margin, where it is supported by the unmined seam in the forefield (Fig. 14). The ratio $z = v_{z \text{ dyn}}/v_{z \text{ end}}$, as in Eq. (305), is not constant along the profile of a trough, with the result that, using a positionally constant time factor z from Eq. (323), delayed subsidence of ground points within the subsidence zone is in some places overestimated and in others underestimated when calculated with Eqs. (305) and (322).

As a variation on the procedure outlined in Sect. 10.2.1.3, when calculating active and delayed subsidence during phases I and II by Eqs. (307b) and (307c), even in salt-mining, at least two differing time factors, one for the inner zone and

one for the margin, should be introduced. Both time factors should adjust increasingly to $z = 1$ by the time that the ground comes to rest again on the closing-up of all chambers. This is the case if it is assumed that the time factor in a 150 m wide margin zone increases, for example, quadratically on the basis of z (margin) = z^2 (centre) – i. e. as a function of the value of the time factor applying in the inner zone at the time t (see Appendix IV).

Example:

$t = 20$ years;	z (centre) = 30%;	z (margin) = 9%
$t = 40$ years;	z (centre) = 50%;	z (margin) = 25%
$t = 70$ years;	z (centre) = 80%;	z (margin) = 64%
$t = 100$ years;	z (centre) = 90%;	z (margin) = 81%
$t = 150$ years;	z (centre) = 100%;	z (margin) = 100% .

In the case of workings under 300 m wide, the time factor z (margin) is applied to the whole working.

Particular difficulties arise in the calculation of delayed post-mining subsidence if, in addition to chamber pillars, there are residual pillars, entry pillars, barrier pillars, or borehole-protection pillars left in place. These pillars retard the subsidence of strata overlying the workings, with the result that dynamic subsidence over these wide pillar areas is overestimated by Eq. (305). If the areas of these residual and barrier pillars are regarded as extraction areas, the local correction values $-v_{zd} = D e d$ can be calculated with an integration grid as “negative” subsidence troughs, which decrease to zero with time t through the introduction of a delay factor $d = 1 - z$ (Fig. 274). The influence factor e obtained with the integration grid has the effect that the correction value, like the supporting effect of the residual or barrier pillar, decreases with distance from the pillar. The initial magnitude D of the delay in subsidence, which corresponds to critical subsidence aM in Eq. (164) and depends in part on the size of the pillar area, is obtained from the difference between ground subsidence measured over the pillar centre at a point in time t and the subsidence as calculated by Eq. (307 b):

$$D(t) = v_{z \text{ dyn}} (\text{calculated}) - v_{z \text{ dyn}} (\text{measured}) .$$

To conclude this chapter, the following comments may be made on the accuracy to be expected in pre-calculating dynamic ground subsidence.

The complicated, three-dimensional development of movement in strata, from the working area up to the ground surface, does not lend itself easily to mathematical description without making certain approximations and assumptions. One possibility of calculating the change in position of strata points and surface points for selected mining and time stages, in accordance with kinematic laws, is offered by the finite-elements method. Assuming an adequate computer capacity, this method of calculation enables convergence in the workings to be predicted and the deformation of elements in the rock mass to be adapted to a rheological model.

It should also be borne in mind that, in the prediction of ground movement over time, other errors besides the uncertain time factor arise. According to in-

vestigations in the Ruhr District, seam areas actually mined differed in size by $\pm 10\%$ from the extraction panels on which the calculations in mine planning were based. The resulting error is connected with the position of the calculation points. For example, where the face height M taken from the mine plan and used for calculation is wrong by approximately 10% in relation to the seam thickness later mined, this error falsifies the calculated subsidence of all surface points by the same margin of $\pm 10\%$.

11. Impact of Ground Deformation on Structures

11.1 Types of Stress on Structures

The components of ground movement arrived at in earlier sections – namely, subsidence v_z and horizontal displacement v , together with the factors of tilt v'_z , surface curvature v''_z , and linear change ϵ derived from them – serve the subsidence engineer as a basis for determining surface damage. Here again, as in consideration of the interaction between shaft and surrounding strata, the question arises as to how far a given structure follows the movement and deformation of the ground. We know that the lie of the structure alters with the movement of the ground – it subsides, shifts laterally, and tilts with the ground on which it stands (Fig. 276); we also know that a stiff but elastic structural foundation undergoes less bending, stretching, and compressing than the ground; but we do not know the magnitude of the difference in deformation as between ground and structure in each case. There is a need therefore for closer investigation into the soil mechanics of the transmittal of deformation forces from ground to structure.

In addition to a change of position, structures undergo flexural, tensile, and compressive strains. Many structures are unable to bridge a sunken stretch of ground, either because they lack the necessary rigidity, or because they are linear in form and actually lie on the ground, like the communications networks (roads, railways, canals), or underground, like the pipes and cables of the public utilities. They bend under their own weight and follow the depression in the ground; or if their foundation is rigid and becomes partially exposed, there is a redistribution of the ground pressures under it. The majority of structures, however, succeed in resisting horizontal deformation, as they are harder than the surface layer of loose soil or weathered rock. Consequently, as the subsoil underlying a structure's foundation expands or contracts horizontally, it will slide along the foundation in places and thereby transmit frictional forces to the structure itself.

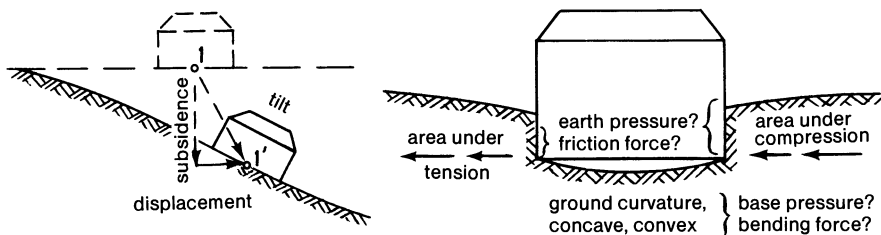


Fig. 276. The influence of ground movement (*left*) and of ground deformation (*right*) on a building within the zone of influence of a mine working

Moreover, in the zone of compression, shifting ground will exert a pressure on the sidewalls of a foundation lying across its direction of movement – the so-called passive earth pressure, or earth resistance.

There are thus four ways in which mining subsidence affects the interaction between ground and foundation:

- a) the structure changes position to exactly the same extent as the ground;
- b) there is additional earth pressure on the sidewall of the foundation;
- c) frictional forces are transmitted to the foot of the foundation; and
- d) flexure of the ground under a bend-resistant foundation produces changes in pressure on the foot of the foundation.

Of these, only the factors governing changes in position can be regarded as resolved and understood up to now (Chaps. 8 and 9). The earth pressure, the frictional drag, and the uncovering of the foot of the foundation all act to distort the structure. In contrast to the changes in position at (a), however, the tensile and compressive strains in the structure are smaller than the stress-relieving horizontal movements of the ground. The frequent practice of equating linear changes in the structure with predicted horizontal expansion or contraction in the ground thus produces too high a value for strains in the structure. It is true that this provides a safety margin if used as the basis of design, but it represents a crude and not always necessary oversimplification.

The transmittal of forces between the surface layer as it changes shape and the foundation of a structure can be calculated accurately enough today in simple, basic cases (as will be shown) by soil-mechanics formulae. These working hypotheses and empirical rules will serve even for quite complicated problems, provided the uncertainties and assumptions contained in them are borne in mind. To be able to predict the effects of mining on surface structures in terms of horizontal ground movements, however, it is necessary first to consider the stress/strain behaviour of soils and then to calculate the earth pressure and friction forces.

11.2 Stress/Strain Behaviour of Soils

In soil mechanics, the soil – or earth materials like gravel, sand, silt, and the clay resulting from the chemical weathering of rock – is defined as a natural rock pile consisting of mineral grains which can be separated according to grain size by agitation in water. By contrast, in solid rock, both massive and stratified, the minerals are held together in a lasting bond by strong cohesive forces. A distinction is drawn between dry, non-cohering sand or gravel soils (pore volume 30% – 45%, angle of friction 30° – 40°) and cohesive soils of clay or silt (pore volume 25% – 75%, angle of friction 10° – 30°, cohesion $1 - 2.5 \text{ N/cm}^2 = 10 - 25 \text{ kN/m}^2$).

In a *non-cohering material* like sand, a proportion of the grains changes position even on a small increase in loading: the grains seek, as it were, to escape from the pressure by turning and shifting and readjusting their position until

each grain has found a new state of stable support suited to its shape (Fig. 277). This is the state when equilibrium is reached between, on the one hand, normal directional forces σ_n exerted perpendicularly on the points of contact of each grain by its neighbouring grains, and on the other hand, tangential forces τ acting on the same points of contact up to the limit of frictional adhesion. During this so-called readjustment of support among the sand grains, a sand sample exposed to equal pressure from all directions – i.e. to hydrostatic stress, where $\sigma_z = \sigma_x = \sigma_y$ – is reduced in volume; by contrast, with unequal side-loading – i.e., when $\sigma_x \neq \sigma_y$, a state of shear stress – it is also possible to open up a dense sand. An ideally elastic soil on the other hand, when exposed to shear stress, would change its shape, not its volume.

It follows that, where sandy soil is concerned, there is no area of elastic response, i.e. no area in which the sand grains do not undergo a plastic displacement in relation to each other, even on small changes in compressive stress. Sandy soils display an overwhelmingly plastic behaviour: elastic deformation ϵ_e is so slight that scarcely any recovery takes place on removal of the load, and the readjustment of support that has occurred among the sand grains is largely maintained. This explains the large element of plastic deformation $\epsilon_{\text{retained}}$. If the load is reapplied in the same way, and at the same intensity, the result – since the grains of sand do not need to change position again – is additional compaction of the sample and a slightly greater change of shape ϵ_2 as compared with that under the initial load ϵ_1 . Only a higher load, or one applied in a different direction, will bring about a fresh readjustment of support and further changes in the shape of the sample.

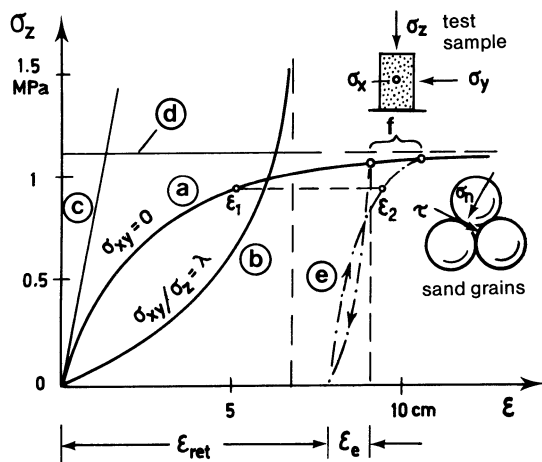


Fig. 277

Fig. 277. Deformation of a sand sample under vertical pressure. **a** Unimpeded lateral expansion; **b** encased sample; **c** elastic deformation, limit of shear strength; **d** plastic deformation, limit of shear strength; **e** load relief and renewed loading; **f** additional compaction after load-relief

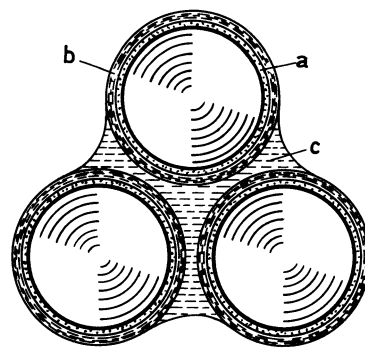


Fig. 278

Fig. 278. Water bound into the sand mass. **a** Hygroscopic water on surface of sand grains; **b** water of adhesion; **c** capillary water

In *cohesive soils* having many small grains and a large grain-surface area, a relatively large proportion of the pore water is firmly attached to the grains by molecular forces. As a result, momentarily high shear stresses are taken up by this bound water, or “water of adhesion”, and the grains are prevented from coming into contact with each other (Fig. 278). This is what produces, on the one hand, the cohesiveness of these soils and, on the other, their high workability and low frictional resistance as compared with dry sand. Under high stresses, however, both these characteristics of cohesive soils disappear, as their cohesion is not enough to prevent the displacement of grains when the loading is both high and persistent. In other words, under strong pressure, cohesive and sandy soils behave alike as regards deformability.

Elastic phenomena and the storing of potential energy thus play no significant part in the deformation of soils. The law of superimposing forces, which in mining subsidence engineering is bound up with elasticity and is attributed to the mutually superimposing effects of extraction, does not apply to the stress/strain behaviour of soils. The characteristic curves in Fig. 277 indicate, for pressure tests with restricted lateral expansion (i. e., when the sample is encased in a stiff material) a less-than-linear increase in compaction (b) on the application of a load, and for compression tests with unrestricted lateral expansion, a more-than-linear increase in compaction (a).

In the area of concave curvature of the ground surface above a working face, the surface layer may expand both upwards and downwards (an increase in thickness, or vertical extension) under the horizontal bending pressure (compression). At the convex rim of the subsidence trough, this surface layer may move towards the trough centre under the perpendicular abutment pressure (tensile strain) with simultaneous reduction in thickness (vertical compaction). In the course of this flexural deformation, the sand grains in a cohesive soil (idealized in Fig. 279 as spherical in shape), which are embedded in mineral admixtures and water of

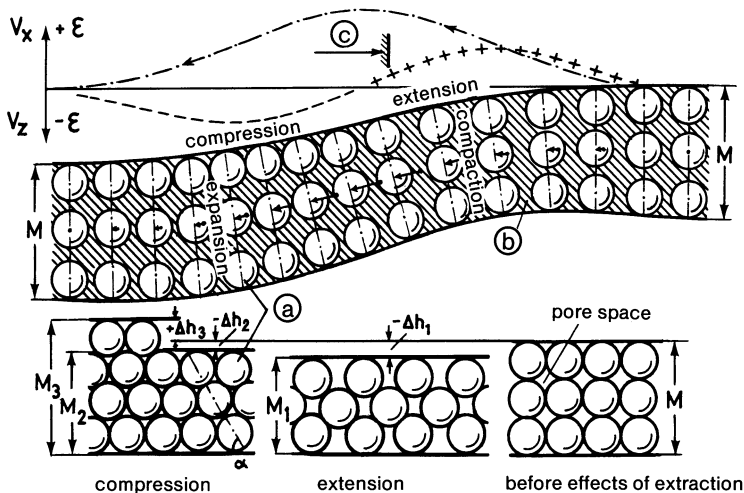


Fig. 279. Deformation of a cohesive layer of ground above an area of extraction (critical area) through change in the position of grains (top). Bottom: cubic (right) and rhombic stacking pattern of grains of sand. a Sand grains; b pore water, mineral impurities; c extraction front

adhesion and in part surrounded by free pore water, change position in such a way that, in the extension zone, their spacing is increased horizontally and decreased vertically, and in the compression zone, their spacing is increased vertically and decreased horizontally. Between the rim and the centre of the trough, all the sand grains change their horizontal position, as indicated by the arrows in Fig. 279, in accordance with the displacement curve.

If the surface layer is non-cohesive sand, originally densely stacked in a cubic pattern, it is opened up in a horizontal direction in the extension zone, and is reduced in thickness by the amount Δh_1 ; in the compression zone it is compacted to a rhombic pattern, with one or more additional layers being formed by grains forced up between others, thereby increasing the thickness by the amount Δh_3 . A straightforward, horizontal shear stress, without flexural deformation, would reduce the thickness by Δh_2 , however. This realignment of grains from a cubic to a rhombic stacking pattern, which can reach an angle of orientation α of 60° , allows a considerable displacement of individual points in the compression zone, depending on the thickness of the surface layer affected.

To sum up, the *behaviour of soils* under pressure can be stated thus:
 the individual grains making up a sandy soil shift their position in a plastic manner (readjustment of support);
 the volume of a given body of soil is not constant (density and pore volume can alter);
 the curves representing application and removal of load are not identical (minimal elasticity);
 after the removal of load, the original stacking density is not restored (permanent deformation);
 strain is not proportional to stress;
 water of adhesion and friction between grains both act against shear strain.

11.3 Earth Pressure on a Foundation Sidewall

On convex bending of the surface layer, the sidewalls of a structural foundation standing in the ground experience a relief of pressure; on concave bending, they experience a pressure additional to the pressure of earth at rest (Fig. 280). In extreme cases, if bending of the surface layer is sufficiently intense, this pressure E on foundation sidewalls may, in the extension zone, drop from earth pressure at rest E_0 to the level of active earth pressure E_a , or in the compression zone, rise to the level of passive earth pressure E_p , also known as earth resistance: $E_a < E_0 < E_p$; $E < E_p$ (see Fig. 284).

Phenomena of active earth pressure are to be observed when *cracks* in the ground form in the extension zone. If tensile ground strain as a result of convex curvature exceeds the separation strength (ultimate extension value) of the surface layer, which lies between 5 and 7 mm/m, a crack 1–3 m deep and pointing obliquely to the mine working opens up at the surface (Fig. 281). The loosened and cracked ground on the hanging-wall side of this crack can then slip

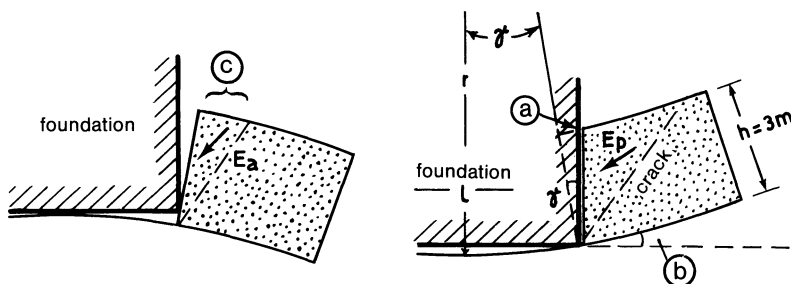


Fig. 280. Earth-pressure forces acting on a foundation wall, given anticlinal (*left*) and synclinal (*right*) curvature of the ground. **a** Extent of compaction of the ground – e.g. 1.5 cm; **b** tilt $\tan \gamma$ – e.g. 5 mm/m; **c** wedge of ground slides down along fracture plane (*broken line*)

or subside as a wedge along a second crack and towards the main crack, if the dead-weight force of the sliding body overcomes the shear resistance along the opposing fracture plane. As a result of the slipping or subsiding of this wedge of ground, a contrary crack, or slope crack, is formed at a distance of 30–60 m from the main crack. Since the main crack can be followed down only a few metres when dug up, it must be assumed that, where the distance between cracks exceeds 10 m, there is either a very deep zone of loosened ground or as very deep jointing. This movement process leaves behind, at the main crack line, a step in the ground several decimetres high or, between the two crack lines, a trench-like depression. Such steps in the ground are greatly feared as constituting a subsidence danger to traffic installations and residential structures (see Sect. 7.3).

Earth resistance E_p , as an additional stress on a structure, is of particular interest to the mining subsidence engineer. Its name, stemming from soil mechanics, indicates that earth pressure is produced by the movement of a supporting wall against the soil, causing the latter to react passively and offer resistance to deformation. Since the ground is the active element of movement in the sphere of mining influence, the reverse situation, in which the ground pushes against a foundation sidewall, also occurs in subsidence engineering. In the compression zone, for example, given a ground displacement v_x from left to right and steadily

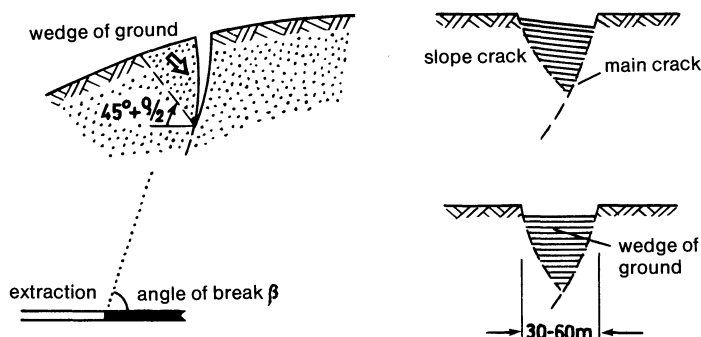


Fig. 281. Crack (*left*), step, and trench (*right*) in the ground, within the area of the angle of break (maximal extension and convex curvature)

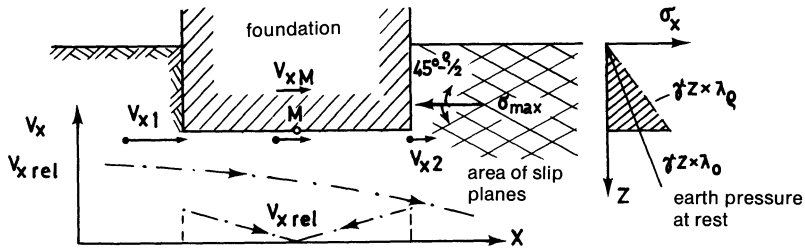


Fig. 282. Relative displacement as between the ground and a foundation shifted by the amount v_{xM} (compression)

decreasing, as in Fig. 282, the ground can press against the left sidewall of the foundation and become compacted by $v_{x1} - v_{xM}$, while on the other hand, if the rigid structure is carried along complete with the middle point M of the subsoil by the distance v_{xM} , the sidewall on the right-hand side will push the ground back by $v_{xM} - v_{x2}$.

The pressure E exerted on a foundation sidewall by ground which is exposed to concave curvature and thus compressed has reached its maximum value E_p when, under increasing shear force, all factors in the granular make-up of the soil offering resistance to movement have become fully effective, and when the ground, as this limit stress is exceeded, shears off along two groups of slip planes enclosing, with the greater horizontal main stress, an angle of $45^\circ - \rho/2$.

These slip planes form in the ground if the ratio of shear and normal stress along them amounts, in non-cohesive soil, to

$$\tan \rho = \frac{\tau}{\sigma_n} \quad (328)$$

and in cohesive soil to

$$\tan \rho = \frac{\tau - c}{\sigma_n} \quad (329)$$

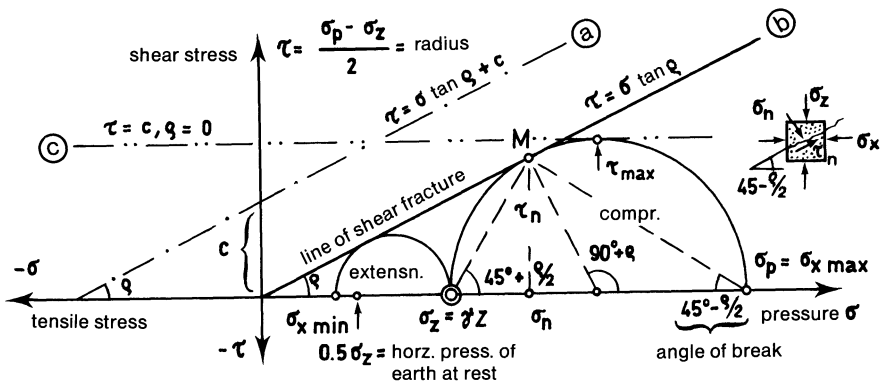


Fig. 283. Lines of fracture for **a** cohesive, **b** non-cohesive, and **c** plastic soils; also stress circles for a point in the ground at depth z

where ρ signifies the angle of inner friction and c the cohesion or shear strength when normal stress is zero. Graphical representation of the equation $\tau = f(\sigma, \rho, c)$ gives the so-called Coulomb fracture line (Fig. 283).

According to Mohr, the horizontal limit stress $\sigma_p = \sigma_{x\max}$ which precipitates fracture in the compression zone can be determined for any desired point in the ground (depth z) by first calculating the vertical compressive stress $\sigma_z = \gamma z$ from the weight force $\gamma = 18 \text{ kN/m}^3$ for the ground point concerned and then describing a semicircle¹ in the diagram, starting from this stress value σ_z and tracing it towards the right above the σ -axis (pressure). This semicircle touches the line of fracture at the point M and intersects the σ -axis at the required point of limit stress $\sigma_p > \sigma_z$ (passive earth pressure).

By *Rankine's theory*, if shear stress at the foundation sidewall (angle of wall friction $\delta = 0$, for $\delta = 15^\circ$ doubled values) is ignored, the critical ratio of principal stress is

$$\lambda_p = \frac{\sigma_p}{\sigma_z} = \tan^2 \left(45^\circ + \frac{\rho}{2} \right). \quad (330)$$

For example, this gives for $\rho = 20^\circ$ approx. 2.0 (clay), for $\rho = 25^\circ$ approx. 2.5 (loam, silt), and for $\rho = 36^\circ$ approx. 3.85 (sand, gravel). Thus, for the horizontal limit stress in compressed ground the following relation applies:

$$\sigma_p = \lambda_p \sigma_z = \lambda_p \gamma z. \quad (331)$$

For the whole foundation sidewall of height $z = h$, this gives a horizontal, passive ground pressure (earth resistance) in non-cohesive soil of

$$E_p = \int_0^h \sigma_p \, dz = \frac{1}{2} \sigma_p h = \frac{1}{2} \gamma h^2 \lambda_p \quad (332)$$

and in cohesive soil of

$$E_p = \frac{1}{2} \gamma h^2 \lambda_p + 2ch \sqrt{\lambda_p} \quad (333)$$

per metre length of wall.

For example, with a 2 m high and 15 m long sidewall to the foundation of a residential house lying across the direction of displacement in non-cohesive sand, this horizontal ground pressure can, in the compression zone, be eight times greater (2080 kN) than the pressure of earth at rest

$$E_0 = \frac{1}{2} \gamma h^2 \lambda_0 \quad (334)$$

of 207 kN (1 kN = 1000 N), if the coefficient of earth pressure at rest $\lambda_0 = \sigma_x / \sigma_z$ is approximately equal to 0.5.

According to *Coulomb*, for a cohesive soil, or in the case of wall friction resulting from ground sliding away sideways before the wall, even higher values are

¹ Statement: $\sigma_p = \sigma_z = 2 \tau_{\max}$ (diameter).

obtained than in the above example. In calculating the strength of foundations in mining therefore, the peak earth pressure arrived at by Eq. (332) should be doubled, i. e.

$$E_p \approx \gamma h^2 \lambda_p. \quad (335)$$

A static calculation should be undertaken to check whether or not this earth pressure exceeds the bending strength of the foundation sidewall (see the construction guidelines in the Appendix).

These *peak horizontal pressures* are reached only if, given a sufficiently large displacement movement of the soil, all obstacles to movement in the sand grains are mobilized (shear strength and cohesion), and only if, after the limit of shear strength has been exceeded, slip planes form in the soil (deformation boundary condition according to Rankine). As regards the amount of relative displacement as between foundation wall and ground needed for this, varying values are mentioned in the literature – from 2 cm in the case of very tightly compacted ground (Terzaghi and Peck, 1961) to 10 cm for a smooth wall, or 15 cm for a rough wall, and less solid compaction (Franzius, 1928), given a wall height of $h = 2$ m.

For a wall of sheet-piling 3 m high and 5.6 m long in cohesive soil, a displacement movement of 21 cm on break-up of the soil under a pressure of 3920 kN was measured (Zweck, 1957).

On average therefore, peak earth pressure E_p on a foundation wall will develop when displacement as between wall and ground amounts to between 0.05 and 0.1 h . This observation takes account of the fact that, on the one hand, the ground is compacted not only as a result of the obstacle presented by the wall but also as a result of the influence of mining, even without the presence of a foundation; and, on the other hand, that the construction trench around the foundation walls is filled with loose earth which is less densely compacted than the original, native earth. These two phenomena will cancel each other out in their pressure effect on the foundation, and so the figure for displacement travel given above is an acceptable calculation basis for E_p in mining subsidence engineering. In a surface layer showing concave curvature as in Fig. 280, right, this peak pressure for E_p presupposes a displacement travel of the ground before the walls of a 50 m long and 2 m deep foundation amounting to a > 10 cm – i. e., a radius of curvature in the foundation subsoil

$$r = \frac{hl}{2a} \quad (336)$$

of under 500 m, or a ground compression of over 4 mm/m. It must also be noted however, that the amount of compaction travel in the soil further depends on whether the ground lying across the axis of the structure is compressed or extended (see Fig. 142).

A relationship, found in trials, between proportional compaction movement and earth pressure is shown in Fig. 284. For a foundation of depth $h = 2$ m, maximum compaction travel until ground failure (E_p) at 0.1 h comes to approximately $s = 200$ mm. With the given values, passive earth pressure is obtained by Eq. (335) as 560 kN/m width of the structure. Given 40% of the maximum

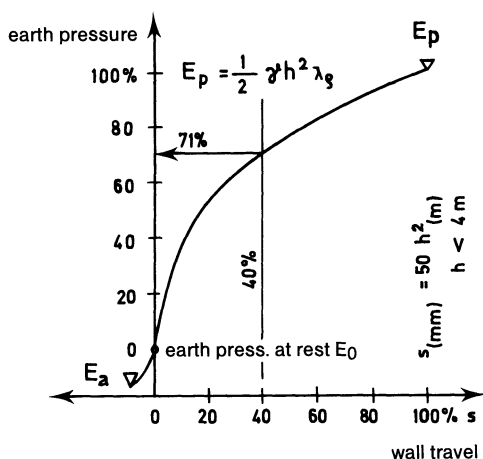


Fig. 284. Graph of force against travel in earth pressure (after Franzius, 1928). Example: $h = 2$ m, $\gamma = 20$ kN/m³, $\lambda_p = 7.0$, $l = 40$ m, compression 4 mm/m, $E = 0.71 E_p$

possible compaction travel, this earth pressure is attained only as to 71% – i.e., $E = 400$ kN per metre of wall width.

Finally, from these mechanical properties of the top layer of ground the following conclusions can be drawn for the purposes of subsidence engineering:

- Earth pressure E increases with the square of the depth of foundation h ;
- before the effect of mining is felt it is earth pressure at rest E_0 that bears on the vertical foundation walls ($\sigma_x = 0.5 \sigma_z$);
- where the ground is under tensile stress, pressure on the foundation wall is less than earth pressure at rest (active earth pressure $E_a < E_0$);
- in the compression zone, ground pressure on the foundation wall is higher than earth pressure at rest and reaches its maximum value (passive earth pressure $E_p \approx 10 E_0$) on a relative displacement as between wall and ground of approximately $0.1 h$, provided the buckling or bending strength of the foundation wall has not been exceeded first;
- the denser the soil, the greater the ground compression, and the greater the length of the structure in the direction of displacement, the sooner this compaction travel (wall travel) of $0.1 h$ associated with the maximum value E_p is reached;
- in mining in general, a compaction value of only 10% – 40% of the limit value of $0.1 h$ is reached (e.g., 2 cm wall travel as against 20 cm limit value for compaction where $h = 2$ m, $l = 20$ m, and ground compression is 2 mm/m); in practice therefore, an earth pressure E of only 30% – 70% of the maximum value E_p is to be expected (Fig. 284).

Earth pressure on the foundation has not yet been measured in mining districts and compared with ground compression. An overthrust of the ground, corresponding to soil failure and passive earth pressure, has been observed in the case of an inhomogeneous subsoil and very intense mining influence (Fig. 153).

11.4 Friction Forces Acting on the Structure

As the ground over a mine working deforms, it makes spasmodic shear movements under the base and along the sides of the foundation and in doing so creates a dynamic friction force F_t (index t = tangential). On the one hand, this force seeks to impede the movement of the ground, and on the other, it carries the structure along and deforms it. This friction force is that part of the perpendicular, or normal-acting, force F_n which is determined by the coefficient $\mu = \tan \rho$:

$$F_t = \mu F_n. \quad (337)$$

Figure 285 illustrates the connection between the perpendicular force and the base area of the foundation. The endless belt, which represents displacement of the ground in the direction of working, carries along the weights placed on it and connected to a coiled spring (which represent buildings) up to the point where the tensile force in the spring, which increases linearly with its extension, is equal to the friction force F_t . It can be seen that, with the same base area A , the 2-kg weight has been displaced twice as far as the 1-kg weight and has created double the tensile force in the spring before coming to a halt. The friction force is therefore proportional to the normal (perpendicular) force. The normal force and the friction force remain unchanged even when, with unchanged weight, the base area of the body is reduced. This is shown by the 1-kg weight at the rear, which, despite half the base area and therefore double the soil loading (in N/cm^2), has been displaced by only the same distance as the 1-kg weight at the front. Consequently, given the same structural mass, it is immaterial from the point of view of friction force whether the architect decides, in a mining area, on a narrow skyscraper or a single-storey building of great extent.

The *coefficient of friction* μ for sandy soils amounts on average to 0.65 ($\rho = 33^\circ$) and for firm marl and clay soils to approximately 0.36 ($\rho = 20^\circ$). With lightweight buildings (base pressure $< 10 \text{ kN}/\text{m}^2$) on cohesive soils, adhesion stresses (cohesion) of the order of $10 \text{ kN}/\text{m}^2$ of contact area must be taken into account. Accordingly, it is only under a high loading (over $100 \text{ kN}/\text{m}^2$) that a cohesive soil, despite its smaller coefficient of friction, exhibits lower friction than a granular soil and behaves as a “slip layer”.

For the vertical *normal force* F_n (in N) acting on the base plane A of the foundation, the sum of all the bearing forces of the structure – the weight force, or

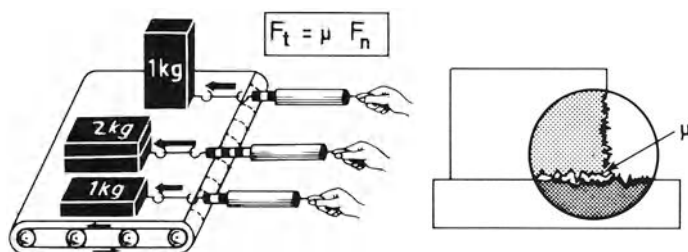


Fig. 285. Measurement of friction force

contact force – is used; and for the horizontal normal force acting on the foundation walls, the earth pressure at rest E_0 , which increases with the square of the depth (h^2). Where the depth of foundation is small, however, friction force on its walls can be ignored. With high buildings, the dead weight per floor, including the cellar level but excluding the foundation itself, is put in the literature at between 7 and 11 kN/m² – say 10 kN/m² as a round figure. In the case of single foundations keyed into the building ground and not secured against changes in distance apart, special calculations have to be employed.

In general, the middle of the structure M, as the focal point² of resistance to sliding movement, will participate fully in displacement of the ground at this place – i.e., relative displacement v_{xrel} there is nil. Away from this middle zone, relative displacement as between ground and foundation does take place. In the compression zone, the ground under the left-hand side of the structure moves ahead of related points in it, as shown in Fig. 286, whereas under the right-hand half, lying towards the trough centre, the ground lags behind the structure.

In the extension zone, relative movements of ground and foundation are reversed as between the two halves of the structure. They are thus directed either towards the middle of the structure (compression) or away from it (extension), and are greatest at the end of the foundation (Fig. 286). The friction force transmitted to the structure as a result of this relative movement therefore has the effect, in the compression zone, of reinforcing the stress on the foundation imparted by earth pressure E_p (Fig. 282); in the extension zone it acts as a tensile force on the foundation (see also Figs. 307 and 308).

The friction force transmitted to the structure accumulates in its stressed longitudinal or transverse axis as elastic deformation force; and so long as the foundation does not fracture, this force increases linearly with the angle α (derived from $\tan \alpha = \Delta \tau / \Delta x$) to a value of

$$F_{tmax} = \frac{1}{2} F_t = \mu \frac{1}{2} F'_n \quad (338)$$

up to the middle of the structure (Fig. 286). F'_n denotes the uniformly distributed construction load per metre of structure length, in N per running metre (see Fig. 287, left). Tensile or compressive friction stress thus increases in proportion to the length l of the foundation taken in the direction of displacement. This is the reason why, in mining areas, buildings with long frontages are divided up into separate units. Should compressive or tensile strength be exceeded in a homogeneous foundation, fracture would start from the middle of the structure if there were no weak spots such as door and window openings, which in practice determine the location of fracture zones and lines.

Elastic *deformation of the structure* in mm/m reaches a value of

$$\epsilon_{lmax} = \frac{F_{tmax} 1000}{E S}, \quad (339)$$

² Parts of the building projecting deeper into the ground, such as central-heating basements, anchor the structure to the ground ($v_{rel} = 0$). Where parts of the building are higher than others, the centre of gravity, or focal point, shifts from the middle towards the heavier section of the building.

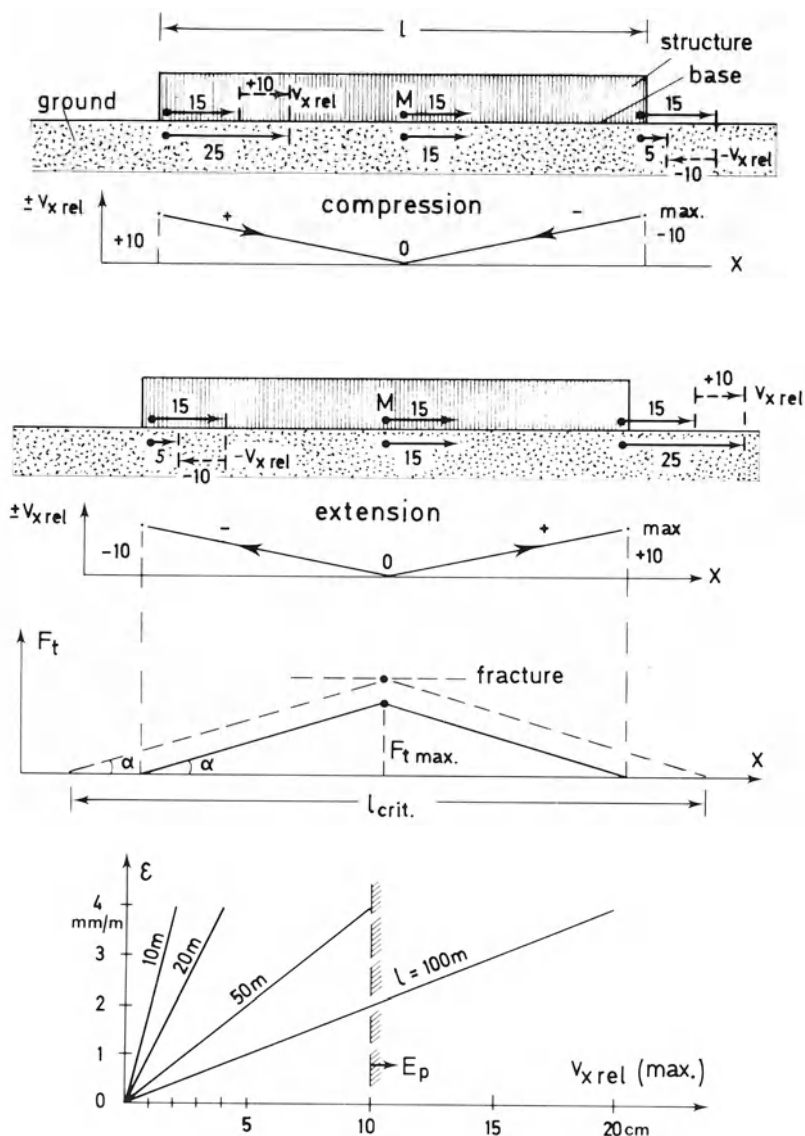


Fig. 286. Increase in friction force F_t towards the centre of the structure; and relative displacement at the end of the structure, as a function of extension or compression of the ground

at the centre of the foundation, where $F_{t \text{ max}}$ denotes the transmitted maximal friction force in N at the foundation centre, S the cross-sectional area of the foundation in m^2 , and E Young's modulus of elasticity in N/m^2 (in the case of concrete, $25,000,000 \text{ kN}/\text{m}^2$). This enables the elastic linear change in mm of the whole foundation to be calculated from

$$\Delta l = \frac{1}{2} \epsilon_{l \text{ max}} \cdot l \quad (340)$$

Relative displacement in mm as between a foundation stressed up to $\epsilon_{l\max}$ and ground which is uniformly compressed or extended by ϵ rises to a value of

$$v_{x\text{rel}} = \frac{1}{2} \left(\epsilon - \frac{1}{2} \epsilon_{l\max} \right) \quad (341)$$

up to the end of the foundation.

As a *calculation example* a 50 m long, 15 m wide, three-storey structure with a foundation 2 m deep and standing in an area affected by mining (Fig. 287) may be taken. Assuming $\mu = 0.65$, $\lambda_0 = 0.5$, and $\gamma = 18 \text{ kN/m}^3$, this gives an earth pressure at rest, per running metre in the longitudinal axis, of

$$E_0 = 0.5 \gamma h^2 \lambda_0 = 18 \text{ kN/running metre}$$

and a compressive or tensile friction force of

$$F_{t\max} = \mu E_0 l/2 = 0.65 \times 18 \times 25 = 293 \text{ kN}$$

transmitted to the longitudinal sidewall of the foundation and increasing up to the middle of that wall. The structure will thus be exposed to a friction stress acting on the two foundation sidewalls and amounting to a maximum of 586 kN at the centre of the foundation.

To this must be added the friction force acting on the underside of the 750 m² foundation baseplate. Assuming a base pressure F_n of 10 kN/m² multiplied by 4 (the number of storeys including cellarage) plus 3 kN/m² (for the 15 cm thick concrete baseplate of the foundation), making 43 kN/m² or – in terms of the longitudinal axis – $F'_n = 43$ times 15 equals 645 kN per running metre, this friction force will amount to approximately 10,500 kN (tension or compression)

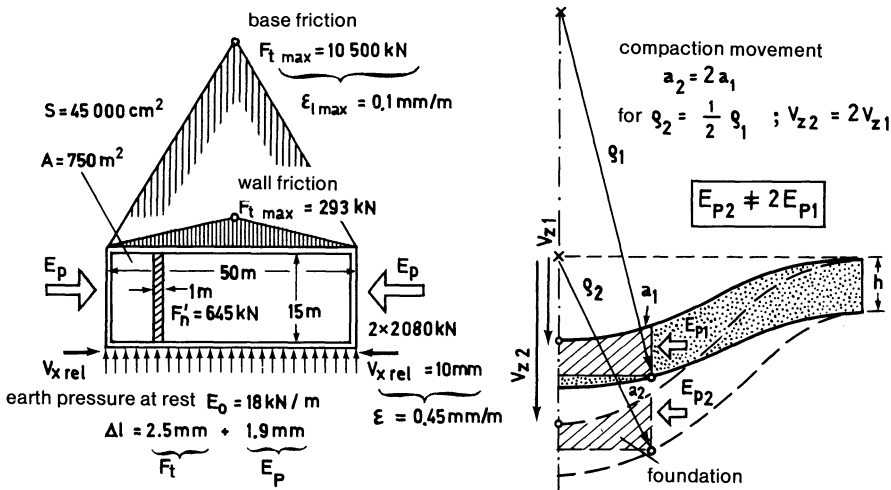


Fig. 287. Left: plan of the foundation 2 m deep for a three-storey structure, showing ground forces acting on the side of one wall. Right: earth pressure E_p on a foundation wall when ground subsidence and curvature double (seen in cross-section)

at the middle of the foundation baseplate, calculated as follows [Eq. (338)]:

$$F_{t\max} = \mu F'_n l/2 = 0.65 \times 645 \times 25 \approx 10,500 \text{ kN}.$$

The friction force on the two sidewalls of the foundation, which are only 2 m high, is thus very small in comparison with the base friction (in this case 6%). With thick foundation baseplates it should be noted that marginal compressive or tensile stress on the base caused by friction force is four times greater than the average cross-sectional stress.

Friction force arising at the base of the foundation causes an elastic linear change at the centre of the structure amounting to

$$\epsilon_{l\max} = \frac{F_{t\max} \times 1000}{ES} = \frac{10,500 \times 1000}{25,000,000 \times 4.5} \approx 0.1 \text{ mm/m}.$$

For $S = 4.5 \text{ m}^2$ here, the alternative cross-section, related to concrete, of the foundation (in this case 15 cm thick), including the longitudinal sidewalls and the ceilings (an additional 15 cm thickness in all), must be used – i.e. steel cross-sections are to be multiplied by the factor 10 in accordance with the elasticity moduli of concrete³ and steel. The length of the whole foundation alters by

$$\Delta l = \frac{1}{2} \epsilon_{l\max} = 25 \times 0.1 = 2.5 \text{ mm}.$$

For comparison, linear change in the foundation resulting from earth pressure $E_p = 2080 \text{ kN}$ may be calculated thus:

$$\Delta l = \frac{2E_p \times l}{ES} = \frac{2 \times 2080 \times 50}{25,000,000 \times 4.5} = 1.88 \text{ mm}.$$

This additional and uniform linear change in the proportion of $1.88 \div 50 \approx 0.04 \text{ mm/m}$ is of no significance for the compression of the foundation, apart from the fact that the earth pressure, because of the friction resistance of 645 kN per running metre and the cohesion force of approximately 10 kN/m^2 (clay), cannot affect the whole length of the foundation by deforming it but tends more to place the foundation sidewalls under bending stress at the point where it acts on them (Fig. 287).

The end of the structure becomes displaced in relation to ground that is uniformly extended or compressed by, say, $\epsilon = 0.45 \text{ mm/m}$ by an amount of

$$v_{x\text{rel}} = \frac{1}{2} \left(\epsilon - \frac{1}{2} \epsilon_{l\max} \right) = 25 \left(0.45 - \frac{1}{2} 0.1 \right) = 10 \text{ mm}.$$

In this case the peak of earth pressure E_p ($v_{x\text{rel}} > 10 \text{ cm}$) would not have been reached (Fig. 286, bottom).

Calculations show that, in long, heavy structures within areas affected by mining, friction force outweighs earth pressure. Linear change in the structure

³ With concrete, Young's modulus differs in value as between compression and tension and moreover depends on the degree of loading. Part of the deformation is plastic (shrinkage and creep).

lags behind extension or compression in the ground. Whereas the passive pressure from outside them tends to compress the foundation walls, friction force in the direction of ground displacement acts on the whole length of the foundation, and indeed with increasing deformation effect towards the centre of the structure. The foundation therefore breaks from the centre outwards.

In conclusion, attention should be drawn to one qualification. In the calculation of friction and of relative displacement, it was assumed for simplicity that the plane of the foundation became displaced and horizontally deformed to the same extent as was calculated for a ground surface subjected to no load. In reality, however, the foundation carved into the ground constitutes a foreign body which resists the pressure of the ground on the outer walls and impedes the displacement of sand grains underneath itself, through shear stresses and increased perpendicular loads. A structure exposed to deformation forces in the ground thus reacts on the ground itself. The heavier the structure, the deeper the zone in the ground affected by the reaction forces. Together with the loaded ground, the foundation forms a pre-compacted block (Fig. 288, left) which offers resistance to the compressing movement of the ground and to its further horizontal compaction. As a result, both shear movement of the ground along the foundation and friction are kept to a minimum. In ground subjected to extension, the loaded seating under the foundation counteracts loosening up of the soil (Fig. 288, right). That part of the ground forces which is held back by the foundation

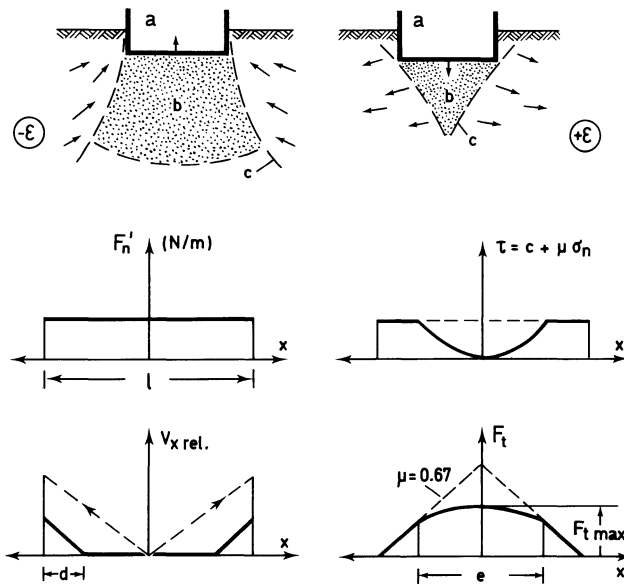


Fig. 288. *Top:* deformation of ground in the vicinity of a foundation when under compression (*left*) and extension (*right*), as observed in a sand model (after Lütgendorf, 1978). **a** Foundation; **b** zone compacted by weight of structure; **c** shear planes. *Bottom:* Slope of increase in tensile force F_t in a structure of length l and of load distribution F'_n , if relative displacement $v_{x \text{ rel}}$ as between ground and foundation takes place only in the margin zone d , while in the inner zone e , where ground and foundation remain bonded together, only cohesive shear stresses are transmitted to the foundation. (*Broken line:* limit values in accordance with guidelines for $\mu = 0.67$)

becomes evened out through relative displacement of soil particles along shovel-shaped paths, directed upwards in the case of compression, or downwards in the case of extension. This balancing of forces through movement along the slide paths is assisted by the fact that the foundation, along with its seating, subsides somewhat into ground exposed to extension, while ground under the foundation exposed to compression expands vertically less than the surrounding ground.

Ignoring the reaction forces exerted on the ground by the structure thus leads, in calculating friction forces and earth pressure, to the adoption of ground compression and extension values and relative displacements which are too large. In relation however to other soil-mechanics assumptions made in subsidence engineering for purposes of assessing deformation forces, these errors are not significant, and indeed they constitute a safety factor.

The transmission of friction forces to rail tracks and to conduits is dealt with in Sect. 12.4.

11.5 Changes in Base Pressure as a Result of Ground Curvature

Before the foundation (e.g. concrete plate, cellular raft or concrete beams) is constructed, its seating of native earth is levelled and covered with a layer of sand or gravel. If this foundation seating later deforms under the influence of a mine working, forces are transmitted to the structure, in part directly as earth pressure and friction force through linear deformation of the ground, and in part as bending forces through curvature of the ground. As a result of upwarping or downwarping of the ground built on, the underside of the foundation becomes detached from the ground in places and stands free of it, thereby losing some of its support (Fig. 276). As these free-standing areas of the foundation cannot transfer their supporting force to the ground, or can do so only in part, the load that they represent has to be taken over by those areas of the foundation still in contact with the ground and be borne by them in addition to the load they already bear. These readjustments of load have the effect of producing bending stresses in the structure, leading to a fresh distribution of base pressure at the seating plane and to the formation of cracks (Fig. 289). In ground loosened up by being in a zone of extension, delayed settlement can occur in parts of the foundation placed under such additional load. For many buildings therefore, ground curvature is an important dimension in the prediction of mining damage. An

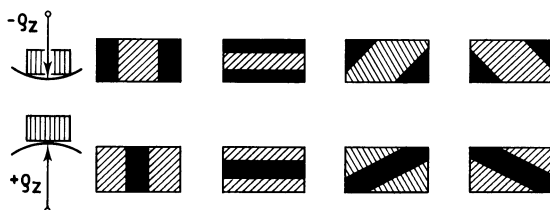


Fig. 289. The underside, or base, of foundations detached and standing free in places (*hatched areas*) as a result of ground curvature. *Top:* downward curvature, forming a trough. *Bottom:* upward curvature, forming a saddle

altered distribution of vertical base pressure can also sometimes develop if a tilted structure cuts into the ground on one side [see Eq. (366)].

Buildings are in general neither so loosely constructed that their bearing members experience no stress on the bending of the base of their foundations, nor so rigid that an alternation in vertical ground pressure has no influence on their stability. Experience shows that they can endure a small degree of ground curvature without suffering damage. What interests the subsidence engineer therefore is the amount of difference in settlement or of bending that structures can tolerate. From an evaluation of settlement measurements made on over 100 structures in an area free from mining (Skempton and Donald, 1956), it is found that visible cracks develop in masonry walls given a radius of curvature of 1 km, corresponding to a sag f of 2 cm in a chord of 12 m length (Fig. 303 d). In bending trials with masonry walls in residential buildings standing on continuous joists, cracks over 1 mm wide appeared in downwarping from $r = 5$ km onwards, and in the case of upwarping from as early as $r = 10$ km onwards (Rausch, 1955). Curvature caused by settlement will therefore cause no damage to a masonry structure of length L , provided $\rho_z > 500 L$.

To be able to judge the effect of ground curvature on a structure, it is necessary to know the value of the vertical *base-pressure stresses* on the underside of the foundation and the extent to which the structure settles into the ground, whether this is deformed downwards into a trough or upwards into a saddle. The settlement of a foundation, given a known base pressure of q , is ascertained either from the rigidity or stiffness index $E_s = q \div \Delta h/h$ of the ground – in the case of sand, $3 - 10 \text{ kN/cm}^2$ ($30 - 100 \text{ MPa}$) – or from the bedding index $C_b = q \div v_z$, expressed in N/cm^3 , which gives the increase in pressure in N/cm^2 necessary to produce an increase in settlement of 1 cm. The rigidity number corresponds to the E modulus of solid materials and relates compressive stress to relative settlement $\Delta h/h$ as a percentage of the original bed thickness h . The bedding index, on the other hand, depends not only on the nature of the soil but also on the thickness of the yielding bed.

For calculating the distribution of base and subsoil pressure, there are two methods used in constructional engineering – the bedding-index theory, and the theory of second order. By the bedding-index theory, the settlement v_z of a structure is proportional to the base pressure q at every point in the foundation (Fig. 290, left):

$$v_z = \frac{q}{C_b}. \quad (342)$$

The *bedding-index theory* is sufficiently accurate for long foundations resting on thin, yielding layers of soil. In the second-order theory – also known as the *rigidity-index method* – the overlapping of the stress fields of neighbouring loads is taken into consideration (Kany, 1959). The foundation is divided up longitudinally and transversely into ten equal sections resting on imaginary yielding supports (the ground), the subsidence movements of which can be determined, just as with a beam on an elastic underlay [see Eq. (23)], from the rigidity of the structure/ground system and the overlapping of compressive stresses (Fig. 290, right). Underneath a rigid building structure therefore, stresses in the base

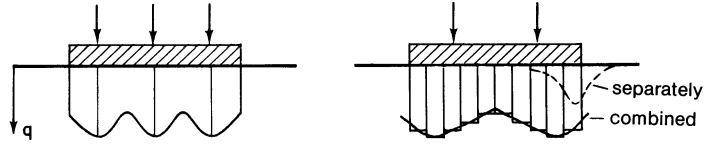


Fig. 290. Distribution of base pressure (soil pressure) under a flexible beam, by the bedding-index theory (*left*) and the rigidity-index theory (*right*)

of the foundation become increasingly displaced towards its edge. Consequently, the thicker and the more yielding the underlying layer of ground, the greater is the bending stress on the structure. The peak limit of marginal stress at the end of the foundation is set by the plastic flow of the ground (soil failure).

Because of interaction between deformation of the ground built on and rigidity of the structure, the rigidity-index method assumes that the line of curvature of the foundation coincides with that of the ground built on when this is behaving passively. In a mining area this condition is not fulfilled, owing to deformation of the ground. In the case of active ground deformation therefore, base-pressure distribution cannot be calculated from the rigidity of the system. For example, in cases where a rigid structure is exposed to upward warping of the ground, the concentration of stress at the edge of the foundation is compensated by the increased support at its centre.

A method of *base-pressure calculation* which can be used in mining districts has been devised for rigid foundations on the basis of the bedding-index theory (Rausch, 1955). For a foundation of length L , width b , and transverse load Q , the length a of flattening of the ground, the maximum amount by which it is compressed δ_0 , and the new base-pressure stresses q are found in a simple computer run.

As illustrated in Fig. 291, the following values result where a structure is exposed to *upward warping* of the ground:

Compression of the ground:

$$\delta = \frac{\sigma}{C_b} = \frac{q}{C_b b}, \quad (343)$$

content of the parabolic base-pressure area:

$$Q = \frac{2}{3} a q_0, \quad (344)$$

maximum compression given a sag $f = L^2/8 \rho_z$ [Eq. (114)]:

$$\delta_0 = f \left(\frac{a}{L} \right)^2 \quad (345)$$

and the length of ground flattening:

$$a = \sqrt[3]{\frac{3Q}{2bLCf}}. \quad (346)$$

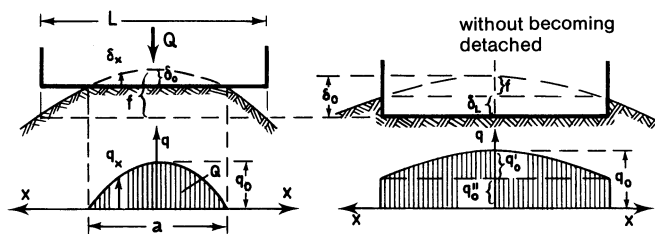


Fig. 291. The base pressure q where the structure rests on an upwarped saddle, by the bedding-index theory. (After Rausch, 1955)

In terms of the mean base pressure $\sigma_m = Q/L$ and of the mean subsidence $\delta_m = \sigma_m/C_b = Q/bLC_b$, the length of ground flattening is

$$a = \alpha L, \quad (347)$$

where the factor α has the value (Fig. 291, left):

$$\alpha = \sqrt[3]{1.5 \frac{\delta_m}{f}}. \quad (348)$$

This relationship whereby ground flattening decreases with increasing sag and ground curvature applies only so long as a is not greater than L .

The maximal values of base pressure and vertical ground compression are

$$\sigma = \frac{q_0}{b} = \frac{1.5}{\alpha} \sigma_m \quad (349)$$

and

$$\delta_0 = \frac{\sigma_0}{C_b} = \frac{1.5}{\alpha} \delta_m. \quad (350)$$

Finally, from the general formula

$$q_x = q_0 \left[1 - \left(\frac{2x}{a} \right)^2 \right] \quad (351)$$

the base pressure can be obtained for any desired position at a distance x from the focal-point axis. Thus the bending stress on the foundation can be calculated from the weight forces emanating from the structure and the ground's reactive pressure.

If α is greater than 1, the foundation no longer lifts off from the ground at its ends and, given upward curvature of the ground, a higher base-pressure stress occurs in the middle of the foundation than at its ends. In such a case the following values obtain, as in Fig. 291, right:

$$Q = \frac{2}{3} q'_0 L + q''_0 L \quad (352)$$

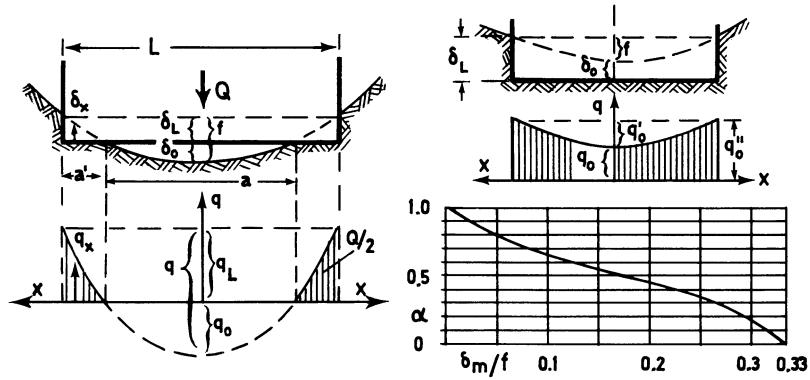


Fig. 292. The base pressure q where the structure rests on a downwarped trough, by the bedding-index theory. (After Rausch, 1955)

and

$$q_0'' = \frac{Q}{L} - \frac{2}{3} b C_b f. \quad (353)$$

In the middle of the foundation the greatest ground pressure and the greatest subsidence amount to:

$$q_0 = q_0' + q_0'' = \frac{Q}{L} + \frac{1}{3} b C_b f \quad (354)$$

and

$$\delta_0 = \delta_m + \frac{1}{3} f. \quad (355)$$

Subsidence at the edge of the foundation is:

$$\delta_L = \delta_m - \frac{2}{3} f. \quad (356)$$

Where *curvature is downwards* as a trough, the edge of the foundation sinks into the ground up to the point where the assumed triangularly shaped base-pressure stresses correspond to the load Q (Fig. 292). As this happens, the structure experiences a subsidence of

$$\delta_x = f \frac{4x^2 - a^2}{L^2} \quad (357)$$

at the position x . With the factor derived from the relation δ_m/f , the degree of stress and settlement at the edge of the foundation can be obtained:

$$\sigma_L = C_b f (1 - \alpha^2) \quad (358)$$

$$\delta_L = f (1 - \alpha^2) = f - \delta_0. \quad (359)$$

If these marginal stresses are greater than the plastic limit stress of the ground σ_{Br} , the triangular stress figure must be altered in accordance with $\sigma_L \leq \sigma_{Br}$ (so

that $Q/2 = \sigma_{br} a' b$). With a shorter trough length a , the bending stress on the structure become less intense.

Where the structure no longer lifts off from the ground when this is warped downwards, then – as in Fig. 292, right – the following values apply:

$$q'_0 = b c f \quad (360)$$

$$Q = -\frac{2}{3} q'_0 L + q''_0 L \quad (361)$$

$$q''_0 = \frac{Q}{L} + \frac{2}{3} b C_b f \quad (362)$$

$$q_0 = \frac{Q}{L} - \frac{1}{3} b C_b f \quad (363)$$

$$\delta_L = \delta_m + \frac{2}{3} f \quad (364)$$

$$\sigma_L = \sigma_m + \frac{2 C_b f}{3}. \quad (365)$$

The above sequence of calculations lends itself to estimating the bending stress imparted to rigid structures of all kinds by ground curvature resulting from mining, and to establishing whether foundations are sufficiently rigid. The overlapping of pressure as between different parts of the foundations does not have the effect of reducing accuracy in cases where the structure is an extended one standing on a layer of soft ground only a few metres thick, such that no significant trough of settlement can form. Where the subsoil is soft and extends to some depth however, the calculated results must be refined to take account of the trough of settlement, because the base-pressure stresses resulting from upward curvature of the ground (Fig. 293 b) can offset or even cancel each other out in a thick layer of ground (Fig. 293 c). On the other hand, where the ground is warped downwards, the two influences reinforce each other as the structure settles, and this leads to increased stress on the foundations.

In this connection it is also worth mentioning the relationship between the tilt of the ground and the *tilt of a building*. The inclination of a building in an area

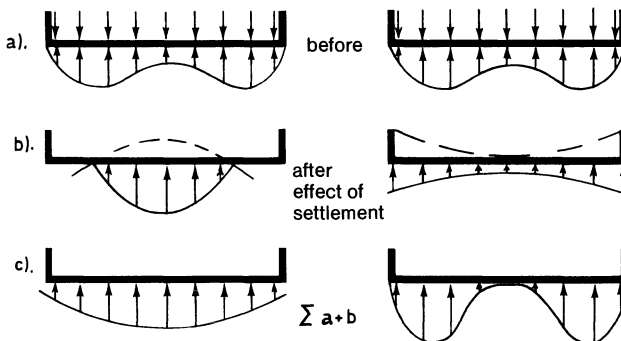


Fig. 293. Correcting the calculated base pressure, taking account of settlement of the structure. *Left*: on ground upwarped into a saddle. *Right*: on ground downwarped into a trough

affected by mining deviates from the tilt of ground that is under no load in proportion to the magnitude of the load represented by the structure and the relative hardness of the ground, since these two factors determine the extent to which the structure presses into the ground on one side. From trials with sand models (Ryncarz, 1963) the following statistical formula was found for the tilt of a building as a factor of its length L and its height H :

$$v'_{z\text{structure}} = v'_{z\text{ground}} \left(0.033 \frac{L}{H} + 0.45 \right) + 0.6 \times 10^{-4}. \quad (366)$$

11.6 Time-dependence in the Transmission of Forces

In contrast to elasticity, the passage of time has a considerable influence on the state of stress and strain in the ground.

Under a constant load, deformation at first continues to increase (creep): under constant deformation, ground stress recedes somewhat (relaxation). The deformation of ground saturated with water moreover, given a sufficiently rapid increase in compressive stress, is retarded by the flow resistance of pore water (consolidation) in pore channels still partially filled with air (three-phase soil). In cohesive soils, cohesion can diminish under a load that is maintained for a length of time: which can lead to displacement of grains and reduction in pore volume.

These time-related after-effects in the ground suggest that the rearrangement of grains and ultimate consolidation take a certain time t to complete and probably come to an end only gradually, in accordance with an exponential function such as $(1 - e^{-at})$, with a as a time constant. Each phase of movement in the ground set off by mining is thus associated, in the case of increasing compression, with a peak of earth pressure on the foundation walls. When the ground has returned to rest, this earth pressure relaxes somewhat under the delayed effects of changes in the texture and compaction in the granular composition of the soil. It then remains at that level for an unlimited time when the ground is completely at rest, provided there is no further rearrangement of grains as a result of traffic vibrations, percolating rain-water, frost expansion, or ground-water movements. Even after ground movement has come to an end, a structure can still settle to some extent (creep).

Where *ground compression* grows uniformly, the horizontal compressive stress on a foundation wall, starting from the level of earth pressure E_0 at rest (see Fig. 284), increases at first in a concave curve (curve b in Fig. 277), since the loose bed of sand poured into what was the construction trench must first be compacted to the density of "native earth". It then rises in a convex curve (curve a) to the point of rupture on relative displacement of the ground by 1 or 2 dm along the foundation wall. The passive earth pressure E_p reached at this point is incapable of being exceeded on further compression and curvature of the ground. Thus ground compression of double the intensity, which can be expected in the case of double the amount of subsidence and curvature after the mining of a second seam (Fig. 287, right), does not mean double the pressure on the foun-

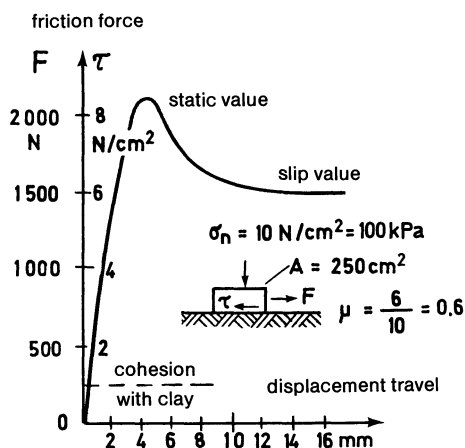


Fig. 294. Friction with a concrete slab moving on sand. (After Petermann, 1933)

dation wall, since there is no linear proportionality between ground deformation and earth pressure (Fig. 284).

On regressive ground curvature, in the sense of a decrease in ground compression, the load on the foundation wall drops very rapidly to a fresh value of earth pressure at rest, because of the only slightly elastic return-deformation of compressed ground; on renewed ground compression, however, the curve of earth pressure climbs steeply back to the original loading, because there is no longer any need for compaction (see Fig. 277). Later mine workings having an assumed compression effect of equal magnitude on the construction ground can thus produce both a small increase in pressure on foundations in the course of the initial loading movement and a very great increase in pressure on renewed loading, if the ground has been exposed to extension in the interim.

It is much the same with the *friction force* transmitted to the foundations. Trials with a concrete slab moved uniformly on sand, and loaded with a normal stress of $\sigma_n = 100 \text{ kN/m}^2$, show that friction and applied tensile force, after a very small displacement of 4 mm, reach a maximum value (static value) which derives from the interlocking of sand grains with the concrete surface and, in the case of cohesive soils, also from cohesion or bonding (Fig. 294). As soon as the interlocking resistance has been overcome on displacement of over 1 cm, and the grain axes have become arranged on the slip surface in the direction of displacement under the influence of translation and rotation movements, friction drops to a constant value (dynamic or slide resistance) which applies to greater displacements.

The force of friction between two solid bodies depends neither on the size of their contact surface nor on the rate of uniform slide movement but only on the force of contact. If the body adheres to the surface beneath, the force required to set it in motion is greater than the force that will maintain uniform motion⁴ (static friction > dynamic friction). In the case of bituminous slide layers having "inner friction", however, slide resistance increases with the rate of movement (see Sect. 14.3.2). The rate of ground displacement amounts at most to 1 mm/h:

⁴ In reality, the body does not slide uniformly but in jerks (stick-and-slip effect).

which implies a relative displacement of 0.5 mm/day at the end of a 50 m long structure on ground deformation of 1 mm/m. In the Ruhr, with rapid rates of face advance (over 3 m/day) a daily increase in compression of a maximum of 0.23 mm/m has been observed.

A relative displacement of 1 cm between a 50 m long, 2 m deep foundation and the ground will, as discussed earlier (Sect. 11.4), appear first at the end walls on linear deformation of the ground amounting to 0.45 mm/m, corresponding to a radius of curvature of 5000 m at the surface of the ground. This 1 cm position becomes displaced with increasing ground deformation towards the middle of the structure – for example, from a 25-m distance from the middle point at 0.45 mm/m linear deformation of the ground to 10 m at 1.0 mm/m, and to 5 m at 2.0 mm/m linear deformation (Fig. 295). From the ends A of the foundations to this position of 1 cm relative displacement at the point B, the local movement resistance, and consequently also the friction force transmitted locally to the foundations, are independent of further increases in relative displacement and of the rapidity of these. The whole friction force transmitted thus increases linearly at the angle α at this outer section A – B of the foundations. From the 1 cm relative-displacement position onwards, local transmission of friction force drops to nil at the middle M of the structure, giving rise to a degressive curve of combined friction force for the middle section B – M of the foundations (Fig. 295).

For this reason, the combined friction force, which increases up to the middle of the structure, is not completely independent of the amount of ground movement over time, although the increase in friction force becomes less and less as the 1 cm displacement position moves towards the centre of the structure. Each subsequent accretion of ground compression or extension thus produces, just as with earth pressure, an ever smaller increment of combined friction force transmitted. In general, where structures exceed 20 m in length and linear deformation of the ground exceeds 1 mm/m, the final value of transmitted friction force as in Eq. (341) can be reckoned with.

When the ground comes to rest, the transmission of friction force ceases. The elastic deformation produced by friction force up to that point in time in concrete foundations reinforced with, for example, steel inserts recedes somewhat in

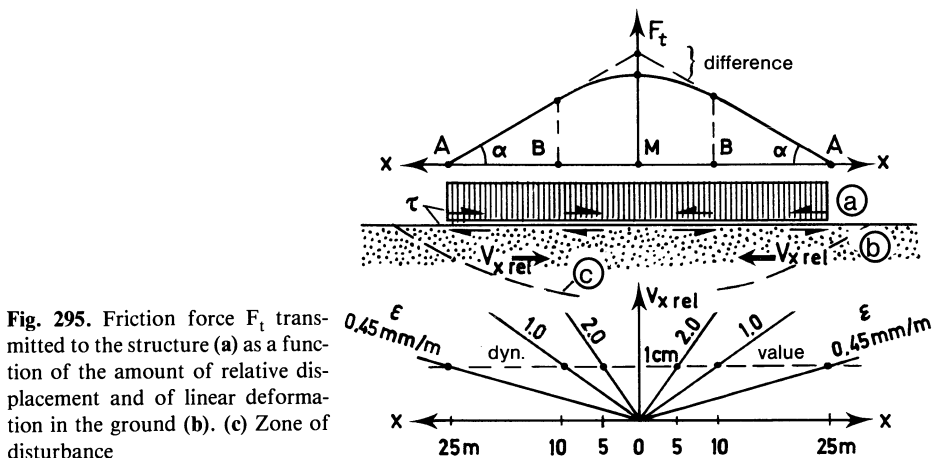


Fig. 295. Friction force F_t transmitted to the structure (a) as a function of the amount of relative displacement and of linear deformation in the ground (b). (c) Zone of disturbance

the event of a gradual decrease in static shear stress. The greater part of the elastic deformation, however, persists for a long time. By contrast, a part of the friction force transmitted to masonry foundations is consumed, even during ground movement, as plastic deformation in the joints and contraction fissures opened up by the bending of the structure, with the result that after abandonment of working only a small amount of elastic deformation energy remains effective in the masonry. A noticeable elastic return-deformation of masonry cellar walls has been observed in the zone of compression after artificial destressing of the ground by means of boreholes.

On a reversal of the direction of displacement, or after a long period of ground repose, a heightened initial resistance (repose value) can set in again, though on the short friction path of $v_{xrel} = 3-6$ mm (Fig. 294) it no longer greatly influences the total friction force. With decreasing ground compression or extension, elastic deformation of the foundations is reduced in like degree (relaxation). The friction forces exerted on foundations by several mine workings, either simultaneously or consecutively and all in the same direction, overlap degressively – i.e., the linear deformation of the ground induced by a second mine working places less stress on the foundations than does that induced by a first working. The law of superposition thus does not apply to an increase in either earth pressure or friction force. A distinction should therefore be drawn between, on the one hand, a time factor for horizontal ground movement as a function of the spatial position and size of the mine working and, on the other, a time factor for the force transmitted to the foundations in relation to the magnitude of ground linear deformation and curvature, as well as the extent, thickness, and depth of the foundations themselves. These considerations apply also to bending forces in a structure exposed to either upward or downward warping of the ground.

At the trough centre the ground becomes compacted under concave curvature, and earth pressure on foundations increases correspondingly. Concave curvature v_z'' will, like tilt v_z' , increase linearly with the intensity of mining influence, when subsidence of the three surface points A, B, and C in Fig. 296 doubles from 7–13–10 cm to 14–26–20 cm and trebles to 21–39–30 cm. If elastic behaviour is assumed in the ground, the bending stress – σ_x compacting the ground horizontally can be taken as proportional to curvature, since both depend on the

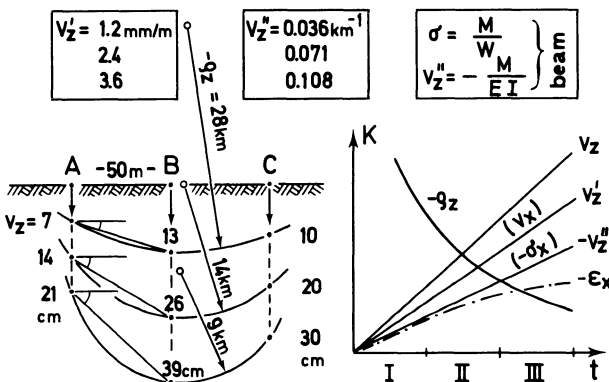


Fig. 296. Linear increase in tilt and curvature on uniform increase in subsidence at the points A, B, and C

bending moment M . Compaction force and therefore earth pressure thus increase linearly with mining influence. Compaction travel, however – i.e., compressive strain $-\epsilon_x$ in mm/m – will depend to a less than proportionate degree on curvature and compaction force, according to the stacking pattern of sand grains and the volume of pore space (curve b in Fig. 277).

Figure 297 illustrates the development over time of ground deformation forces in a long structure on which three mine workings act one after the other, exerting either a compressive or a tensile stress. In this *example* the transmitted friction force, in contrast to earth pressure which recedes only to E_a , drops to nil after the transition from compressive to tensile ground movement and creates a tensile force in the structure so long as tensile stress predominates in the ground. While friction force is increasing, earth pressure remains at an unchanging minimum. The peak friction force in the middle of the structure, which exceeds earth pressure on the foundation walls, is reached at around 2 mm/m linear deformation of the ground and before the state of passive earth pressure is reached. On the reimposition of load, the earth pressure which was acting at the position c is recreated at a ground compressive strain of 1 mm/m instead of only at 2 mm/m as previously. Because of sand compaction and minimal elastic return-deformation, the repose pressure for earth reappears, during the load-relief phase, as early as a ground compressive strain of 0.5 mm/m. The sector of slip surface 1 shown in the ground section depicted at bottom right shears off during the initial loading phase a–c. During the load-relief phase d–e there appears in the loosened ground the slip surface 2, along which the wedge of ground slides down to the foundation. During the phase of second loading f–g the ground in front of the foundation, which has been contracted by more than 10 cm, is caused to

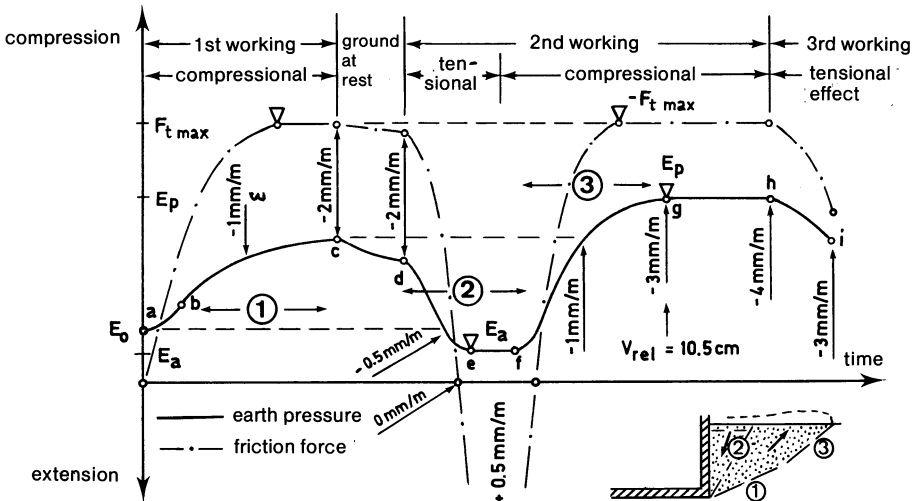


Fig. 297. Ground forces acting over time on a 70 m long structure during the period of influence of two mine workings. a–b Pre-compaction; b–c rearrangement of sand grains; c–d relaxation; d–e load relief to the level of active earth pressure, and reversal of relative displacement; f–g reimposition of load up to passive earth pressure; h–i load relief. *Bottom right:* formation of wedges of ground under passive (1 and 3) or active (2) earth pressure

well up and produce a ground failure at the rupture surface extending to position 3.

The action of deforming ground on the foundations of a structure, the theoretical bases of which have been solved, leaves a number of *questions* unanswered if, instead of the peak values E_p , F_{tmax} , and q , the values for deformation over a period have to be found. Whereas the structure follows ground movement closely, and changes in its position (subsidence, tilt, displacement) can thus be predicted sufficiently accurately, the factor governing the transmission of ground movement (curvature, compression, extension) to structures is not known in all cases. The following are some examples of what is still lacking in the way of numerical values derived from measurements on structures in areas affected by mining:

ground forces E below the level of passive earth pressure E_p which act on foundation walls on very slight compaction of the ground ($a < 10$ cm);

the delayed effects over a period of ground deformation and the transmission of forces (creep, relaxation);

elastic deformation forces left behind in the structure, and how they overlap with newly transmitted forces;

the increase in friction force with increasing linear deformation of the ground;

stresses on the structure and the ground which alternate in direction and magnitude (transition from extension to compression, from convex to concave curvature, and *vice versa*);

ground movement in a foundation's zone of disturbance; and

changes in base pressure underneath rigid structures as compared with calculated ground curvature.

Here then is a wide field for future research.

12. Mining Damage Above Ground

12.1 Types of Damage

Movements and deformation of the ground produce various forms of damage at the populated surface, depending on whether residential houses, industrial installations, communications networks, public utility supply mains, or agricultural land are concerned. Damage to buildings results from tilting, curvature, and linear deformation of the ground built on. With the majority of communications installations and canalized waterways, it is subsidence and the compression or extension of the subgrade which alter the gradient of their alignment or deform the rails, road surfaces, and conduits concerned to the point of failure. With underground cables and pipelines, on the other hand, it is only linear deformation which causes damage. Subsidence on its own can cause mining damage to fields, meadows, drainage channels, canals, and watercourses. Damage resulting from ground displacement is found only in service piping branching off to individual buildings and in extensive conveyor-belt installations (Table 20).

In hilly mining districts, damage to buildings and communications installations will be more important than damage connected with ground-water levels. In flat land, on the other hand, approximately half of all *mining damage costs* arise from putting right interference with the natural flow of surface waters. The impact of mining on the natural waterway system, which appears locally as a relative rise in the groundwater level¹ and on a wider scale as a reversal of the flow gradient, can in flat land occasion just as much damage in terms of cost as the effects of mining on structures of all kinds. If it is further remembered that a large part of the damage to structures can be ascribed to differential subsidence – tilt in buildings, alterations to gradients in railways, and changes in altitude in sections of canal – it will be clear that subsidence constitutes by far the most damaging component of ground movement.

By contrast, the cost of repairs to fractures in buildings, railway lines, and pipelines resulting from ground curvature, compression, and extension is put at a maximum of 30% of total expenditure on damage in flat land. This contrast between 70% subsidence damage and 30% deformation damage underlines the importance of the pillar and stowing method as the only means by which subsidence, the largest item of mining damage costs, can be reduced (cf. Sect. 14.2). In the next few sections below, the various forms of damage at the surface when affected by mining are discussed in the order in which they appear in Table 20.

¹ The impression that the ground-water level has risen arises from the fact that the ground surface, as it sinks, is brought nearer to the water table, which for its part is maintained at the same level by water flowing in from the surrounding mining-free district.

Table 20. Mining damage at the surface, and the influence of the separate elements of ground movement on surface objects

Surface object	Sub-sidence	Tilt	Flexure (bending moment)	Displace- ment	Exten- sion (friction)	Compression (friction, earth pressure)
Residential houses		X	X		X	X
Office blocks		(X)	X		X	X
Row of houses		X	X concave		X	XX
Machine shops			(X)		(X)	(X)
Furnace installations		X			XX	X
Coking plant		X	X		XX	X
Blast furnaces		XX			X	X
Machinery		XX				
Assembly lines		X				
Rolling mills		XX				
Conveyor belts				X	X	X
Chimneys		XX				
Pipeline crossings				X		
Crane installations		XX		X		
Reservoirs, tanks		X	XX		X	
Cooling towers		X	XX			
Water treatment plant		XX			X	X
Railway lines	X				X	X
Railway stations	X	(X)			X	X
Roads	(X)				X	X
Canals	XX					
Locks	XX	X	X		X	(X)
Bridges	X	(X)		X		
Tramways					X	X
Underground railways	X		(X) tunnel		X	X
Pipelines					X	X
Water mains				(X)	X	
Sewers	XX			(X)	X	X
Gas mains	(X)			branch-off	X	X
Underground electri- city cables					(X)	X
Fields	X					
Meadows	X					
Woods	X					
Drainage trenches	XX					
Outfalls and the natural flow of surface water	XX					

Key: X medium, (X) slight, XX high sensitivity

12.2 Mining Damage to Buildings and Fittings

A building is not only caused to subside and be horizontally displaced by mining; it is also tilted, bent, and compressed or extended. Bending and linear deformation (extension, compression) always affect the structure simultaneously.

Figure 298 illustrates the *change in stress* on a house before, during, and after passage of the face under it. At mining stage I, the house stands ahead of the extraction front in the zone of maximum tensile strain and maximum convex curvature. Through the friction force resulting from displacement of the ground, which over the forefield increases in the direction of the area of extraction, the structure is exposed to tensile stress. This tensile stress is intensified by convex curvature of the ground over the forefield ahead of the face. At mining stage II, shortly after passage of the face, the structure is at maximum tilt and is also displaced towards the area of extraction. Curvature in the vicinity of the profile transition point is nil. At mining stage III, the structure is exposed to concave curvature and is compressed by earth pressure. In addition, the friction force resulting from ground displacement, which over the working area decreases in the direction of the centre of extraction, exerts a compressive stress. At the centre of the trough, deformation and tilt later recede, and subsidence is at a maximum

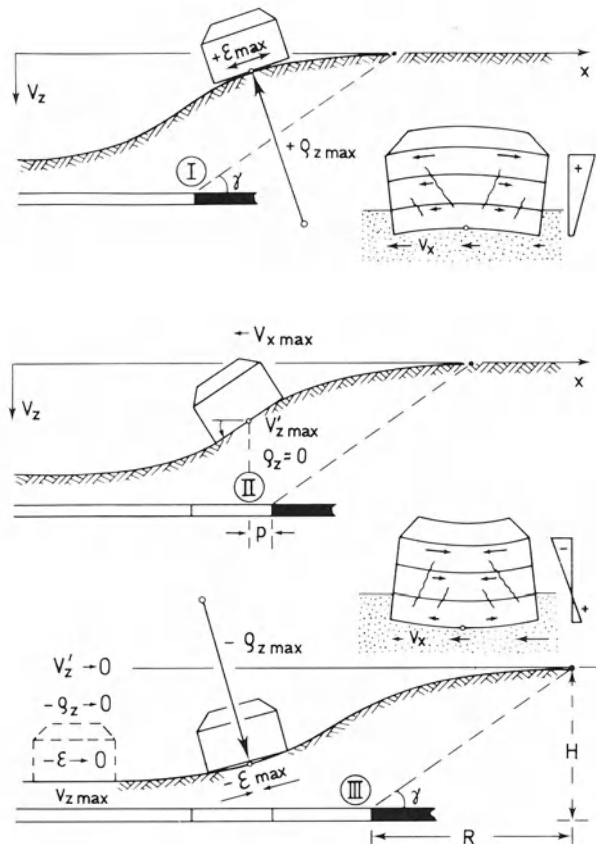


Fig. 298. Movement and deformation of a structure as the face passes under it

there. If the structure lies, not ahead of the approaching extraction front but, from the beginning of mining, over the working area or over the zone outside the stationary ribside (Fig. 142), then the nature of stress on the structure undergoes no change (Fig. 123).

A structure is subjected to no static stress if subsidence of its foundation seating – the levelled subgrade – is completely uniform; mining damage can nevertheless occur if the supply arrangements (approach ramps, water mains, etc.) are adversely affected by changes in gradient. Furthermore, if there is a relative rise in the ground-water level, cellars that have not been sealed off may become flooded, and damage costs may arise for pumping out water, dealing with damp in walls, or compensating for loss of resale value.

Where the site subsides evenly but at an angle, the dead weight of the tilted structure and its service load will no longer sit plumb and centrally on the bearing members. Only with very severe tilt or very high buildings, however, will the disturbed equilibrium lead to reciprocal displacement of bearing members like columns and beams and thus to a significant reduction of stability. If the foundations are not righted and levelled with hydraulic presses or screw jacks, the resulting complications in utilizing the slant-lying floors of the building will represent permanent damage (decrease in value). Damage of this nature makes itself adversely felt, in the case of a residential building, only from 2 mm/m tilt onwards (= 1‰ decrease in value), but in the case of a preparation plant with machinery sensitive to deviations from the horizontal, from as little as, say, 0.5 mm/m onwards. In such installations therefore, devices for the lifting of machinery or even for levelling the entire machine shop must be incorporated, at great cost. A tilt of the order of >5 mm/m can affect the working of lifts.

12.2.1 Structural Dislocation and Expansion Cracks

Dislocation, leading to gaping cracks or visible displacements, of closely joined, bearing members such as foundations, walls, joists, floors, or roof trusses, and of their constructional elements such as bricks, concrete, steel, or timber – known as “structural dislocation” – can occur in the event of warping, compression or extension of the foundation seating (Fig. 299). *Structural dislocation* can bring about lasting damage (decrease in value) to the load-bearing framework, or remediable damage (repair costs) to non-bearing partition walls and to plumbing and wiring. First the material-used for effecting joints – mortar, nails, rivets, braces, clips, and straps – will come apart at the contact surfaces and joints of the individual structural elements and members; in cases of severe mining damage the line of fracture will also pass through structural elements (bricks) and even through some bearing members. The loosened cohesion of the individual bearing elements and members reduces the stability and life of the structure, which could otherwise be expected to last 100 years. The resulting material loss of several percentage points in resale value must be borne by the mine owner.

In the case of residential buildings of small ground plan and up to three storeys in height, structural dislocation in the load-bearing framework is usually not significant: its contribution to lasting damage is included in the decrease in

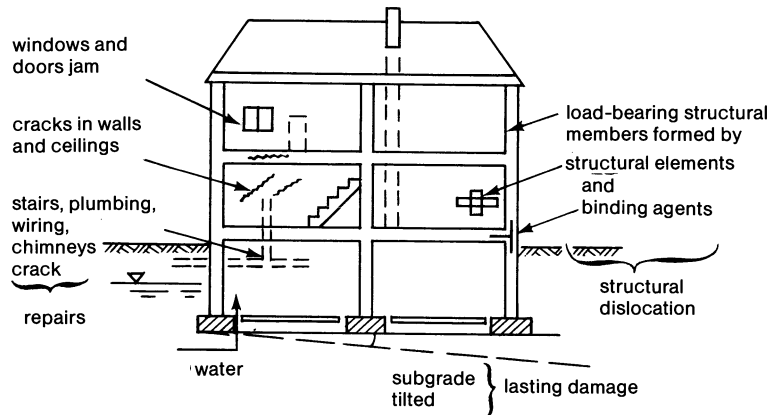


Fig. 299. Types of damage to a structure caused by mining

value as measured by the amount of tilt (Sect. 13.3). Damage to non-load-bearing parts of the building can usually be put right (restoration of appearance) and does not constitute lasting damage carrying the duty of replacement. In Germany in 1979, the cost of repairs to residential buildings associated with ground compression or extension of >1 mm/m is estimated to have been DM 4000 per house on average, for work done to structural framework, masonry, and paintwork².

Since the number and closeness of wall and ceiling cracks does not readily permit conclusions to be drawn concerning the degree of lasting damage to the load-bearing frame of a building, permanent dislocation to bearing members in large structures must be investigated separately in each case. Only in this way can conclusions about loss in value be derived from shortened expectations for the life of the structure. The investigation must be extended to all bearing parts. Identified structural dislocation is classified into repairable damage and permanent damage. *Repairable damage* includes plaster cracks and transverse cracks in bearing masonry or concrete walls, ceiling cracks in the plastering, and roofs rendered leaky by bending. Renewal is generally required for longitudinally and transversely cracked masonry pillars, tilted and cracked staircases, jammed and draughty windows and doors, and plaster-covered piping and wiring strained by bending or by axial forces (which can have serious consequences). There is *permanent damage* to the foundations if the reference pegs reveal a crack of more than 5 mm width which is found to continue under the cellar floor when this is opened up, and if reciprocal and persistent displacement of parts of the foundation is visible, hollows or fissures have formed in the ground under the foundations, and large cracks appear under beam abutments. Where a wall of quarry stones has fractured along slip planes, or the webs of hollow blocks in a wall have broken and become mutually displaced, the stability of the wall concerned is always reduced. Furthermore, in the case of floors under shear or tensile stress, irreparable dislocation of the structure is recognizable in the reduced bearing-surface of joists, panels, caps, or finished parts of flanges.

² In 1979, DM 100 was equal to £25 sterling or US\$ 55. Altogether 30,000 cases of damage in 1981 in the Ruhr District.

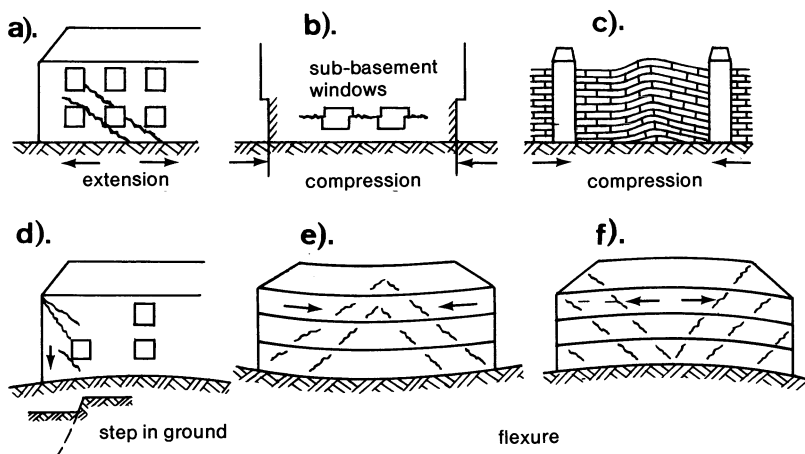


Fig. 300. Development of cracks in the walls of a house

Chimney cracks extending through several storeys are a sign of structural dislocation in the rest of the masonry and reduce the value of the building, even after gas-tight repairs (Drisch, 1972).

A characteristic of tension cracks in walls is their mostly steep path at angles of $60^\circ - 70^\circ$ (Fig. 300). By contrast, cracks attributable to compression run more or less horizontally, while in severely compressed walls or floors, the brick or tile courses are folded upwards. In walls with existing weak zones at the openings through them, cracks run diagonally from the corner of one window or door to the corner of another above or below, following the line of least resistance. In structures or parts of buildings of considerable linear extent there are in addition bending cracks, which follow a bell-shaped course in longitudinal walls in the case of downward curvature. Upward curvature of a wall finds expression in cracks running diagonally outwards, and in buildings with no stiffening in the ceilings the corners threaten to break off in wedges. A symmetrical pattern is thus characteristic of bending cracks (cf. the photographs in the Appendix).

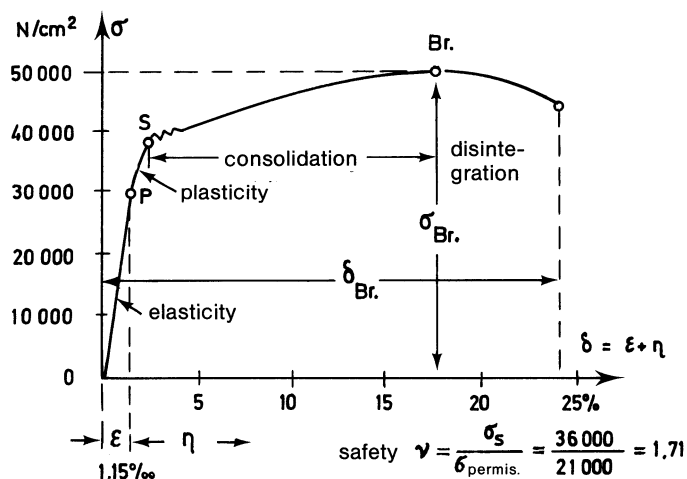
Under uniaxial stress, deformation-at-rupture and *strength limits* are approximately as follows, if the first visible cracks in the material or in the bonding are taken as the criterion of rupture:

for *brick walling* with lime mortar (modulus of elasticity $E = 1,200,000 \text{ N/cm}^2 = 12,000 \text{ MN/m}^2$): 1 mm/m compaction at rupture, or $1,200 \text{ N/cm}^2$ (12 MPa) compressive strength, and 0.5 mm/m extension at rupture, or $600 \text{ N/cm}^2 = 6 \text{ MN/m}^2$ tensile strength;

for *mass concrete* ($E = 2,500,000 \text{ N/cm}^2 = 25,000 \text{ MN/m}^2$): 1.5 mm/m compaction at rupture, or $3,750 \text{ N/cm}^2 = 37.5 \text{ MN/m}^2$ compressive strength, and 0.15 mm/m extension at rupture, or $380 \text{ N/cm}^2 = 3.8 \text{ MN/m}^2$ tensile strength;

for *structural steel* in concrete work ($E = 21,000,000 \text{ N/cm}^2 = 210,000 \text{ MN/m}^2$; Fig. 301³): 12 mm/m flow boundary, or $50,000 \text{ N/cm}^2 = 500 \text{ MN/m}^2$ stretching

³ Elongation of 1 to 2 mm/m is acceptable in structural steel.



limit (point S), and 140 mm/m breaking limit, or $70,000 \text{ N/cm}^2 = 700 \text{ MN/m}^2$ compressive or tensile strength (point Br.).

In general, bending strength amounts to 1/20 of compressive strength.

For the deformation-at-rupture ε_{Br} and strength limits σ_{Br} mentioned, the critical length of structure producing rupture at its centre (see Fig. 286) can be determined using the formulae in Sect. 11.4:

$$l_{\text{crit.}} = \epsilon_{\text{Br}} \frac{2ES}{\mu F_n' 1000} \quad (367)$$

$$= \sigma_{\text{Br}} \frac{2S}{\mu F'_n}. \quad (368)$$

In this it is assumed that linear deformation of the ground exceeds the rupture-deformation limit of the structure, and that relative displacement at the end of the structure amounts to at least 1 cm.

Thus, for *example*, for a three-storey, 15 m wide structure ($F'_n = 645 \text{ kN/running metre}$) with a concrete cross-section of $S = 30 \text{ cm}^2/\text{1 cm}$ foundation width, one obtains a critical structure length, by Eq. (367), of 0.15×530 or, by Eq. (368), of $380 \times 0.21 = 80 \text{ m}$ at which the concrete breaks in the middle of the foundation under the tensile force of friction. For a ten-storey block ($F'_n = 1,650 \text{ kN per running metre}$) the critical length of structure, given the same foundation cross-section, is reduced to approximately 32 m.

These rough estimates make it clear that friction forces can in general compress or extend only very long and low-strength or heavily loaded foundations to the point of rupture. Many instances of damage to foundations or cellar walls thus arise, not through any overstressing of constructional elements in terms of tension or compression, but through bending, arching, or tilting of the component parts.

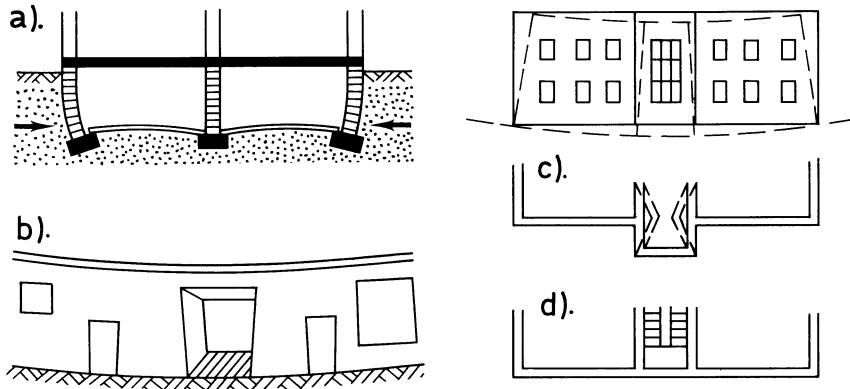


Fig. 302. The buckling of sub-basement walls (a) and of door pillars (b) under earth pressure; also deformation of a projecting structure (c) on flexure of the ground. (d) Correct ground plan with internal stair-well. (After Luetkens, 1957)

For example, if the footings of walls and supports are not based on a continuous raft and are unable to oppose sufficient resistance to horizontal *earth forces*, they will be set aslant and bent inwards or outwards (Fig. 302). Thin cellar walls lying in the direction of displacement may buckle inwards under severe compressive stress. Cellar walls standing at right angles to the earth pressure, on the other hand, are only caused to bulge inwards and not set aslant if the cellar base and ceiling are stiffened horizontally. Tar paper laid in the lower and upper courses of the brickwork as protection against rising damp may act under earth pressure as slip joints, along which the cellar wall may be pushed inwards by as much as a decimetre if it is not bonded or anchored in a form fit with base and ceiling. Strip foundations on a grid pattern become stressed through lateral bending at right angles to the direction of displacement and through axial thrust or pull in the transverse strips. Because of the stiffening effect provided by the ceilings of each storey, damage caused by earth forces is mostly confined to the cellar and the ground floor. In a less rigid structure, by contrast, bending cracks and distortions of the load-bearing frame are propagated through several storeys.

12.2.2 Bending Cracks

Many *settlement cracks* are to be found in structures in areas free of mining where, apart from earth pressure at rest, there are no horizontal earth forces acting on the structure. This would indicate that, in mining districts also, probably far more cracks in buildings are caused by folding of the ground than has been suspected up to now. Experience shows that the first cracks in a building of length l (in metres) resulting from bending, or from differential settlement of sections of the foundations, become visible at an arc height of 0.001 to 0.002 of l , or a radius of structural curvature of 30 to 60 times l . Accordingly, for a brick-work house front 35 m long (see Sect. 11.5), settlement differences of around

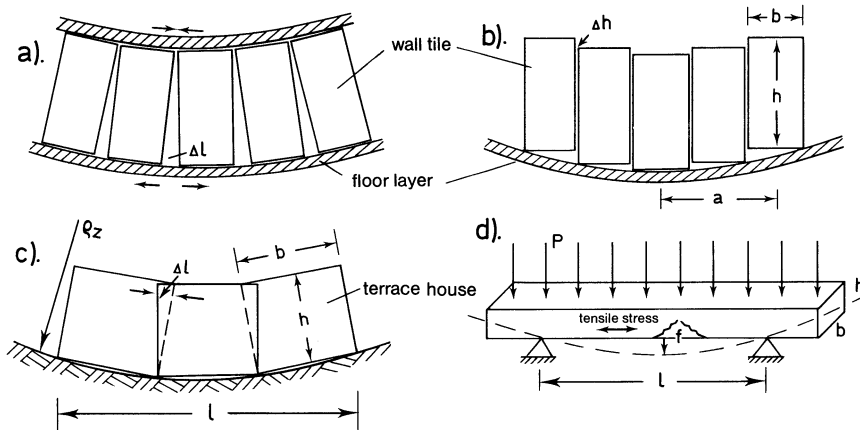


Fig. 303. Spreading of wall slabs (a), and vertical displacement of structural elements (b) or compression (overlapping) of gable ends of terrace houses (c) on flexure of the ground. Tensile stress in a bent beam (d)

3.5 – 7 cm or radii of curvature of around 1 – 2 km will cause damage. On the bending of a 60 cm thick strip of foundation, damaging extension of 0.15 mm/m at its upper or lower surface will also, by Eq. (251), occur at a radius of curvature of 2 km (see Fig. 219, top). Folding of the ground such as this can take place in the event of near-surface working or of repeated working through the rock mass at different levels. In underground mining at depths of over 500 m, folding of the ground acts not so much on individual walls as on the whole bearing structure of long and high buildings with insufficient horizontal stiffening. It also adversely affects concrete structures having no bending strength and sensitive to bending tension, as well as terrace housing.

Thus, on downward warping of the ground, severe damage occurs at the gable ends of a row of terrace houses where they touch and press against each other. Horizontal contraction of the building ground at the cellar level, on the other hand, often cancels itself out in weak spots of the foundations such as openings and passages (Fig. 302) and, once a horizontal shear surface has formed below the level of the ground-floor ceiling (Fig. 300b), is not transmitted to higher parts of the structure. For two adjoining, 20 m long and 10 m high terrace houses, horizontal overlapping Δl in the case of downwarping of the ground, or spreading in the case of upwarping, amounts to as much as 2 cm at radii of curvature of $\rho_z = 10$ km. The resulting relative extension or compression

$$\pm \varepsilon = \frac{\Delta l}{b} = \frac{h}{\rho} \quad (369)$$

in a terrace house of width b , or in a storey wall, increases in proportion to the height h (Fig. 303 a, c). This strain is nevertheless independent of the length of the structure and amounts, with a storey height of $h = 3$ m and a radius of curvature of $\rho = 10$ km, to approximately 0.3 mm/m. Extension of 0.3 mm/m, if distributed uniformly along brick walling, gives rise to hair cracks of 0.08 mm width in mortar joints between bricks of length 25 cm.

Adjoining parts of a building of width b can moreover be displaced vertically along vertical joints as a result of ground flexure (Fig. 303 b). The amount of displacement Δh arising in this way, which increases linearly in proportion to the width b and the distance a from the centre point (apex) of the structure, is given by

$$\Delta h = \frac{a b}{\rho}. \quad (370)$$

For example, in a row of structural elements of width $b = 5$ m, the vertical displacement produced as between the third and fourth element counting from the centre ($a = 15$ m) amounts, at a radius of curvature of $\rho = 10$ km, to 7.5 mm.

These horizontal tilt movements and vertical shifts of position in the bearing parts of buildings can cause severe mining damage to joints and junctions. Damage frequently also occurs in residential buildings if, in designing the horizontal layout of the planned structure, no account is taken of later warping of the ground and the front exterior is interrupted by projecting structures – for example, for a stair well. If the ground later becomes warped in the longitudinal axis of the building, the projecting part will become separated at the corners from the house front, because it will be able to deform unhindered as a separate element owing to the gap in the wall (Fig. 302 c).

From the knowledge that structural damage resulting from linear deformation ε of the ground increases with the length l of the structure, whereas the *degree of damage* stemming from flexure ρ_z depends decisively on the height h of the structure, and that both components share in the damage in a definite relationship, the attempt has been made to gather both values into a single formula

$$l \left(\varepsilon + \frac{h}{\rho_z} \right) \leq 100 \text{ mm} \quad (371)$$

with the object of assessing the permissible measurements of a bearing masonry wall, given pre-calculated ground deformation (Belyaev, 1968). Conversely, a rough estimate of acceptable ground deformation for an existing structure can be given. Thus, for $l = 20$ m, $h = 5$ m, $\varepsilon = 2$ mm/m, and $\rho = 5$ km, one obtains, using Eq. (371):

$$20,000 \left(2 \times 10^{-3} + \frac{5,000}{5,000,000} \right) = 40 + 20 = 60, \text{ i.e. } < 100 \text{ mm}.$$

As compared with the acceptable deformations listed separately in Table 30, Eq. (371) has the advantage of taking account of the interaction of linear deformation and flexure in the causing of damage.

Since it was shown in Sect. 11.4 that frictional tensile force does not come into consideration as a cause of damage for many buildings, and in particular for light buildings and short buildings, the magnitude of *bending tensile forces* to be expected on flexure of the ground will now be examined. A sagging concrete beam, if its tensile strength is exceeded, fractures starting from its upper or lower external surface. At that point in the case of a reinforced concrete beam, it is the

reinforcing steel alone which takes up the tensile forces. An exact calculation of the sagging of a concrete foundation is not possible, because of the dependence of concrete's modulus of elasticity on the loading, and because of shrinkage cracks and local variations in bedding index. As a rough estimate, the span or self-supporting length l of a sagging concrete beam can be obtained from the formula

$$\sigma = \frac{M}{W} \quad (372)$$

for a particular line load p , cross-sectional width b , and height h at which the tensile strength of concrete $\sigma_{Br} = 380 \text{ N/cm}^2$ (3.8 MN/m^2) is reached on the underside of the beam (Fig. 303 d). For this purpose we introduce

$$M = \frac{p l^2}{8} \quad (373)$$

for the bending moment M in the middle of the beam and

$$W = \frac{b h^2}{6} \quad (374)$$

for the moment of resistance $W = I \div h/2$ in the case of the rectangular cross-section, and the formula is solved for l as follows:

$$l = \sqrt{\frac{4}{3} \frac{\sigma b h^2}{p}}. \quad (375)$$

On this basis, a concrete foundation wall 50 cm wide and 2 m high, when loaded with a uniform pressure of $20 \text{ N/cm}^2 = 0.2 \text{ MN/m}^2$ (a line load of $p = 1000 \text{ N}$ per running cm or 100 kN/m), will fracture on the underside at a span of as little as 10 m. Subject to elastic behaviour, and ignoring bedding pressure, the wall will bend in the middle by

$$f = \frac{5}{384} \frac{p l^4}{E I}. \quad (376)$$

For the surface moment of inertia of the rectangular cross-section the following relation, as is known, applies:

$$I = \frac{b h^3}{12}. \quad (377)$$

Subject to the above qualification for concrete beams, this elastic sagging would correspond to a radius of curvature of

$$\rho_z = \frac{l^2}{8 f} = 7.8 \text{ km}. \quad (378)$$

On even more severe bending, the wall fractures up to the neutral axis. *Clearance* under a foundation wall such as this therefore needs to amount to only

$f = 1.6$ mm for the stress to reach the limit of concrete's tensile strength. For a 30 cm high foundation crosspiece of similar width and loading, the limit values are $l = 1.5$ m, $f = 0.25$ mm, and $\rho = 1.1$ km: from which the influence of foundation height is evident. Structural elements in concrete are thus very sensitive to bending. Concrete will crack at arcs only mm high – a fact which underlines the importance of flexure as a contribution to mining damage.

Where ground built on is torn open under severe ground flexure and a high tensile strain of around 7 mm/m, particularly serious consequences for structures will follow as soon as a step-like drop in height develops and a part of the foundation stands free (Fig. 300d). There is then a fear that a corner or side of the building lying over the step in the ground will break off, unless the floor over the sub-basement is underpinned in good time with a jacking device and cracks are prevented from developing in the brickwork higher up. Devices such as this are built into several recesses of the statically indeterminate structure at a distance apart of over 1.5 m. As the foundation subsides over the step in the ground, the apparatus (load capacity 45 Mg) can automatically even up a difference in height of up to 2 cm between the subsiding sub-basement roof and the ground floor above. It then has to be readjusted. In this way the part of the building which otherwise would break off over the ground step can be maintained at its original height, until the separation joint can be filled with concrete when the ground ceases to move and returns to rest.

12.2.3 Alternations in Stress

As shown in Fig. 298, a structure lying ahead of the working front is exposed first to tensile stress and convex curvature, then to compressive stress and concave curvature. Its constituent material is thus subjected first to tension and then to compression. Joints and cracks are opened up, only to be closed again later.

This change from tension to compression is more damaging to a structure than stress of a single kind which gradually increases with enlargement of the area mined, allowing the rearrangement of forces in the structure to proceed slowly. The rate at which external forces like earth pressure, friction force, and bending forces acting on the structure increase, and the duration of tensional or compressional action, depend in part on the mining depth, thickness of seam mined, and position of the structure in relation to the working but principally on the rate of daily face advance.

Rapid *face advance*, at several metres a day, has the effect of increasing the damage to structures, because the alternation between tensile and compressive stress is then confined within a relatively short time span, as shown diagrammatically in Fig. 304. When face advance is at 5 m/d, the change in loading from the peak of tensile stress to the peak of compressive stress is spread over only 60 days, as compared with 313 days when advance is at 1 m/d. Within as little as 15 days after passage of the face – i. e. at a distance of 80 m behind the face edge – the extension goes over to compression, and this rises rapidly to its critical value. At the same time an even greater, permanent compression develops in the transverse direction, which the structure frequently cannot withstand.

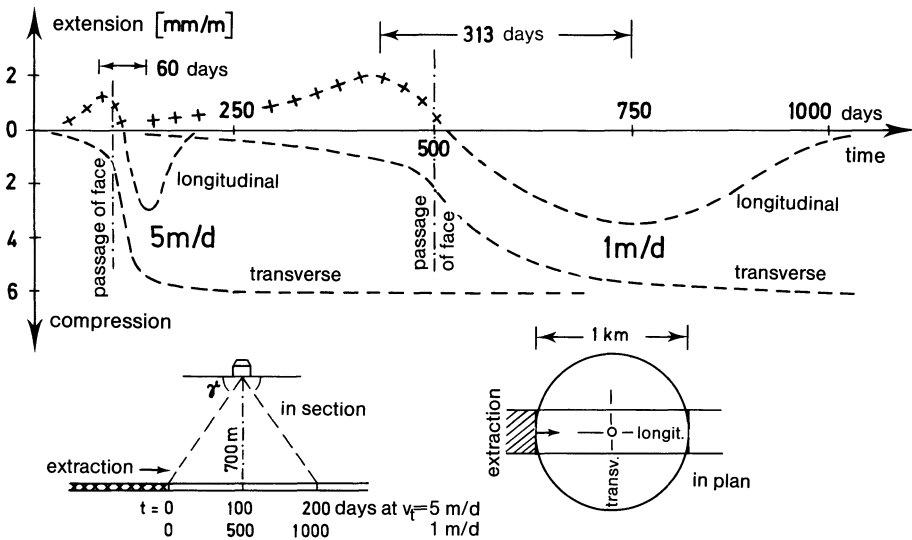


Fig. 304. Dynamic extension and compression longitudinally and transversely to the mine working under a daily face advance of 1 m and 5 m

The rapid change in loading leads to damage in the structure if

- the transition from extension to compression, and from convex to concave curvature, during the two months (60 days) in question coincides with expansion in the structure in the same direction resulting from heat or humidity;
- the ground built on and the soil adjoining the foundations do not have sufficient time for consolidation, adjustment of grains to the change in stress field, and reduction in pore volume;
- the mortar in joints is left insufficient time for a reduction in stress through plastic deformation; and
- as a result of the rapid flexure-deformation of the structure, a few large cracks develop instead of a network of finely distributed cracks.

Damage in the structure will develop above all in a transverse direction, because the rise in compression is particularly strong parallel to the narrow side of the panel (Fig. 304). The change in the magnitude and sign takes place in a similarly rapid way over the advancing face in the case of tilting and curvature (Fig. 298). At shallow depths with little overburden, a rapid face advance in jointed, thickly bedded strata can even lead to a shear movement (de-consolidation) reaching up to the base of the structure (see Fig. 149).

To a certain extent, rapid advance, by promoting fracturing and ease of bending in the strata, also contributes to the formation of a funnel-shaped trough outline with steep sides after repeated working through the same block, and thus to costly mining damage resulting from the greater tilt, flexure, and linear deformation (see Sect. 6.5 and 10.2.2). On the other hand, where delayed subsidence in unfractured strata is considerable, the greater the rate of face advance, the smaller is the degree of ground deformation over the extraction front (Fig. 254).

It is only on the basis of the foregoing observations, which today are still not plentiful, that a decision can be reached in individual cases as to how far a more rapid rate of face advance will produce flatter or steeper trough sides over the working face.

12.3 Mining Damage to Industrial Installations

Mining damage to industrial installations differs from more general damage to buildings in that, when repair or preventive measures become necessary, production frequently has to be reduced or even suspended – a fact which may have heavy consequential costs for the mine causing the damage. Furthermore, restoration work in manufacturing plant is much more costly and protracted than “cosmetic repairs” to residential and administrative buildings, because it mostly has to be carried out while work continues in order to avoid a loss of output.

Damage of *furnace installations* is especially onerous. Among these may be counted in particular coke ovens, metallurgical furnaces, ceramic firing kilns, and brick-making tunnel kilns. “Hot repairs” to such installations while in full operation are seldom possible, and so the loss of output from cooling down to reheating – usually a week each time – has to be compensated for by the mine owner as indirect damages. In the case of thoroughgoing repair or renewal of furnaces, age depreciation or the so-called campaign life of a furnace until the next relining – which for a glass-melting furnace, for example, may be 5 years – should be taken into account as a compensatory factor.

In the case of batteries of 40 to 50 *coke ovens*, often erected on concrete rafts 50 m long and 2 m thick, cracks can appear in the brickwork, in inspection holes and waste-heat flues, and also in the refractory silica lining, when mining takes place in the shaft protective zone. Between oven chambers and heating flues, and also between regenerator chambers, harmful gas flows can develop if separating walls are cracked. Gas mains in the coking plant can be protected against damaging axial forces by an adjustment of gas-tight sleeve joints (by ± 40 cm) from time to time. The tracks for the coke pusher assembly must be raised and levelled in the event of a major alteration in gradient. In the absence of expansion joints, the coal tower also suffers compression. Tilts of 3 to 4 mm/m in a transverse direction do not interfere with operations; in the longitudinal direction of the battery, even a radius of curvature of as little as 200 m appears to have done no damage in an isolated case (Luetkens, 1956). Coking plant are usually renewed after 20 years.

Chimneys and *towers*, because of their small horizontal area, are only minimally affected by linear deformation of the ground; they are, however, subject to tilting (Fig. 305, left). If the upper axis point has migrated into the outer (hatched) third of the horizontal ring area formed between the chimney's or tower's external circumference a at the bottom, near the foundation slab, and its internal circumference b at the top, then the upper part of the structure must be removed.

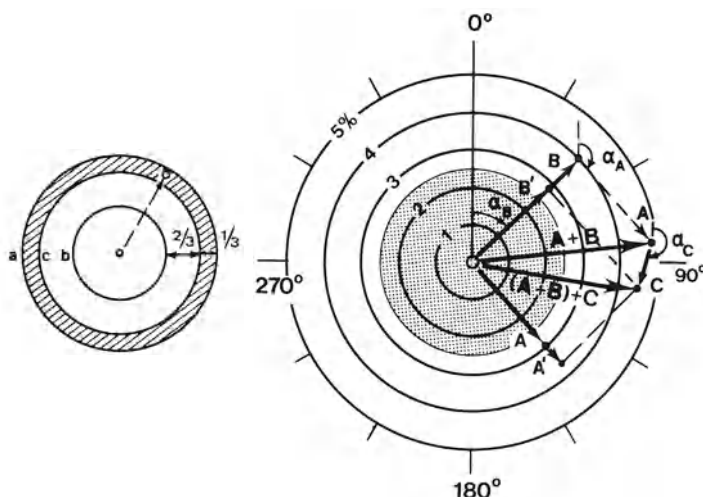


Fig. 305. *Left:* tilting of a chimney up to the permissible limit c ; a External diameter at bottom; b internal diameter at top. *Right:* circular grid for graphical determination of the change in tilt of a plane (vector $A + B$) resulting from tilting of the ground through mining (vector C)

Reservoirs for water or oil are subject to obvious hazards due to tilting, and may easily be cracked to the extent that loss of water or oil becomes serious.

The effects of tilting on gasholders, when severe, can be detrimental both in the risk of losing the water seal and in the excessive binding of rollers on guide columns.

Inclined or bent *crane tracks* render operation difficult: the crane may run downhill on its own or may no longer lie horizontal. The bearing supports of the track must then be raised or lowered, usually while the crane is in operation. If the track is bent out sideways due to a compression effect (rail buckling), the gauge may be altered, with the result that the track wheels jam or the crane even capsizes.

Turbines are particularly sensitive to bending effects. A radius of curvature of over 25 km is harmless, but a tilt in the longitudinal axis of over 4 mm/m can affect lubrication of the axle bed, as this is then unevenly loaded.

Machine shops erected as frame constructions are generally easy to level or put right in the event of ground movement. Cracks in non-bearing wall elements are of no consequence. Often there are recesses and purchase surfaces for lifting apparatus incorporated in the load-bearing frame at the design stage. In ductile pipelines, containing many bends, within the works area, there will be little damage if they remain freely movable in wall apertures and clamps and are provided with adjustment sections. Such pipelines should not be laid under plaster. By contrast, the levelling of heavy machinery, rolling plant, and long production lines, which have to be laid in concrete foundations and steel frames that can be raised, is a costly procedure because of the need to halt production temporarily. The tilting of factory machines and plant can be classed as damage when the degree of tilting renders the equipment inoperable and cannot be rectified by "built-in" methods. The latter are in fact quite common and consist

of either screw feet which can be adjusted to relevel the machine or instrument, or of holding-down bolts which can be loosened for the insertion of wedges or shims. For many sensitive machines, even a sag of 1/50,000 is harmful.

On acceptable *degrees of tilt*, plant managements often, in the absence of relevant experience, quote values which are too low (e. g., 0.1 mm/m), which can appear on changes in temperature, even without the influence of mining. Furthermore, in view of the multiplicity of types of production unit, the subsidence engineer can only judge the degree of mining damage, including consequential damages, for each case separately on the basis of construction plans, production processes, and information gained on inspection. It is a well known fact that materials in the construction field are often less susceptible to fracture, structures themselves more stable, and production processes more able to adapt to disturbance than might be expected on the basis of strength calculations and works visits.

12.4 Damage to Communications and Public Utilities Installations

From the subsidence-engineering point of view, the noteworthy aspect of communications installations and public utility mains is their elongated form. Friction forces can act on them over a great length and become cumulative. They may also subside over long distances. Types of installation coming within this category include railway tracks, canals, roads, pipelines, and cables. In their strung-out construction, they lie either on the surface of the ground (railway lines, roads), or cut into the surface (canals), or are laid in the ground (pipes, cables).

12.4.1 Railway Tracks

The effects on railway tracks produced by mining include, amongst other things, damage from subsidence. Alteration of the specified gradient of – in many countries today – over 1:80 (12.5‰) for unrestricted main lines, and a minimum of 1:400 for railway stations, can impair orderly traction and shunting activity and necessitate additional propulsion and braking forces or reduced payloads and train lengths. The flow of water in drainage ditches and outfalls alongside the track is interfered with. Lowering a ridge uplift in the track profile, or raising a sunken stretch (usually done in steps of 20 cm), including raising bridges, to restore an “economically acceptable” gradient, entails an interference with traffic. The costs of the slowdown in traffic and of attention to the track have to be borne by the mine.

Very high costs arise if intersection complexes, complete with embankments, underpasses, and culverts, have to be raised, or large marshalling yards including loading platforms, signal installations, and sheds have to be reconstructed (Fig. 306). The overhead contact wires of electrified sections of the railway are relatively insensitive to mining damage, but they increase the cost of remedial

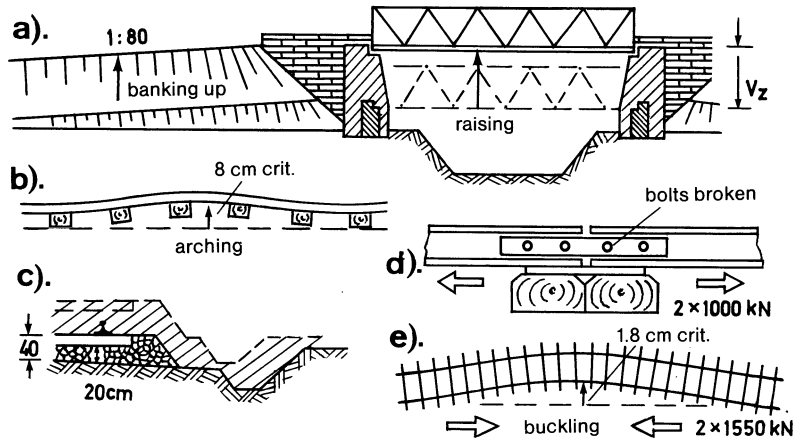


Fig. 306. Damage to railway tracks. **a** Lifting bridges to their old level; **b** arching of track under compression; **c** raising bed of ballast in 20 cm steps; **d** fishplate failure under tensile stress; **e** lateral buckling of track under compression

works, because they have to be raised along with the subsided stretch of line that they serve. Pit sidings which have subsided to a gradient of $1 : 50$ can only be used for one wagon at a time, for reasons of safety. Where mining has caused a tilt at the surface of $2 \text{ mm/m} = 1 : 500$ along the track axis, the effect is an increase in gradient from, for example, $1 : 100$ to $1 : 83.3$ for a main line, or from $1 : 1000$ to $1 : 333$ for station track. The extra height of 20 to 150 mm in the outer rail in a curve, to counteract the centrifugal force of the train, is altered by more than 7.5 mm (where track gauge is 1.435 m) only in the case of a radial tilt of more than 5 mm/m. It is only when working is near the surface that ground displacements of 1 m and more across the track axis will produce visible warping sideways with radii of 10 to 20 km, which are insignificant when compared with the prescribed minimum radii of curvature of 180 to 1070 m required for speeds of 60 to 150 km/h.

Ground displacement parallel to the track axis, on the other hand, can lead to separation fracture in junction fishplates, to the closing-up of expansion gaps, and to *lateral warping* or *arching* of the track (see Appendix). The 30 m long rails must then, starting in the zone of ground extension, be “distorted” back to the average expansion gap width, which is 8 mm at a temperature of $+10^\circ\text{C}$. Sleepers that have been displaced in the underlying bed of ballast have to be repacked. Continuous-welded track has not yet been laid in German coalmining districts, for reasons of safety and because there is a lack of suitable trial stretches. Expansion-gap track, however, no longer meets the technical and economic demands of today in track construction and is inferior to continuous track in both travel comfort and maintenance. The four fishplate bolts at the rail joint shear under an axial force of 1000 kN. Rails of profile S 49 (cross-sectional area $S = 62.3 \text{ cm}^2$), on the other hand, rupture only at around 2500 kN (stretching limit 40 kN/cm^2). The rigidity of the track-frame towards lateral track warping amounts with normal sleeper bedding at the ends to around 1,550 kN longitudinal pressure per rail. The connection at the expansion gap thus consti-

tutes the weakest member of the track unit. Some 60% of all breaks in the track are fishplate fractures. A calculation of the longitudinal forces to be expected in a mining area is highly complicated, as will be shown below, because of the overlapping of temperature stress, traffic load (e.g., 100 kN/rail on rapid braking), expansion-gap relief, and additional forces resulting from mining.

On displacement of the ground in a mining district, *friction forces* over the bed of ballast as it moves with the ground are transmitted to the sleepers bedded in it, which in their turn are firmly fixed to the rails. Since the thrust resistance of a rail on the bedplates of the sleepers is, at approximately 33 kN/m, four times as great as the displacement resistance of the track frame in the ballast as recalculated for the rails, the sleepers are obliged to shift in the bed of ballast, and do so to the limit of elastic deformation of the rails in the longitudinal direction. The sleepers thus cleave immovably to the foot of the rails. The friction or shear forces F_t transmitted from them to the rails at an interval of 60 to 70 cm can be replaced by a uniform line-load, which can be empirically ascertained from the displacement resistance of the track. Presses are placed against both rails of a track of length l , and the pressure is increased until the track shifts in the bed of ballast. This happens when the pressure F_t on each rail reaches $8 \text{ kN} \times l$ (in metres), the limit value of the transmitted thrust force.

With the beginning of ground movement, *rail stresses* rise at first only to the level of fishplate friction, amounting normally to 50 or at most 400 kN. After fishplate friction has been overcome, the individual rails move in the slots of the fishplates, while rail stresses remain constant. These stresses begin to increase again only when the expansion gaps are fully extended, in the case of tensile action, or completely closed up, where the action is compressive. In this state the string of rails is able to absorb axial forces (from temperature, traffic, ground friction) of up to 1000 kN ($= 0.76 \text{ mm/m}$ rail elongation) in the extension zone before the fishplate bolts fail, and in the compression zone up to 1500 kN ($= 1.18 \text{ mm/m}$ rail compression) before the track warps laterally. If, for simplicity, it is assumed that, given a sufficiently large displacement of the ground, the *displacement resistance* of the track amounting to $2 \times 8 \text{ kN/m}$ will be fully mobilized by the friction forces on the sleepers, then the track would be wrecked by a force-increase travel of length 250 m in the extension zone, and of length 380 m in the compression zone. Where the graph of the distribution of transmitted friction forces in the rail string peaks in a triangular, "rooflike" shape (see F_t in Fig. 286), the extension zone must be $l_{\text{crit}} = 500 \text{ m}$ wide as an outside limit, and the compression zone 760 m wide: which presupposes a mining depth of over 700 m, given a limit angle of 60 gr.

It can be shown on a *model* how elongated elastic bodies (rail tracks, pipelines, building structures – (1) in Fig. 307, left) deform through friction force if the bed on which they lie (2) is extended or compressed, like the ground in an area affected by mining. In a body delimited on both sides, the line of tensile strain ε_1 rises from both a' and c at the angle α to form a ridge at the middle-point b . In the process, the body is deformed less than the underlying bed (curve ε_2) if the deformation resistance of the body is greater than the friction force [Eqs. (86) to (88)]. The middle-point b of the body has undergone the full amount of the underlying bed's displacement (curve v_2), while the left-hand side of the body has

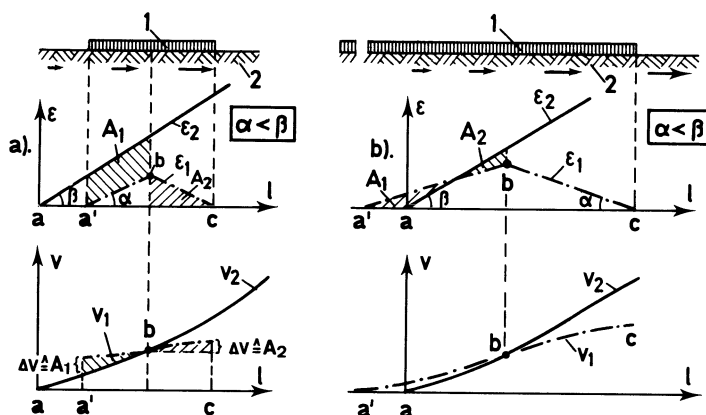


Fig. 307. Deformation through frictional force of (1) an elastic body on (2) an underlying layer experiencing extension. **a** Body delimited on both sides; **b** body infinitely long on the left. (After Thoma, 1956)

been displaced more, and the right-hand side less, than the underlying bed (curve v_1). The difference Δv in displacement at the end positions a' and c of the body are obtained by integration of the differences in extension within the areas A_1 and A_2 over the length of travel l . When the body is not delimited on the left (Fig. 307, right), it begins to expand from as early as point a' , although tensile strain in the underlying layer begins only at point a . On the right-hand side, tensile strain in the body, as in the first case, will decrease from the point b at the angle α , with the result that the body is obliged, because of its greater deformation resistance ($\alpha < \beta$), to move ahead of the underlying layer in the area $a' - a$, only to be overtaken in displacement by the underlying layer at the position b . The difference in displacement Δv is nil at point b only when the areas A_1 and A_2 are equal. In constructing curve ε_1 , the line $a' - b$ must be shifted parallel until a real equality $A_1 = A_2$ is obtained.

If the deformation resistance of the body when unlimited on the left is equal to the friction force ($\alpha = \beta$), the body differs from the underlying layer in expansion and displacement only on the right-hand side (Fig. 308, left). A very weak body, with a deformation resistance less than the friction force ($\alpha > \beta$), is deformed on the left-hand side to the same extent as the underlying layer. The line of tensile strain ε_1 in the body rises there not at the angle α but, like the underlying layer, at the angle β . The point b therefore shifts somewhat towards the end c of the body (Fig. 308, right). Deformation produced by friction force in the overlying body is thus at its greatest where relative displacement Δv is nil (point b in Fig. 307, zone $a - b$ in Fig. 308). The situation is reversed at the ends of the body: there the deformation force transmitted to the body is nil and relative displacement maximal.

If regard is had to the sinusoidal shape of the curve of ground linear deformation in the mining area, then by the rules in Sect. 4.2 the *line of track deformation* must be drawn at the gradient

$$\tan \alpha = \frac{\varepsilon}{l} = \frac{F_t 1000}{ES} [\text{mm/m per running metre}] \quad (379)$$

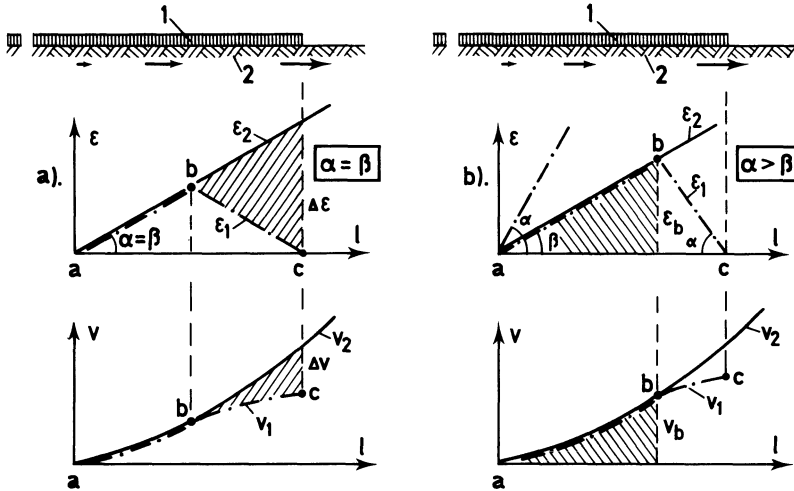


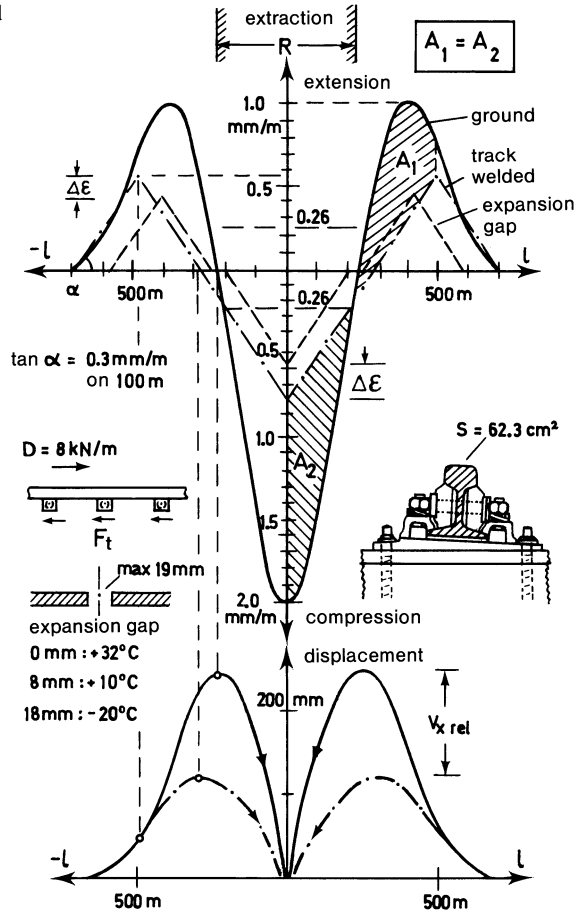
Fig. 308. Deformation through frictional force of (1) an elastic body on (2) an underlying layer experiencing extension. Body unlimited on the left. **a** Angles of deformation of equal size; **b** angle of deformation of body greater than that of underlying layer. (After Thoma, 1956)

from 0.3 mm/m to a length of 100 m⁴, as shown in Fig. 309 for a continuous-welded track (Thoma, 1956; Hilbig and Kratzsch, 1964). The values obtained in this way, if they are to be applied to the expansion-gap track (8 mm average gap-width, 30 m length of rail), must be reduced on either side of the “roof-shaped” line of track deformation by – very roughly – a maximum of $8/30 = 0.26$ mm/m (Fig. 309). This example makes it clear that a welded track over half a critical area of extraction at a depth of 700 m, given maximal ground extension of 1 mm/m and ground compression of a maximum of 2 mm/m, is extended by only up to 0.55 mm/m and compressed by up to 0.82 mm/m. For an expansion-gap track, peak values may be estimated at 0.45 mm/m extension and 0.55 mm/m compression. If ground deformation increases to 5 mm/m extension and 10 mm/m compression, these peak values rise only slightly to 0.7 or 0.55 mm/m extension and 1.0 or 0.75 mm/m compression in the rails. On the other hand, they increase linearly with widening of the subsidence trough.

Thermal stress accessory to the force of friction reaches 15.7 kN/1°C with impeded linear deformation of the rail. For the maximum and minimum temperatures of +60°C and –30°C found in Germany, forces of 540 kN compression and 50 kN tension were measured in 30 m expansion-gap rail, and of 750 kN compression and 650 kN tension in welded line. In expansion-gap track therefore, practically no tensile forces develop in winter, since the maximum pull-out scope of 19 mm is not quite used up by linear deformation, because the fishplate and sleeper friction impedes the temperature compression of the rails. For the same reason, complete closure of the expansion gap in summer does not occur until +32°C instead of theoretically at +26°C. If it is assumed that ground movement in summer begins only after complete closure, and in winter after

⁴ $F_t = 8 \text{ kN} \times 1 \text{ m}$; $E = 21,000 \text{ kN/cm}^2$; $S = 62.3 \text{ cm}^2$. Distinguish here and in the next chapters between (ex)tension/compression in mm/m (= strain, lengthening, shortening) and in N (= force) or in Pa (= stress).

Fig. 309. Action of ground affected by mining on welded track, and on track consisting of 30 m rails with expansion gaps



complete opening up, of the expansion gaps, lateral buckling of track in summer under the additional influence of mining can be expected at a friction force of $F_t = 1550 - 540 = 1010 \text{ kN}$ or rail compression of

$$\epsilon = \frac{F_t}{SE} 1000 = 0.77 \text{ mm/m} . \quad (380)$$

For welded track (neutral temperature $+20^\circ \text{C}$) the values are $1550 - 750 = 800 \text{ kN}$ friction and 0.61 mm/m rail compression.

Accordingly, in the example shown in Fig. 309, only the welded track would be buckled at the height of summer. Fishplate bolts become sheared in winter at a level of mining influence of $1000 - 50 = 950 \text{ kN}$ friction, or rail tensile strain of 0.72 mm/m , which should arise only on ground tensile strain of over 5 mm/m . On the other hand, welded track with a stretching limit of 2500 kN per rail and a welding factor of 1 would begin to crack at a rail tensile strain of 1.4 mm/m (cold brittleness), which is hardly likely to be reached even in an area affected by mining.

Measures taken against the dreaded *buckling* of tracks include a widened sleeper bedding course, rigid-angled bracing of rail and sleeper, laterally placed piles, raising the neutral temperature, a heavy permanent way, de-stressing the rails by loosening sleeper fastenings in the adjoining zone of tensile strain, and cutting open the rails with a welding torch in conjunction with remote sensing of rail stress (strain gauges).

The calculated limit values set out in Table 21 for ease of reference apply, it should be remembered, to rectilinear tracks laid under normal conditions. Owing to variations in the direction of ground movement, sleepers in an area affected by mining may lie loosely in the bed of ballast or over hollows in the ground, and this may reduce track resistance to lengthwise and transverse displacement. The risk of the track buckling increases if it becomes warped sideways or upwards by

Table 21. Forces in the railway line when affected by mining and temperature stress. Limit load values

Season, and type of damage	Expansion-gap track with 30 m rail	Continuous-welded track
Summer + 60°C	540 kN → ← temperature compression → ← mining 1010 kN ↓	750 kN → ← temperature compression → ← mining 800 kN ↓
Buckling of track at 1550 kN compression	Permissible: 0.77 mm/m compression of rail by mining Attained: 0.55 mm/m (with 2 mm/m ground compression) 0.75 mm/m (with 10 mm/m ground compression)	Permissible: 0.61 mm/m compression of rail by mining Attained: 0.82 mm/m (with 2 mm/m ground compression) 1.00 mm/m (with 10 mm/m ground compression)
Winter – 30°C	50 kN ← → temperature tension ← → mining 950 kN ↓	650 kN ← → temperature tension ← → mining 1850 kN ↓
Failure of fishplates at 1000 kN tension	Permissible: 0.72 mm/m extension of rail by mining Attained: 0.45 mm/m (with 1 mm/m ground extension) 0.55 mm/m (with 5 mm/m ground extension)	Permissible: 1.4 mm/m extension of rail by mining Attained: 0.55 mm/m (with 1 mm/m ground extension) 0.70 mm/m (with 5 mm/m ground extension)

Additional forces: traffic load, braking forces, local concentration of stress, inherent stress from laying of track.

Strength values reduced by points of material weakness, unequal packing of sleepers, low buckling strength of fishplates, inadequate anchoring of rails, sleeper bedding loosened by effects of mining.

ground movement. Axial forces do not always show a uniform linear increase with effective length of action on the track but may be concentrated locally as a result of damage or wear to materials or of uneven packing and anchoring of sleepers. Linear deformation of rails, which increases with length of frictional contact, may finally amount to 1 mm/m or more, if working proceeds in several adjoining panels simultaneously, and if the zones of compression and extension, which are shown in Fig. 309 as being almost 500 m wide, extend in practice to over 1 km in width. The calculations also ignore traffic and braking forces, curves and switch points, inherent tensions in the rails stemming from the track-laying process, and the safety factor prescribed for top-category track – namely, $v = \sigma_{\text{stretch limit}} / \sigma_{\text{permissible limit}} = 2.67$.

Streetcar, or tramway, *tracks* are as a rule bedded within the road surface on concrete, and are either continuous-welded or provided with equalizing gaps in the form of wedge-shaped rail ends which can be pushed together or pulled apart by 70 cm (see Fig. 379). Bedding within the road has the advantage that part of the rail heat is conducted away into the bed – resulting in a maximum temperature difference of only 50°C (thermal stress 13 kN/cm²) as compared with 90°C with bedding on ballast – and that tracks are prevented from buckling laterally under compressive stress. Significant damage resulting from relative displacement as between track and road-paving arises only in the paving above the level of the gauge rods, which move with the tracks, and at the switch points (see Appendix). Rain-water percolating down through gaps in the paving formed in this way may wash under the track, hollow out the ground beneath it, and destroy its bond with the concrete. Because of the relatively low traction load and short train length, alterations in gradient have little adverse effect on the operation of streetcars or tramways.

12.4.2 Roads and Bridges

Roads are constructed of a load-bearing “roadbase” course of cement-bound or bitumen-bound, or sometimes unbound, rock material, laid on a firm formation consisting of a levelled “subgrade” (earth, rock, artificial embankment), and of a surface layer of asphalt concrete, or coated macadam (see Fig. 380, left). Surfacing with paving stones is now rare in main highways. The lowest bearing course – the “sub-base” – serves as a protection against frost, and the lower bituminous layer of the surfacing – the “base course” – as a binding course. The thickness of the bearing course of 10 to 30 cm depends on the traffic loading and the method of construction (categories I to V in Germany). The thickness of the bituminous surface layer (4 cm) and binding course (up to 8 cm) and also of concrete surfacing (up to 22 cm) is constant in each construction category, even though the method of bearing-course construction may vary (Kutscher, 1980).

Concrete surfacing is laid with three types of joint running across or parallel to the axis of the road. The blind joint passes through at least a quarter of the thickness of concrete and serves as a transverse joint at a 5 m interval providing a straight line of fracture for the concrete when it shrinks on setting or in winter. Clearance joints, the second variety, placed every 15 m at right angles to the road

axis, have a width of gap of 2 cm. They are tar-filled and pass through the whole thickness of the concrete deck. Their function is to take up the temperature-induced expansion of the concrete in summer and thereby minimize longitudinal forces. The latest roads now mostly do without such clearance joints. Butt joints, the third variety, which delineate concrete slabs prefabricated by sections, reveal no gap. Longitudinal joints parallel to the road axis are either blind joints or butt joints. To ensure that adjoining concrete slabs are at the same level, the transverse joints between them are dowelled in. The concrete slabs are anchored at the longitudinal joints, to prevent reciprocal displacement.

Bituminous surfacing contains no constructive joints apart from the so-called work boundaries, which mark off the sections completed daily when the road is first constructed. Surfacing is applied in layers at 130°C, with each new layer binding to the one beneath. The mixture of materials is arranged in such a way that strength increases towards the top layer. According to the grain-size distribution of the mix, a distinction is drawn between coated macadam and voidless mastic asphalt.

Mining extraction affects the road both in terms of position (subsidence, tilt, transverse displacement) and in terms of shape (flexure, extension, compression). The result is that driving dynamics, load-bearing capacity, durability, and drainage are all affected. Ground subsidence affects the road through ground water rising at the trough centre and soaking the subgrade and the roadbase bearing course. As a consequence, the subgrade freezes and heaves in winter, and in the spring the thawing ground softens and loses in load-bearing capacity. The mean distance between the ground-water table and the road formation should therefore be at least 1.2 m in winter. The gradient of the road both lengthwise and crosswise is altered most over the advancing or stationary edges of the workings, being either increased or reduced according to the direction of ground tilt. The simplest way to determine the new *inclination* of the road is by a graphical method. On the circular grid of Fig. 305, right, the components of the existing gradient both across and along the road are traced as vectors A and B from the centre and added geometrically to give the resultant vector A + B. To this vector is joined vector C, the pre-calculated principal tilt of the ground. The resultant (A + B) + C represents the subsequent principal inclination of the road surface, which is analysed into the components A' and B' to enable the required subsequent gradient, both across and along the road, to be read off from the circular grid.

All new roads are laid out, for drainage reasons, with a transverse slope of 2.5‰ = 25 mm/m. If the slope of the road top is too flat, the rate at which surface water flows off is slowed down, and the film of water on the road increases in thickness from 1 mm to 2 mm, creating a danger of skidding. The slope should therefore not be allowed to drop, as a result of mining, below 0.5‰. The piping and ditching in which surface and ground water from the road is gathered should therefore still show, after the ground has subsided, a gradient of between 0.2‰ and 2‰, in order that water shall flow neither too slowly (banking up, sedimentation) nor too rapidly (erosion, turbulence).

If the transverse slope of up to 6‰ acceptable in the road curve is reduced by 1‰ or 2‰ as a result of tilt in the ground, driving dynamics are adversely

affected. In the longitudinal direction, a change in gradient within an area of subsidence is noticeable only in steep stretches of road by its effect on braking distance and speed of lorry traffic if, for example, the maximum acceptable longitudinal gradient of 4% for fast roads is increased by subsidence to, say, 5.5%. In a hilltop or trough area, driving dynamics and sight distance require that the radius of vertical curvature shall not drop below 10 or 5 km as a result of overlapping ground flexure.

Cracks in the road surface caused by ground flexure are conceivable only in the event of flexures of under 1 km radius in a zone of geological faulting. Even linear deformation of the ground will lead to cracking in the concrete deck only if there are weak spots like sewer manholes, road joints, or embedded piping at the point of maximum longitudinal force, or if large temperature strains (tensile in winter, compressive in summer) supervene. For the frictional force arising at the interface between bound and unbound courses (roadbase/subgrade or surfacing/roadbase) can, by Eq. (338), create at the centre of an $l = 200$ m wide extension or compression zone only a maximum tensile or compressive force of $F_{t\max} = 602$ kN in a 1 m wide strip of road ($\mu = 0.7$; $F'_n = 8.6$ kN/m). Even if this force is absorbed by the concrete deck (thickness, e.g., 22 cm) alone, because the roadbase bearing course (thickness, e.g., 15 cm) is not strong enough to do so, it is not damaging to the cement concrete, even as a tensile stress $\sigma = F_{t\max}/A = 602/2200 = 274$ N/cm² or 2.74 MN/m² (where A = cross-sectional area).

With bituminous surfacing, strength depends on temperature and rate of deformation. The binding base course prevents displacement as between the surface wearing course and the roadbase bearing course (see Fig. 380, left). As temperature falls, the flexibility of bituminous surfacing decreases, and bitumen becomes brittle below 0°C. Consequently, crack damage occurs in bituminous surfacing more frequently in winter than in summer. Strength drops noticeably if the bitumen is deformed slowly. The slight time-dependent linear deformation of approximately 0.2 mm/m per day of the ground in an area affected by mining thus reduces the strength of bitumen. On the other hand, this low daily rate of deformation favours constant equilization of stress in the road top. Elongation at rupture of bituminous surfacing undergoing slow deformation lies between 1% for mastic asphalt and 1.5% for coated macadam. Therefore bituminous surfacing in a mining area will crack only where the underlying roadbase bearing course has formed a wide split which the surfacing courses cannot bridge without rupturing, or where tensile strain in the road is concentrated over cracks in the ground (see Fig. 150b). In an area affected by mining therefore, the bearing roadbase course should be bitumen-bound and not cement-bound.

Where linear deformation of the ground takes place as a result of mining, the most common outcome is not cracks in the road surface but the following *types of damage*:

On extension:	Dilatation of joints in the concrete deck Pulling open of butt joints in kerbstones Pulling apart of sewer pipes Flexural deformation of bituminous surfacing

On compression: Heaving of bituminous surfacing
Forcing up of kerbstones
Spalling of edges in joints
Expressing of joint fillings
Compaction fractures in sewer pipes.

If cracks in road surfaces are not quickly filled in, rain-water soon washes through to the underlying bed. In areas of intense compression, small paving blocks (pavé stones) have proved very effective. The emergence of steps in the ground or of an undulating pattern of subsidence in roads is a danger to traffic.

The supports or piers of a *bridge* can move towards or away from each other according to the type of horizontal strain, but they can also tilt in different directions or twist in the horizontal plane, with complex and often serious effects upon the decking or arches.

The NCB has reported that in long bridges where there are a number of spans, the bridge length may be greater than the subsidence wave above a shallow working, and adjacent pairs of supports will suffer more relative movement than the net overall strain from one end of the bridge to the other.

The appearance of subsidence damage on bridge arches is often a complex of cracks, and the precise originating movements are difficult to visualize. Compressive damage often appears as crushing and spalling-off of brick facings and of horizontal joint openings.

Extension can cause the bridge supports to move away from each other to such an extent that the arch loses shape by flattening; but many arches unaffected by subsidence show open joints and flattening due to loss of tightness in the keystones. Perished mortar is one cause of this (NCB 1975).

12.4.3 Cables and Pipes

Among the many cables, pipes, sewers and drains lying under a town's streets, the electric *cables* laid in long lines in loose earth are sensitive to tensile stress in their joint boxes and the shorter, form-fittingly bent sections to compressive stress; pipelines on the other hand are sensitive not only to axial linear deformation but also – apart from freshwater mains – to alterations in gradient (Fig. 310). In the case of triple-jacketed high-voltage cables, the external frictional forces acting on the jute covering or plastic sheathing are transmitted to jacket, insulation, and conductor. Usually only the conductor has any significant mechanical strength. In the extension zone, insulation in a curved section can be cut through by the conductor, and in dense ground the compacted conductor can bore through the insulation, with earthing or short-circuiting as the result. In dry-core cables in particular, insulated with paper and lead, the conductors and even the individual wires in their insulation can be seriously displaced. There is advantage in using externally smooth "Protodur" cables with a friction-resistant intermediate jacket of rubber or glass-silk ribbon, which prevents undesirable relative displacement as between insulation and jacket. Good results have also been had with stiffened fabric at the cable end and the joint-box neck, with spiral

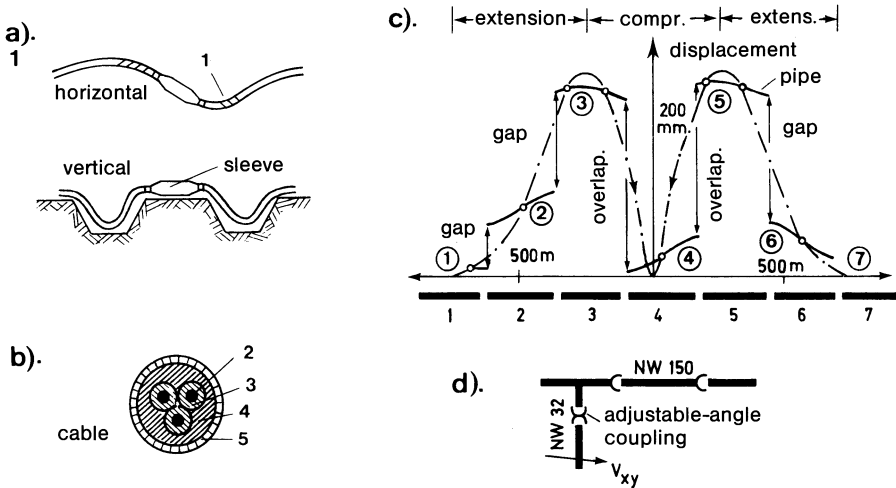


Fig. 310. The effects of mining subsidence on cables and pipelines. **a** Laying cables in loops as a precaution against linear deformation – the upper joint box stiffened with fabric (1); **b** cross-section through an electricity cable – (2) conductor; (3) insulation; (4) jacket; (5) sheath; **c** determining the width of gaps between the butt-ends of a pipeline from individual pipes 1 to 7, given the ground displacement; **d** adjustable-angle gas communication pipe at the branch connection to a house (NW = nominal width, bore)

conductors which can be axially compressed, and with form-fitting joint boxes of casting resin, which prevent conductors from being pulled out under tensile action or bent into loops within the joint box under compressive action. Cast-iron joint boxes for cables should be used in an area affected by mining only if they incorporate an expandable spacer piece. Since the cable follows ground displacement only partially, and since the splaying of conductors and separation-rupturing occur only at a linear deformation of around 40 mm/m, damage to cables, particularly in their joint boxes, bends and kinks, can be expected only in the event of very intense ground extension or compression. When 150 kV cables are laid in channels or steel pipes, or are hollow-bedded under shaped bricks, they experience no stress from mining-induced ground movements.

In the case of public utility mains and branch *supply-pipes* of cast iron, steel, asbestos-cement, or earthenware, a movement of elongation will loosen the packing in coupling boxes, and compression will splinter pipe ends, spall off whole pieces of pipe walling, and even burst coupling boxes (Appendix V). Transportation losses of material may then exceed the normal expectation of 4% in the case of gas and 5% for water, with the result that coupling-box packings (lead rings, tarred yarn, lead wool) have to be redone and damaged sections of piping dug up and exchanged, in some cases, for ductile piping. Pipe junction elements sensitive to axial forces may be replaced by sleeve joints capable of adjustment by 1 or 2 dm, wedge-shaped coupling boxes, screw couplings, or retaining rings (removable packings) placed at an interval of 30 to 50 m. The intervals for these expansion pieces and their expansion widths are governed by the slope of the ground displacement curve (Fig. 310).

At particular risk, besides the elbow joints, slide valves, water pots and water hydrants which are so closely bound to the earth, are the junction pieces of the domestic communication pipes branching off from the mains on average every 20 m – unless they have been fitted with axially movable piercing clamps and adjustable-angle couplings to obviate the possibility of lateral bending fracture. These branching-off points act as a ground anchor if the principal main is axially displaced more strongly than the domestic communication pipes. Domestic lead-ins have to be packed with greased yarn and be provided with scope for movement by means of a rubber membrane. A third of all damage to gas pipes in the Dortmund district of West Germany can be traced to mining influences. The squeezing of pipes of small nominal bore (clear internal diameter) may have a cause other than mining – e.g., traffic load or frost pressure.

Long-distance *pipelines* of steel, with an internal diameter (nominal bore) of 200–710 mm or more and laid in the ground in straight lines, are damaged less frequently than pipe networks within towns, because their bitumen insulation has a favourable, friction-reducing lubrication effect, and because the axial resistance to deformation of pipelines increases roughly in proportion to their external diameter. The *displacement resistance* F_t of a pipeline, as ascertained from tests, comes to approximately 4.5 kN per running metre for a nominal bore of 300 mm in sandy soil and with 4 mm thick bituminous winding material (Geilenkeuser, 1962). In the absence of a bituminous outer layer, far higher frictional forces of 50 kN/m in sandy soil and over 200 kN/m in stiff clay must be reckoned with. In these tests an old pipeline of some 50 m length is dug up at both ends and subjected to axial pressure until the whole section of pipes moves (see force F in Fig. 294).

The displacement resistance F_t can also be calculated by the measured extension at the pipe according to Eq. (383) or theoretically according to Eq. (337), $F_t = \mu F_n$ (with $F_n = \gamma \pi r(t+r)(a+K)$ for the normal pressure), if following data are known: the coefficient of friction $\mu = 0.2–0.6$, the weight force of the ground $\gamma = 17 \text{ kN/m}^3$, the depth t of the pipe in the ground (to the crest point) and the parameters $a = 0.9–1.1$ for consolidation of the trench back-fill and $K = 0.4–2$ for the differing horizontal earth pressure at right angles to the pipeline in the extension and compression zones (Meißner, 1978).

The following may be noted in amplification of Sect. 12.4.1. Horizontal displacement of the ground in an area affected by mining causes static shear or *frictional forces* to be transmitted to the pipeline, which in turn responds with an internal, elastic force of deformation. Until equilibrium is created between these forces, deformation occurs in the pipeline as well as reciprocal displacement as between pipeline and ground. If deformation up to the stretch limit of 240 N/mm^2 for piping of type St 35 steel⁵ is assumed and a factor of safety of 1.87 taken, then the admissible stress in the pipeline, given a weld-strength reduction factor of 1.0, will be

$$\sigma_{\text{permissible}} = \frac{240 \times 1.0}{1.87} = 128 \text{ N/mm}^2. \quad (381)$$

⁵ The steel frequently used for welded oil pipelines, namely type API $\times 52$, has a stretch limit of 360 N/mm^2 (point S in Fig. 301), which corresponds to a strain of 1.7 mm/m .

From this can be derived a damage-free axial deformation of the welded pipeline ($E = 210,000 \text{ N/mm}^2$), regardless of diameter and cross-section, of

$$\epsilon_{\text{permissible}} = \frac{128}{2.1 \times 10^5} = 0.61 \text{ mm/m} . \quad (382)$$

The seasonal fluctuation in temperature, which makes itself felt between 1° and 16°C in a natural-gas trunk pipeline under 1.2 m of earth (Kiwitt, 1973), produces linear deformation, depending on the temperature when the pipeline was laid, of a maximum of 0.17 mm/m (for laying at $+16^\circ\text{C}$). The shear forces transmitted from the ground to the steel pipe via the bituminous covering deform the pipeline⁶, in accordance with Eq. (89) or (379) by 0.3 mm/m on a length of 100 m. The *deformation curve* of a welded pipeline drawn in the diagram of ground extension and compression at this slope of 0.3 mm/m for each 100 m is thus similar to the linear-deformation curve of a continuous railway line. Lateral earth pressure on the pipe (Fig. 142) varies according to the lie of the pipeline in relation to the mine working (radial, tangential). Ground compression at right angles to the pipeline will increase normal pressure F_n (factor 2–3) and thereby also, by Eq. (337), the frictional force on the pipe and the gradient $\tan \epsilon$ (e.g. 0.4–0.9 mm/m on 100 m); with ground extension at right angles to the trace the effect is the opposite (0.2–0.3 mm/m on 100 m). These local changes in frictional force have to be taken into account (see Fig. 297). The exact connection between linear deformation of the ground in mm/m and the altered earth pressure in N/m^2 that it produces is still unknown.

How axial pipe deformation relates dimensionally to ground extension and compression is shown in Fig. 311 with the example of a butt-welded, 1240 m long pipeline of 400 mm nominal bore. This is constrained on both sides against movements of the adjoining sleeve-pipeline by means of counterpressure supports firmly anchored in the ground and, in front of them, non-reactive expansion couplings made by Ruhrgas AG (earth cover 1.2 m, pipe wall 8.8 mm thick, external bituminous winding 4 mm thick). Axial deformation of the pipeline was ascertained with transportable extensometers in inspection pits at an interval of 30 to 50 m. Relative displacement between ground and pipeline could be measured from horizontal indicators on standpipes struck 1.2 m deep into the foot of the pit. During the 5 year period of mining and testing, the gas trunk line was in operation with increasing internal pressures of 20 to 40 bar. The internal pressure increased the circumference of the pipe and, with each stage in loading, the pressure of the pipe exterior on the ground altered, and with it the frictional force. Because of these and other unknown influences, frictional force can be determined only by measurements taken on the pipeline itself.

On the basis of Fig. 311, the gradient $\tan \alpha$ of compression (the solid line) over a 100 m length of pipeline amounts to around 0.2 mm/m between stations 25 and 12, to 0.3 mm/m between stations 4 and 12, to 0.4 mm/m between stations 56 and 68, and to 0.65 mm/m between stations 76 and 68. The gradient of the curve of extension between stations 4 and 56 also fits into this scale of values. The increase from station 25 to station 76 in the gradient of the pipe

⁶ $F_t = 4.5 \text{ kN per metre}$; $E = 21,000 \text{ kN/cm}^2$; $S = 74 \text{ cm}^2$ for 300 mm nominal bore.

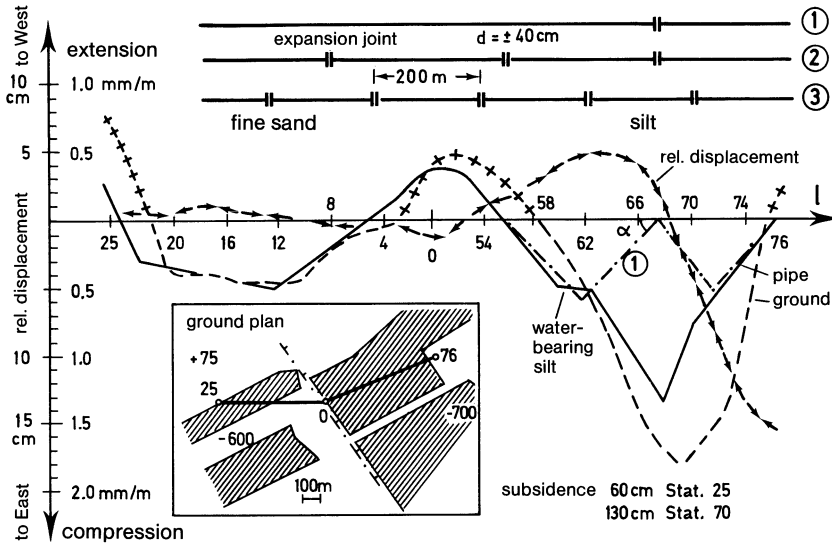


Fig. 311. Measured linear deformation in the ground and in a welded trunk gaspipe (internal diameter 400 mm) between stations 25 and 76. (After Kiwitt, 1973). Curve 1: deformation of pipe max. 0.61 mm/m if one joint is incorporated at Station 68

deformation curve thus depends greatly on the nature of the ground – it is greater in silt than in fine sand. Axial deformation is moreover, as was observed, less by approximately 0.2 mm/m over a length of 100 m when ground water rises and frictional force is reduced, as a result either of moisture in silty soil (e.g., station 62) or of the relief of load on sand grains through the buoyancy effect.

From the deformation gradient, as established by survey methods, of a minimum of 0.2 mm/m for fine sand and a maximum of 0.65 mm/m for silt over a pipe length of 100 m, and given a cross-sectional pipe area of $S = 110 \text{ cm}^2$, it can be calculated that the transmitted *shear force* is per 1 m length of pipe

$$\begin{aligned}
 F_t &= \frac{\epsilon E S}{1000} \\
 &= \frac{0.002 \times 2.1 \times 10^7 \times 110}{1000} = 4.6 \text{ kN/m}
 \end{aligned}
 \tag{383}$$

or per 1 cm^2 of external area

$$\begin{aligned}
 \tau &= \frac{F_t}{A} \\
 &= \frac{4,600}{12,700} = 0.36 \text{ N/cm}^2
 \end{aligned}
 \tag{384}$$

as a minimum, and

$$\begin{aligned}
 F_t &= 15 \text{ kN/m} \\
 \text{or} \quad \tau &= 1.15 \text{ N/cm}^2
 \end{aligned}$$

as a maximum, since the external area A with this pipeline of nominal bore 400 mm comes to approximately $12,700 \text{ cm}^2/\text{m}$. The minimum value shows a good agreement with the value of $F_t = 4.5 \text{ kN/m}$ given above for a pipeline of 300 mm nominal bore. The maximal shear-force value of 15 kN/m between stations 68 and 76 has probably been contributed to by not only the silt but also the ground compression there, because normal pressure F_n [Eq. (337)] is two or three times higher in the compression zone than in the extension zone. Assuming unaltered shear-stress values τ , the shear force F_t is proportional to the external diameter d_a . If it is further assumed that the wall thickness of pipes s is constant for all diameters d_a , then by Eq. (379) ($S \approx \pi d_a s$; $F_t = \pi d_a \tau$) the gradient $\tan \alpha$ of the deformation curve is the same for all nominal bores, given identical materials for pipes and outer wrapping. For other types of steel the following relation applies:

$$\tan \alpha_1 \div \tan \alpha_2 = E_2 \div E_1. \quad (385)$$

In other tests, elastic deformation of the pipe up to 0.8 mm/m was established, which corresponds to a stress of 190 N/mm^2 . Beyond that the pipe is deformed plastically and permanently.

At the top of Fig. 311 the interval between sleeve joints is shown on the basis of various assumptions: (1) with pre-calculated ground displacement and permissible pipe deformation of 0.6 mm/m , as in Fig. 310c; (2) with unknown ground displacement in accordance with the present recognition of a triangular shape for the deformation line (Eq. (404)); (3) in accordance with previous rules when the extensibility of the sleeve joint and the maximum linear deformation of the ground (e.g., 2 mm/m) are given [see also Sect. 14.3 and Eq. (406)].

A further example of the smaller amount of linear deformation in a buried pipeline as compared with the ground in an area affected by mining is shown in Fig. 312. Packing-gland *expansion joints* with an expansion length of $\pm 400 \text{ mm}$

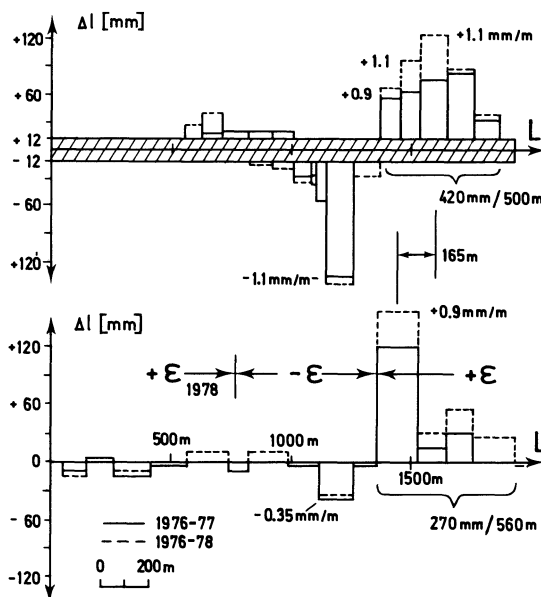


Fig. 312. Measured linear deformation Δl at the surface (*top*), with a confidence interval of $\pm 12 \text{ mm}$, and (*bottom*) at the sleeve joints of a trunk pipeline (after Niedermowwe, 1979). Mining depth 500 m, max. subsidence 70 cm

are built into the 300 mm pipeline every 80 m. In the extension zone on the right of the Figure, the linear change in the ground amounting to 420 mm on a length of 500 m has been transmitted to the pipeline as only 270 mm on 560 m. The peaks of extension in the ground (1.1 mm/m) and in the pipe (0.9 mm/m) are 165 m apart. The peaks of compression over the working area, on the other hand, coincide in position but differ greatly from each other in magnitude, from 1.1 mm/m for the ground to 0.35 mm/m for the pipe.

Linear deformation in a *welded pipeline* of 700 mm diameter, and without expansion joints, situated at the boundary between two workings is represented in Fig. 313. Of particular interest are the measures taken for load relief and their effects. At each second welding seam, at an interval of 24 m, four pairs of measuring markers distributed round the circumference have been welded on. These 20 cm long comparison stretches are measured to the nearest 1/100 mm with a mechanical extensometer. For the calculation of bending stress in the horizontally or vertically bent pipe, a sinusoidal distribution of stress over the circumference is assumed. After the extraction of panel A at a depth of 430 m, the length of pipe between points 163 and 166 is axially compressed by 0.7 mm/m as

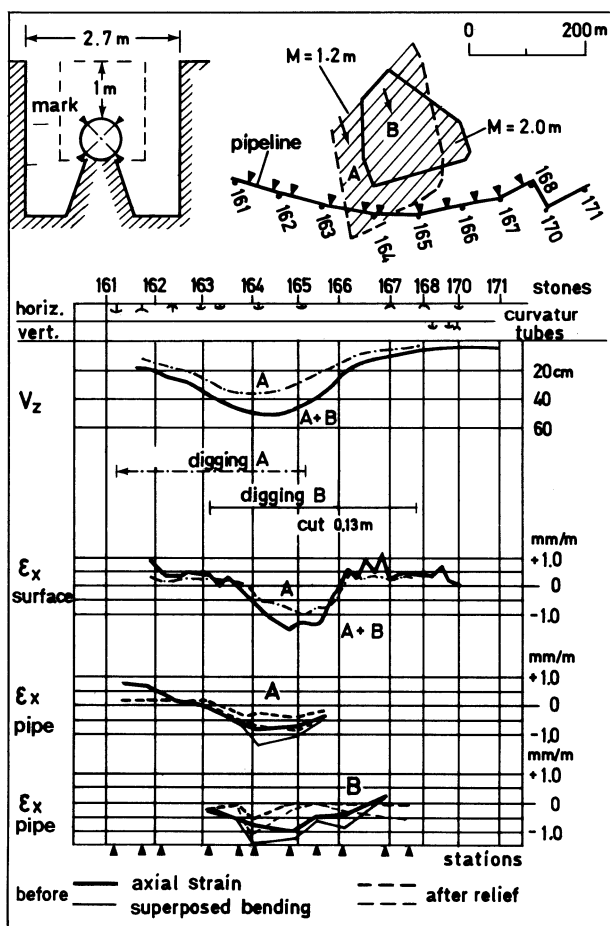


Fig. 313. Measured deformation of the ground and of a buried pipeline (after Lindner, 1976). Top left: measuring points on the pipe. Top right: ground plan of panels A and B and of the pipeline, with ground points 161 to 171 and measuring stations on the pipe (triangles)

a result of ground compression of 1.0 mm/m. To this there is added a compression of 1.4 mm/m caused by the bending of the pipe. After being dug up between points 161 and 165, the pipe becomes de-stressed up to 0.4 mm/m (static shear stress of the ground) and 0.9 mm/m (bending compression). On extraction of the 720 m deep panel B, axial compression in the pipe rapidly increases again to 1.0 mm/m as a result of ground compression of 1.5 mm/m in all (increase only 0.5 mm/m), and bending compression rises to 1.5 mm/m. The pipeline, dug up from point 163 to point 168 and cut open by approximately 13 cm at point 165, becomes completely de-stressed in the area of the gap. Before and behind that area there remains a compression of up to 0.5 mm/m or 1.0 mm/m respectively, owing to the retardation forces in the ground.

Where there are significant changes of direction in the pipeline, special calculations must be undertaken for each straight section, making allowance for the fact that the elbow bends in pipes act as buttresses against axial displacement (Meissner, 1978; Leucht, 1982) and participate fully in local ground displacement. Only where pipelines run in a straight line through the trough are cumulative tensile and compressive stresses theoretically equal. The greater part of compression damage occurs at these elbow bends.

It should also be mentioned that losses in gradient lead to a lowering of temperature and the condensation of water vapour in the product being transported. Water that has seeped down in the subsidence zone must be pumped out of the collecting sumps to free the whole cross-section of the pipe for the gas stream. Ball-and-socket couplings or threaded long-collars (see Fig. 383) permit of bending up to 3° or even more. By a rule of thumb, the smallest radius of elastic bending is the same number in m as the pipe diameter in mm – i.e., approximately 1000 m for a nominal bore of 1000 mm. Radii of curvature as small as this do not usually occur in the ground at today's mining depths.

When subsidence affects the pipe network of a *municipal drainage system*, mud collects and waste water may bank up and flood cellar basements. Linear deformation in pipelines – preferably laid on a firm bed – should cancel out in slide sections provided with plastic tube inserts and accessible via manhole pits. Severe and extensive subsidence may necessitate a dyked polder arrangement to raise water from the main subsidence area and pump it into a new flow channel. In the case of ditches and trenches receiving surface water draining off from an area, a reduction in the natural fall can also be very damaging if settling sediments raise the bed level and thereby cause peaks of run-off water to overflow into fields and cellar basements. If the natural fall is increased, the beds of water-courses can be washed away and lowered (erosion), thereby increasing the distance between the ground-water level and the surface. The new gradient after the pre-calculated subsidence has taken place can be determined graphically in accordance with Fig. 305, right.

12.4.4 Canals and Locks

Shipping canals passing through the zone of influence of a mine retain their original water level even during subsidence, since the banked-up water level will

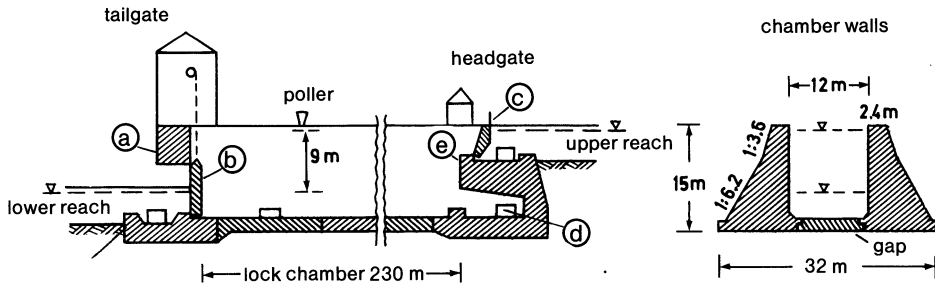


Fig. 314. Longitudinal and transverse section of a lock. **a** Gate beam; **b** lifting gate; **c** clap gate; **d** circulating channel; **e** dock sill

be preserved at the same levels as those obtaining outside the area affected by mining. All canal structures, however, such as locks, embankments, and bridges, subside together with the surface of the ground and in time draw nearer to the unaltered level of the water (see Fig. 306a). The consequences of this situation are an extending of the area of water surface, an increase in its depth (originally 2.5 to 3.5 m), a reduction in the clear passage height, or headroom, under bridges (minimum clearance required 4.5 m), an overflowing on to the otherwise 3 m high towpath, and a shortfall from the minimum height of 1 m above the upper reach of canal required for lock sides and gate baulks (Fig. 314). At the head and tail of the *lock* the depth of water increases to over 3 m as a result of subsidence. Where there is a lifting-type tailgate, headroom can be reduced to under 4 m, and water from the upper reach can flow over the head-bay clap gate when closed. The walls of the lock chamber, which are usually in the form of gravity dams, can be tilted around their longitudinal axis, and this tilting can lead, as chambers are flooded and emptied to a depth of 5 to 9 m, to damage to the vessels of 1500 to 2000 tons made fast with hawsers to bollards on the lockside.

Subsidence of as little as 0.5 m complicates the lock operation, and further downward movements can make it necessary to elevate or even partially to reconstruct the lock. For the lower bay there is advantage in having steel hoisting frames which can be raised. With a view to later systematic lowering of the water level⁷ and dredging out of the canal bottom, the depths of water in the upper and lower bays should be made 1 to 2 m greater than the draught of the barges, and the canal should be lined with sheet piling (Fig. 315). In heightening the sides of a canal which has been excavated out of the surrounding terrain, it can be estimated that raising a 3 m wide towpath by 1 m will require approximately 4000 m³ of earth per km; and to raise by 1 m a 4.3 m wide *embankment* (of slope 1 : 1.5) constructed above ground, approximately double the quantity of bulk material will be required – say 8000 m³/km (Fig. 316) – depending on the height in relation to the adjoining terrain. The higher the existing embankment, the greater the quantities of bulk material required, and therefore the greater the costs. The stone fill and bank-strengthening materials, sloped to 1 : 2.5, as well as the sealing aprons of clay, must also be extended upwards.

⁷ Lowering the water level in a canal installation saves raising the banks and reconstructing the locks (see Fig. 359).

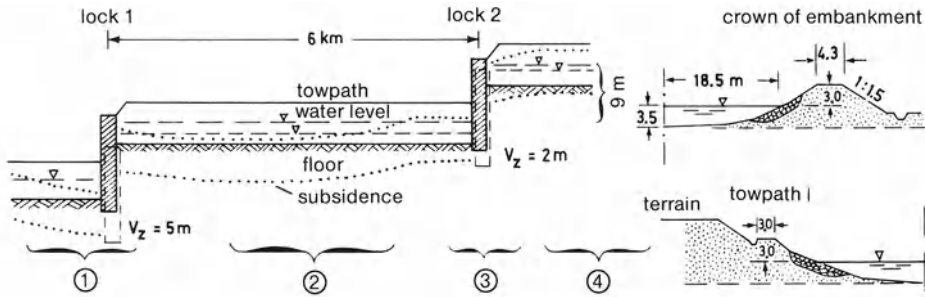


Fig. 315. Corrective measures to locks and canal levels which can be taken in the event of mining-induced subsidence up to 6.5 m. *Right:* profiles of canals constructed either by embanking or by excavation. (1) Heighten lock by 3 m, raise towpath by 5 m; (2) lower water level by 2 m, raise towpath by 4 m; (3) heighten lock by 0.4 m; (4) lower water level by 1 m, dredge canal bottom to 1 m deeper

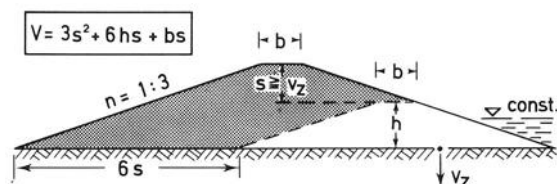
Massive lock chambers constructed of concrete and provided with movement joints every 15 m are largely unaffected by ground extension or compression. At most, thin walls in the area of the circulation channels, and of the pump and operations buildings, may crack and chamber floors may be forced upwards. With major subsidence, canal bridges have to be raised by the same amount to maintain the prescribed headroom clearance of, say 4.5 m. For this purpose, girder bridges of steel, which are easy to oversee in construction, are provided with a special seating to take lifting gear.

With sheet-piled canals, additional costs can arise if the particular piling profile is no longer made, with the result that, in raising the height, flat intermediate plates have to be welded on. In a harbour zone, subsidence can lead to neighbouring industrial installations finding themselves lying lower than quays that have been raised to accord with the apparent rise in the water level. In large rivers, the subsidence trough leads to a more rapid current in the area between the point of zero subsidence and the zone over the face edge, and consequently to a reduction in the depth of water. Filling in depressions in the river bed can provide a remedy.

12.5 Ground-water Damage from Mining

The subsiding of a flat stretch of terrain brought about by mining can have the following consequences:

Fig. 316. Volume V of embankment raising s in m^3 per 1 m length of embankment (after W. Schröder, 1967). h = height of old embankment



- a) alteration of the gradient in the streams, ditches, drains, and sewers which drain off the surface water, ground water, or waste water of townships; and
- b) reduction of the height (distance) between the ground surface or cellar floors on the one hand and the water level of the receiving drainage network and the ground-water table on the other.

Where the gradient in a *watercourse* or a pipeline is reduced, the flow of water is slowed down and diminished, with the result that water in tributary ditches and pipes banks up and silt is deposited. At trough centres and in natural depressions, the subsided ground surface at first comes closer to the ground-water table, and the ground in such places becomes damp and swampy if no steps are taken to drain it ⁸. If the ground subsides to such an extent that it lies deeper than the receiving drainage network or sinks below the ground-water table, the result is the emergence of lakes on the land and the penetration of water into the cellar basements of buildings. Consequent damages can on occasion be considerable if, for example, compensation has to be paid for loss of harvest in fields and pastures rendered temporarily sodden with moisture, or if areas turned into swamp or placed permanently under water have to be kept drained (continuous damages) or exchanged for equivalent land, or if pumps have to be installed in flooded cellar basements.

A watercourse which subsides along with the surrounding terrain overflows its banks (Appendix V), and the water level adjusts itself to a height at which inflowing stream water can drain away at the lower rim of the subsidence trough. If the terrain drops below the level of receiving streams, then natural drainage is no longer possible and water must be pumped up to a higher level (Fig. 317).

The high ground-water table in an area of subsidence, and the disturbance of natural flow in ditches and drains, are particularly harmful to agriculture. In ground that has become too wet, plant growth and yield contract, and it becomes difficult to work fields in the vicinity of steps and waves in the ground caused by subsidence. Ground water may become chemically contaminated by acid water forming in old opencast workings or by drainage wells and waste-water channels affected by mining. It can also happen that mining damage to fields, meadows, and woodlands is caused by excessive compression or extension of the ground, if the effect of such horizontal deformation is to tear the sucker roots of plants away from their main roots.

Where *ground-water* movement is unimpeded, the level of the water-table is maintained despite ground subsidence resulting from mining, because water can flow in from the edge of the subsidence trough and restore the original level. During subsidence therefore, the ground-water table only appears to rise as its distance from the ground surface (its depth underground) is reduced. By contrast, ground water without inflow and flowage, or a ground-water system isolated in lower strata, will go down with the subsiding ground. If damage to crop is to be avoided, ground water must be maintained during the growing season at a definite average depth below the fields – a depth which depends on the length of roots (25 cm in the case of grasses, and 1 m in the case of field crops) and also on the capillary limit of approximately 50 cm, depending on the

⁸ In an infra-red air photo these damp (cold) areas are seen as dark patches.

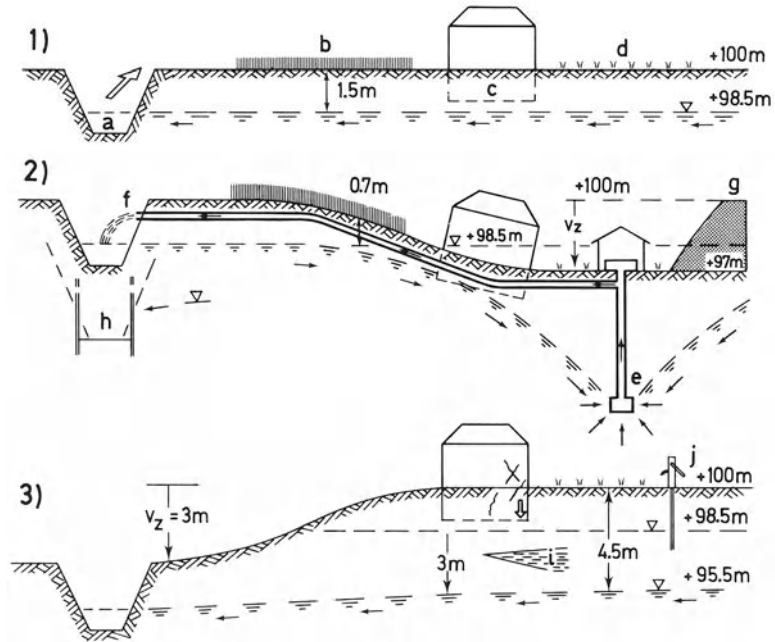


Fig. 317. Alteration in ground-water table's distance below ground. (1) Normal distance below ground before mining affects receiving stream *a*, cornfield *b*, cellar basement *c*, and meadow *d*. (2) Swamping and flooding of subsided terrain near stream. Remedial measures: pumping installation *e* with discharge pipe *f*; embanking *g* of vacant land; and deepening of receiving stream *h*, if necessary with sheet piling. (3) Receiving stream lowered by effects of mining, and neighbouring terrain dried out; cracks in building over dried-up peat bed *i*; no water in well *j*

grain size of the soil. For crops therefore, ground-water requires to be no more than 1.5 m underground, and for pasture land no more than 0.75 m (Fig. 317, top). In this connection it should be noted that the ground-water table can alter by several metres in the spring and summer, and that crop yield is influenced not only by the ground-water situation but also by ground humidity – i.e., by the distribution over time of precipitation and evaporation, or the so-called rain factor:

$$\text{rain factor} = \frac{\text{annual precipitation}}{\text{mean annual temperature}}. \quad (386)$$

If the surface of the ground subsides below the ground-water table, a pond is formed which destroys field produce and, furthermore, places the cellar basements of buildings under water (Fig. 317, middle). A swamped or flooded stretch of land can only be utilized and lived on again if it is linked to a new and deeper-lying drainage channel, or is enclosed and pumped dry, or is filled in and raised in level. Only in small agricultural holdings can the mine operator be absolved from undertaking land drainage, if the costs of connection to a new drainage system, or of continuous pumping, or of filling and raising the land exceed the crop yield and the market price of the land. In other cases, protection of the

terrain against flooding should take priority over economy. (In the Ruhr District, an area of 520 km² which has subsided below the ground-water table has to be drained by 250 pumps.)

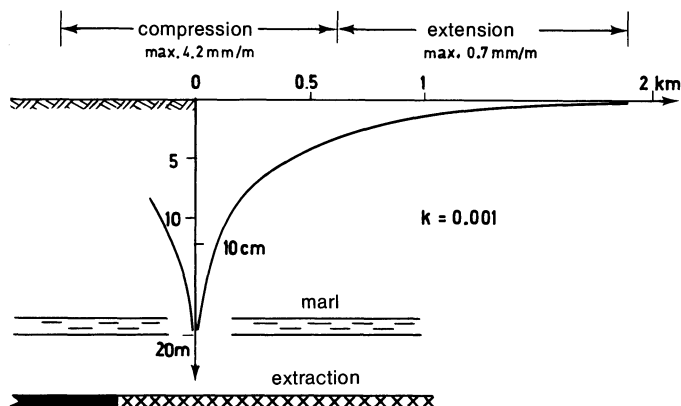
In some coal districts the task of regulating *land drainage* has been taken over by cooperative water associations, to which the mining companies pay a share of mining damage costs according to the degree of subsidence caused. By means of ditches and pipelines, for which land has to be acquired and bridges, underpasses, and culverts have to be constructed, a new discharge of ground water, rain-water, and waste water into a receiving network at a lower level than before is established (Fig. 317, bottom). Where large differences in level have to be overcome, pump stations are erected. Servicing an electrical water system with rotary pumps, assuming that every minute 1 m³ of water has to be raised 1 m, requires 0.25 kW of electric power. Maintenance and interest on capital together cost as much as the electric current. Where the throughput of water is greater – say 50 m³/min (see Darcy's formula) – horizontal pumps are to be preferred to vertical pumps because of their greater effectiveness. The capacity of pumps lying deep in the pump well or sump, and consequently fitted with a vertical drive-shaft, is governed by the normal influx of the precipitation zone concerned and the run-off curve for rain. In the Ruhr, for example, for an area of 500 hectares⁹ a long-term average influx of 12 m³/min and a peak influx of 360 m³/min have been measured, for which a pump capacity (10 pumps) of a total of 220 m³/min has been found adequate. The construction of a small pumping installation for draining a subsided residential settlement would cost approximately DM 2 m (say US \$ 1 m), including a pressure pipeline and a withholding basin placed ahead of the pumps. Lowering the ground-water level by mechanical means, which must be continued even after mine working has ceased, is thus very costly in the long run and incurs the risk in bad weather of an interruption in electricity supply for the pumps or even destruction of the installations. In flat, populated areas therefore, mining should be based on low-subsidence methods generally.

For filling in the flooded central zone of the trough – a step that can only be considered for unbuilt-up areas – mine wastes can be used, provided their rock salt and pyrite content does not significantly affect the quality of the ground water and of the receiving streams. Over the impervious washery wastes a permeable bed of sand, at least 60 cm thick, should be laid to drain the 40 cm thick topsoil after heavy precipitation. If the volume of the subsidence trough to be filled in is taken as being, with caving, nine-tenths of the volume of seam mined, a corresponding cost burden falls on the merchantable coal production in respect of the carriage and introduction of washery wastes. Further costs in relation to the topsoil, the sand bed, and recultivation have to be added.

Mining draws large quantities of water into the underground workings via joints and water-bearing strata – in the Ruhr, for example, approximately 1 m³ of water per 1 Mg of coal produced – if no impervious strata intervene and the safety pillar, consisting usually of marl 20 m thick, begins to let water. Wells, springs, and industrial water-extraction installations then dry up. The ground-

⁹ 1 hectare = 10,000 m².

Fig. 318. Outline of ground-water table and of ground settlement (max. 146 mm) when water seeps away 20 m underground through break in bed of marl over a working. (After data of Kögler and Rom, 1939)



water table drops in a funnel-shaped outline to the areal or linear shaped drainage zone (Fig. 318). The extent R of the ground-water subsidence funnel depends, according to *Sichard*, on the maximum subsidence depth h at the draining point and on an empirically established index of permeability k :

$$R = 3000 h \sqrt{k}. \quad (387)$$

A funnel of depth $h = 10$ m extends approximately 1 km across the sand bed ($k = 0.001$ – i.e., rate of flow is 1 mm/s, given a gradient of 1). The cross-sectional funnel outline is in the shape of a hyperbola which follows a very flat course at the edge of the trough and is only 1 m deep at a distance of 500 m from the point of water extraction in the sand bed. As has been observed, the removal of ground water, even without the washing away of solid and soluble matter in cohesive, compressible strata, causes settlement amounting to several centimetres or even decimetres. The curve of settlement, like the funnel-shaped outline of the lowered ground-water table, is hyperbolic in shape and can be used to derive tensile and compressive strain in the ground. For example, compressive strain at 4 mm/m in the vicinity of the point of water extraction is considerable when compared with a tensile strain of 0.7 mm/m and is undoubtedly harmful to building structures (Fig. 318). Fluctuations of up to 5 m in the water level of great rivers like the Rhine make themselves felt in changes in the marker level of wells several kilometres inland.

12.6 Settlement Damage Ahead of an Opencast Face

In the zone ahead of an opencast face, ground water must be lowered to the level of the floor of the workings by sinking wells to keep the workings, the steep slopes around them, and the coal-seams dry. In the lignite field near Cologne¹⁰ for example, lowering of the ground-water level by up to 300 m was necessary

¹⁰ In the year 1974 some 1,300,000,000 m³ of water was removed there. This had no deleterious effect on plant growth, and surface water was evidently sufficient for the vegetation.

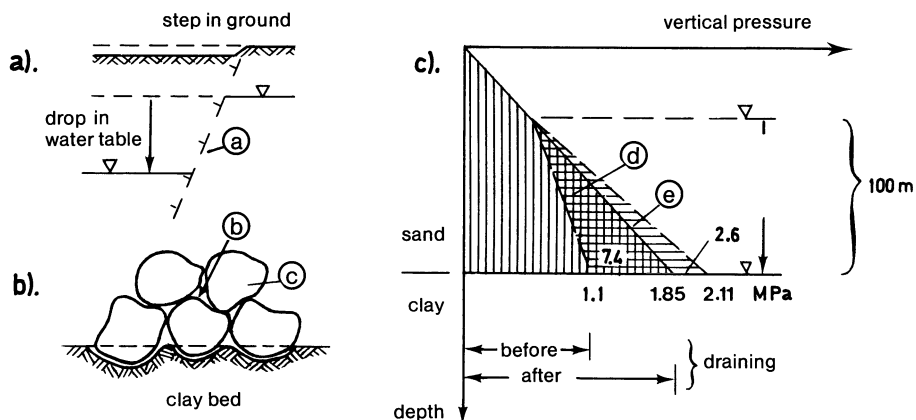


Fig. 319. Ground settlement when ground-water table is lowered. (a) Terrace over impermeable fault a; (b) grains of sand press into bed of clay: b pore water; c sand grain; (c) change in vertical pressure after water drains away: d load relief through buoyancy from water; e weight of pore water

and produced settlement of the ground surface equivalent to 1‰ or 2‰ of the drop in ground-water level – i. e., settlement of the order of 30 to 60 cm. This *ground settlement* is connected with the increase in weight of the sand when drained, which consequently presses more heavily into the underlying bed of clay. For example, a 100 m thick layer of soil consisting of cohesionless grains of equal size in a rhombic pattern, which rests on an impervious bed of clay and exhibits an available porosity of approx. 26%, weighs after draining $2.5 \text{ g/cm}^3 \times 74\% \times 10,000 \text{ cm} = 18.5 \text{ kg/cm}^2$. In the water-saturated state the particles of soil, relieved in load by the buoyancy – the weight of water displaced by solids – previously bore down on the clay footwall by an amount which was less by $1.0 \text{ g/cm}^3 \times 74\% \times 10,000 \text{ cm} = 7.4 \text{ kg/cm}^2$ – i. e., only with a weight of 11.1 kg/cm^2 (Fig. 319). This increase in weight from 11.1 to 18.5 kg/cm^2 and in pressure from 1.11 to 1.85 MPa is transmitted to the particles of clay, which become compacted by the translation movements and force the clay pore water into the bed of sand above. This consolidation process, which is associated with settlement¹¹, is superseded at a certain depth of the clay bed by a comparatively small degree of lift, resulting from the reduction in the combined loading of the clay footwall – the effective pressure and the neutral pressure together – by the weight of water withdrawn, namely $1 \text{ g/cm}^3 \times 26\% \times 10,000 \text{ cm} = 2.6 \text{ kg/cm}^2$ or 0.26 MPa. Among the grains of quartz, by contrast, elastic compaction ($E = 3 \times 10^6 \text{ N/cm}^2$) amounts to only 1 mm. There is usually no compacting rearrangement of grains in the sand bed, and squeezing of the at most $40 \times 10^{-7} \text{ mm}$ thick film of water at the points of contact between sand grains similarly causes only 1 mm of settlement (Fig. 278).

Thus, when the ground-water level is lowered, settlement can be expected only if there are cohesive strata present. If settlement does occur, structures at

¹¹ The stiffness indices E_s needed to assess settlement in individual strata on the basis of Eq. (29) are, in N/cm^2 : lignite 1,000–3,000, loose sand 2,000–5,000, dense 5,000–15,000, gravel 15,000–20,000, semi-hard clay 500–1,000, standard clay 100–250, loam 500–2,000, and peat 40–100.

the surface subside but, because of the very large and flat subsidence trough, in general suffer no damage from tensile or compressive strains in the ground and are not significantly tilted. In lignite districts therefore, the burden of damages payable for mining subsidence is small. Mining damage is considerable in places where pockets of peat have been built over and settlement of up to 1 m occurs, or where, because of geological faults, the water-table sinks only in one block of strata and a settlement step of several decimetres develops at the surface of the ground (Fig. 319). In a diagram recording settlement, the tectonic structure of the drained terrain shows up clearly in steps in the ground above impervious fault planes.

The points of contact between grains of sand, which are in practice areas of contact rather than points, reduce the buoyancy. This is because the grain of sand is not exposed over its whole surface to the water pressure, which acts more strongly on the underside of the grain than on its upper surface, depending on the grain diameter. In the case of a consolidated but porous bed of sand – e.g., an aquifer or a reservoir rock – the buoyancy effect is completely cancelled out by the weight of water in pore channels. From numerous settlement observations it has been possible to derive a formula for ground settlement y (in mm) for the Rhine lignite district as a function of the lowering of the ground-water level x (in m):

$$y = 2.28 a^{-6.89} x^a \quad (388)$$

where a signifies a locality parameter of the order of 0.15 to 1.5 (Varnhagen, 1967). If the drainage of water is stopped, ground water rises again to its previous level and can penetrate into the cellars of newly erected houses if they have not been rendered watertight.

In *coal-mining*, the surface of the ground will rise by several centimetres as a result of buoyancy load-relief if the water-table in abandoned workings rises. In lignite mining, overburden tips will settle by about 1‰ to 3.8‰ of their height over a period of some 11 years. In the Aachen coal district the draining of water has for many decades brought about ground settlement in a ratio of 4:1 – i.e., 4 m³ of water removed is equivalent to a subsidence trough volume of 1 m³.

Structures at the edge of an opencast working can be severely damaged if cracks which develop in the sloping banks around the working extend to the ground under the structure's foundations and produce step-like deformations there. An *embankment slip* such as this can be set off by a fluctuation in the ground-water level, by the development of a slip plane running from the clay footwall to the working, or by a tilting movement in a small-sized coal pillar (Fig. 320). On the removal of water and the consequent elimination of buoyancy load-relief, the weight of the layers of soil, and also the cohesion and friction resistance along the slip plane, are altered. These external and rheological influences can disturb the equilibrium between the impeding forces F and c and the impulse force G – i.e., between the resultant F of the normal force (F_n) and frictional force (F_t) on the slip plane, the resultant of cohesion c and also the resultant of the pressure of water percolating and the weight G of the slip masses. The consequence is that any coal pack present is displaced along an almost horizontal slip plane towards the opencast working, while the masses of soil pressing behind

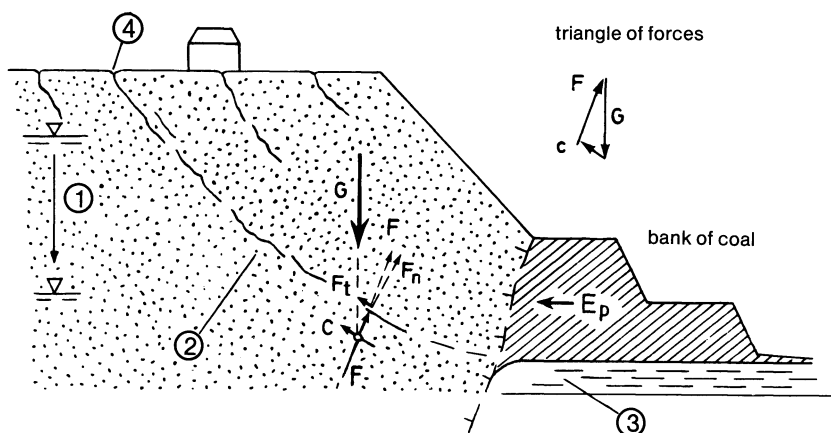


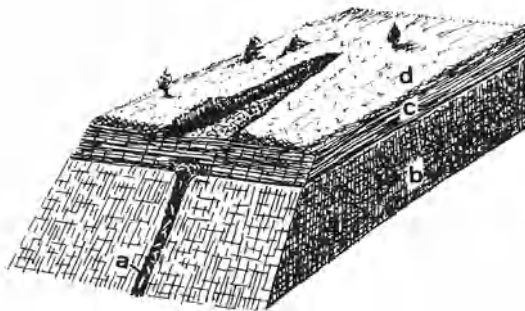
Fig. 320. Cross-section through 100 m high slope round opencast mine, showing formation of crack planes after lowering of water-table. (1) Drop in water-table; (2) slip plane; (3) bed of clay; (4) cracks and steps in ground

it slide sideways into the working along a crack plane inclined above it at an angle of some 55 gr. In the terrain ahead of the face, further and smaller cracks can in certain circumstances lead to the formation of blocks of ground, which then subside individually to the extent that the coal pack, or the forward block of ground, gives way. Calculations of ground statics proceed from a line of slip which in section is either straight, curved, or both straight and curved in parts. More dangerous to structures than cracks in the ground are the decimetre-high steps which develop at those cracks. The ground surface is first extended in the direction of the opencast working by 1 or 2 mm/m and then compressed in places under the influence of the smaller cracks which develop. Besides cm-deep subsidence movements, mm-high heaving of the ground can be seen here and there as blocks of ground behind exert a delayed pressure (thrust). The laws governing the effects of mine extraction do not apply to these complicated movement processes affecting blocks of ground which break off along the high sloping sides of an opencast working.

12.7 Surface Cave-ins over Abandoned Workings

The increasingly dense population in long-established mining districts has in recent years thrown up a new mining subsidence problem. This is the sudden emergence of surface cave-ins and irregular depressions in the form of funnel-shaped glory holes over innumerable, inadequately filled shafts and adits, or over abandoned near-surface mine workings and deeper excavations. As a result of weathering, erosion, and leaching, mine openings such as these can in the course of time lose their stability and collapse or gradually give way. This kind of collapse in room-and-pillar workings, the timing of which cannot be foreseen, makes it appear advisable in populated areas to change to extensive mining on a long front, inducing immediate settlement of the roof in a subsidence wave (Sect. 2.5).

Fig. 321. Gully over an old working. **a** Old, steeply inclined working; **b** Carboniferous strata; **c** caprock from the Cretaceous; **d** layer of soil. (After Hollmann, 1968)



Near-surface working of steep coal-seams constitutes an area of risk for structures, even if the mine excavations are filled in. In the course of time the stowage material in the excavations settles, leaving free space in the upper part into which unmined coal crumbles, until the ground at the surface sinks like a trench along the strike of the seam. Through the action of percolating rain-water, fine particles of soil can be washed down through fissures in a thin caprock into the old mine workings (Fig. 321). At the surface a wide and elongated gully forms without the solid surface layer actually cracking.

Over the edges of flat-lying coal-seams at shallow depths, fissures which developed many decades earlier can become active again, even after workings have been filled in. In areas of disturbance in the extension zone, CH_4 gases often escape and discolour the ground dark blue, damaging plant growth.

In the mining of vein ores also, cracks over 100 m long with almost perpendicular sides can form in stable strata.

Areas of the surface considered to be at risk from old mines are marked out in a profile drawing using angles β of break (Fig. 119). The size of each area affected is established by means of statistical investigations, taking account of type of workings, strength and thickness of roof, fluctuations in ground-water level, and planned utilization of the surface, in such a way that the risk of a cave-in outside the area demarcated is minimal and economically justifiable (Fenk, 1976).

Over mine workings exposed to weathering and leaching, and lying at depths of up to 15 m in the case of vein ores, 60 m with hard coal, and 400 m in potash mining, the effects of mining can be expected to become apparent over an unlimited period of time, implying a permanent risk to the surface.

If the old mine charts in a planned development area are incomplete, an attempt must be made to locate near-surface workings by geophysical means or by drilling on a grid pattern (Bell, 1975). Gravimetric and seismic anomalies resulting from the workings, or differences in ground temperature, are often insufficient for a clear definition of them. In the latest process of acoustic prospecting, compression waves of a length appropriate to the workings being located are used.

Fissure-like openings form at *shaft* heads if stowage material introduced into the shaft well collapses. In measures to preserve shafts therefore, care must be taken to ensure that fill material does not later settle or migrate into shaft stations.

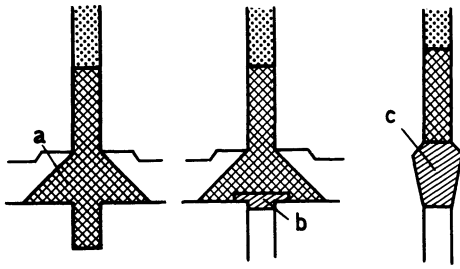


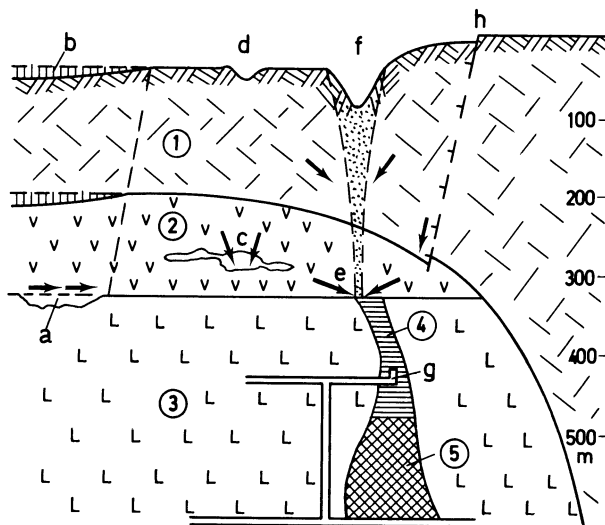
Fig. 322. Filling-in abandoned shafts in the loading-station zone. **a** Bank of rubble; **b** cover plate; **c** solid seal. (After Eckart, Hüls, and Klemm, 1976)

Figure 322 shows how the fill material in the shaft can be sealed from the empty space remaining in the shaft or shaft station. Strata serving as an abutment must be sufficiently stable to absorb the load of the column of fill and, if necessary, the hydrostatic pressure as well. For this purpose a weathering-resistant ballast of crushed rock is well suited and gives a silo effect in dry shafts: that is to say, the increase in vertical pressure γh in the column of ballast resulting from its dead weight is compensated, starting from a certain height in the column, by the increase in horizontal normal force F_n acting on the shaft wall, and in frictional force F_t by Eq. (337) at the shaft wall. Where there is a shaft sump below the shaft station, a sloping bank of ballast in the shaft station is sufficient for lateral support of the upper column of ballast (Fig. 322a). A shaft which is stable and dry below the shaft station can be closed off with a slab of concrete and left unfilled (Fig. 322b). Above the shaft station, a monolithic, tight-fitting concrete seal can be introduced (Fig. 322c). And above the short column of crushed-rock ballast, an inexpensive stowage material should be used as fill.

In the shafts of salt-mines, clay courses in the fill material can serve to hold back the ingress of water. The roof over workings near the surface becomes weakened by weathering and the flushing out of loose constituents to the point that it suddenly collapses (caving to the surface). After abandonment of working the mine structures drown, and in salt-mines the horizontal supporting pillars then dissolve in water and disintegrate. The collapse of individual mine workings produces repercussions at the surface, and where the subsiding panel is extensive, a broad subsidence trough forms through fractureless deformation.

In saline strata, *sink-holes* can arise in various ways (Fig. 323). Flowing ground water can dissolve away whole areas of the salt body (subrosion), and the loss of solid salt rock leads to a trough-like depression at the surface (b). To dissolve 1 m³ of carnallite down to its indissoluble residue amounting to 0.44 m³ (principally sodium chloride, potassium chloride, and kieserite), 587 kg of fresh water is necessary. Disintegration of the saline rock continues until the incoming water has become transformed into a solution (a lye) which stands in chemical equilibrium with the rock in place – i. e., until it is completely saturated. If the saturated solution can escape into the country rock and is replaced by further fresh water, solution of the saline rock continues. Cavities (c) in the gypsaceous caprock may collapse when water escapes, and a funnel (d) then forms suddenly at the surface. Generally, saline strata are protected from solution by a bed of lye at the foot of the ground-water body. If, however, water in the surface layer

Fig. 323. Schematic representation of natural or mining-induced sinkholes over salt bodies. (1) Bunter Sandstone (Lower Triassic); (2) gypseous caprock with saline water in cavities; (3) rock salt; (4) carnallite and kainite; (5) filled-in workings



finds a means of access (e) to the mine, the area of collapse usually expands rapidly, water in the surface layer drags rock down with it by suction, and in a very short time a funnel (f) forms at the surface, often up to 80 m deep and 100 m wide.

In a potash mine near Hanover, some 7,000,000 m³ of water and mud from surface strata flowed into mine workings in this way within a period of only 2 days in 1975. Besides an extensive subsidence depression, several funnels up to 10 m deep and 50 m wide formed at the surface (Appendix V). After 3 months the ground-water table, which had sunk by 40 m, had recovered to its old level, and a drop of up to 10 cm over a large area had entirely disappeared.

In abandoned salt-mines (g), water can penetrate with the passage of time via geological faults (h) lying within the extension zone and so dissolve and weaken pillars that a whole panel of workings can collapse (Baar, 1977).

12.8 Pseudo Mining Damage

Many instances of damage to buildings and pastures in a mining district are not the result of mining activity, as evidenced by observations in areas free of mining. Nevertheless, it is usually very difficult to distinguish such damage not due to mining from mining damage and to expose the non-mining cause convincingly in a court of law, if mining is in progress or has ever been conducted in the area concerned (Table 22). The cause of damage, even for pseudo mining damage, is usually some movement of the ground, caused however not by mining but by a ground-water movement or a tectonic movement (natural causes) or by traffic vibration. In this category of damage belong also inadequacies and errors in building construction, particularly in the matter of foundations, the withdrawal of ground water, as also chemical or thermal influences (artificially induced causes).

Table 22. Summary of non-mining damage to buildings

Pseudo mining damage	Cause	Nature of damage
Natural	Shrinkage of soil in building site: drying-out by sun, or absorption of soil moisture by trees	Spalling-off at house corners
	Building ground varies in consistency: part sandy, part cohesive, with pockets of peat	Bending cracks from heaving in ground
	Changes in ground-water level: rain, flooding, water scarcity, softening of ground	Settlement cracks between buildings of differing weight
	Landslip: thrust effect Frost heave: ice expansion	Tensile cracks Flaking
Artificially caused	Faults in foundations: part-cellaring only varying load varying depths of foundations	Settlement cracks
	Faults in construction: thermal expansion of building materials neglected lack of separation joints	Separation and shear cracks
	Faults in materials: chemical reactions sensitivity to frost absorption capacity poor finish	Spalling Crumbly mortar
	Vibration shocks	Damage to plastering Cracking
	Erosion of building site	Settlement cracks

Examination of every instance of mine damage reported to the mining company should begin by clarifying the following *questions*:

- a) Did the damaged object lie, at the time that damage became evident, within the area of influence of a mine working?
- b) Could older workings have contributed to the damage?
- c) Does the tilt in the structure lie in the direction of the mine working?
- d) Does the damage – from extension or compression, from convex or concave curvature – agree with the position of the building in relation to the mine working according to Fig. 142 ?
- e) Do cracks in the building run approximately parallel to a face edge or, in the case of a hillside position, parallel to the slope?
- f) Within the area of damage, is there knowledge of any anomalies such as lenses of peat, back-swamp areas of a river, or the like in the area built on, or of similar damage to other buildings?
- g) Have fluctuations in the ground-water level been particularly great during the year in which damage occurred ?

- h) Is ground water extracted, or has it ever been extracted, in the vicinity of the structure?
- i) Is the structure of homogeneous construction and materials, and does it have any outbuildings or disproportionately tall sections?

It is only when these questions have been answered that investigation is undertaken at the building itself – assuming that mining damage has not been established beyond doubt. In the case of agricultural damage, a check must be made whether there has been any attack by pests, whether inadequate or faulty fertilizing has taken place, and whether there has been obnoxious penetration originating from any neighbouring industrial installations.

To secure evidence it is advisable to photograph all damaged buildings outside the area of mining influence, in order to be in a position to refute unjustified claims for damages in respect of mining at the time of later mine working.

12.8.1 Natural and Artificially Induced Damage to Structures

Among the *natural* causes of damage not due to mining, the most frequent is a change in the state of the ground built on related to its water content (Fig. 324). Besides settlement of each new building brought about by its own dead weight, which with a magnitude of several centimetres comes to an end about one year after its completion, these changes include:

- subsidence as a result of shrinkage of the cohesive subsoil;
- subsidence through the de-watering of a sandy or fissured building ground;
- subsidence through the softening of a clay building ground;
- subsidence through erosion or suffusion of fine particles in the subsoil;
- subsidence in association with landslips;
- subsidence of parts of a building standing on differing types of ground;
- subsidence as a result of leaching of the ground.

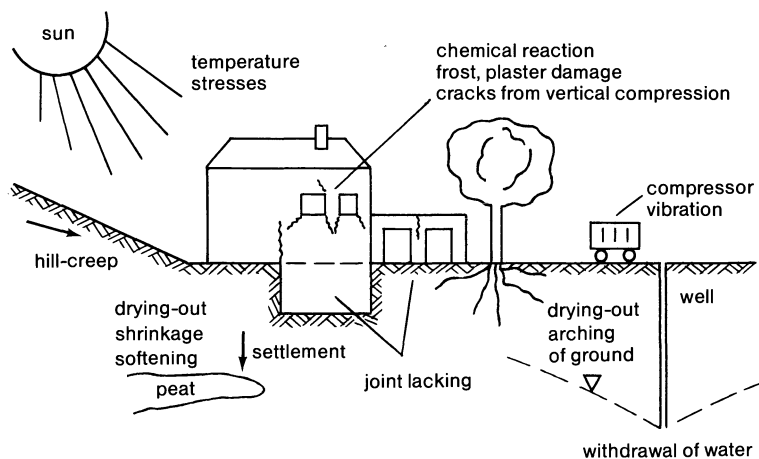


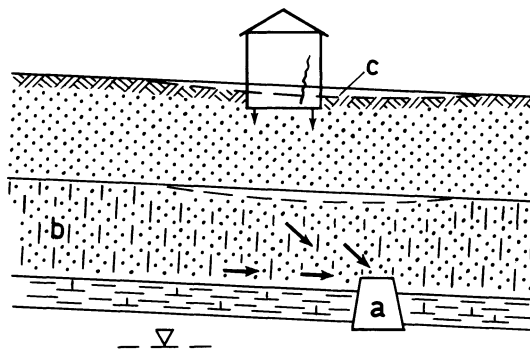
Fig. 324. Cause and effect in pseudo mining damage as seen in a building

In cohesive soils such as clay, loam, gouge, or marl, or in soils containing organic matter, such as peat bogs, if the natural water content is reduced by evaporation, disintegration or vegetation, the loss of water is reflected in a reduction of soil volume. This process, in which soil pores freed of water close up under the dead weight or overlying load, is known as shrinkage or consolidation. The cubic measure of shrinkage of cohesive soils is of the order of 15% – 30%. If the ground takes up water again, its volume increases. This is known as “swelling” and occurs only if stresses in the ground produced by the dead weight or the overlying weight are less than the swelling pressure. Clay shales may swell perpendicularly to the stratification by up to 5%. Non-cohesive soils such as sand, gravel or pebbles, do not shrink on de-watering, but their grain contact pressure increases with the removal of buoyancy. An underlying cohesive stratum is vertically compressed as a result of this increase in pressure. The flat subsidence trough formed in this process, however, presents no risk to building structures. As an example, through the removal of water in calcareous subsoil underlying London, a subsidence basin 20 km wide and up to 20 cm deep at its centre under Hyde Park formed in the years from 1865 to 1931. And as in the extraction of natural gas or petroleum, reservoir rock converges in proportion to the quantity of water, gas or oil removed as the supporting gas or fluid pressure diminishes.

By contrast, shrinkage of cohesive soil in a construction area resulting from locally differentiated drying-out is as a rule uneven, with the consequence that buildings may crack at one corner or suffer damage through the bending of their foundations. On the southern side of houses, in the vicinity of rows of trees or of drainage pipes, at the side of foundations or on lower slopes – these are all locations where the soil in a construction area will dry out quicker and deeper than on a built-over and shady anterior side, or in the centre of foundations. The base of a building will arch as a consequence. Pseudo mining damage of this type is especially to be suspected in damaged buildings having a small plan area.

In river basins and in the catchment area of a carrier watercourse, where the falls in ground water and in precipitation run-off in the direction of the watercourse are determined by the level of the water-table in each case, loose sands can be carried along by the ground-water stream if it flows back to the river under high pressure after being banked up by floods several kilometres inland. Structures on ground affected unevenly by this process can suffer tilting. Fine particles of soil may be washed away if ground water can flow into a tunnel or a deep underground excavation (e.g., for an underground railway; Fig. 325). Because the flow gradient is increased, the drawing power of the ground water is also increased (suffosion). Fluctuations of up to 1 m a month in the ground-water level during the period of crop growth, brought about by the interaction of rainfall (in Germany approx. 60 mm a month on average) and evaporation (monthly average 20 mm approx.), can lead to a shortfall in produce yield quite independently of mining subsidence. On hillsides and slopes, structures may be damaged by “hill-creep” or by landslip in a loam layer lubricated by water absorption. Through frost-heave (ice expands linearly by around 1 mm/m on a lowering of temperature to -20°C) damage is caused in particular to road surfaces laid on insufficiently drained bedding courses.

Fig. 325. Fine soil particles washing away with ground water, and resultant subsidence trough (c). An old mine roadway (a) acts as a drain in an otherwise impermeable bed. Cohesive bed (b) of fine sand



Weathered and softened rock in the building area gives way on an increase in load. Previously dry beds of clay and loess become soft on an ingress of water – e.g., from defective drains or roof guttering – thereby producing settlement in houses built on it. Over leached-out saline or gypseous rock, or over eroded calcareous rock and displaced jointing, sinkholes and step-folds can develop which lead to settlement cracks in the structure. Ground water percolating in saline rock in Lüneburg, in North Germany, led between 1898 and 1970 to settlements of 2 m and to irregular ground displacements of up to 70 cm.

Finally, geologically recent tectonic forces in flexed strata can set off, on the release of stress, ruptures which have the effect of earth tremors at the surface and, at an intensity of 5 on the Mercalli-Cancani-Sieberg earthquake scale, can crack open the plastering on a house. In Upper Silesia, subsidence and linear deformations of the land surface in the order of 15 cm or 0.6 mm/m, which were probably due to tectonic strata movements, were observed over a period of 20 years (Niemczyk, 1949). Slow-developing subsidence movements of decimetre amplitude have been observed even in the Ruhr District away from areas affected by mining. In this connection mention must also be made of smoke damage to the assimilation and transport organs of plants, which affects their growth, and which should not be confused with the reduction in yield associated with ground-water changes resulting from mining. Tar and sulphur-containing waste gases are particularly harmful to grain, meadow grasses and potato, even at a distance of 600 m from the smoke source.

Among forms of damage to structures arising *artificially* and not due to mining can be counted, in first place, inadequate foundations and faults in construction or materials. For example, foundations may be set insufficiently deep and in ground not really capable of bearing the load, or on dumped and uncompacted ground, with the result that settlement cracks occur and pieces break off. Parts of a structure standing on shallow foundations will experience greater settlement than parts on deep foundations, and the structure will shear off vertically (Fig. 324). In winter, where a building has cellar basements under only parts of it, pockets of frost will form under the level sections and exert an upward heave on the structure (Fig. 326, left). Diagonal cracks are the result. If on thawing the water cannot drain away easily, the ground will become softened and the sections of the building not underlain by cellars will sink back below their initial level. The see-saw movement produces crossed cracks.

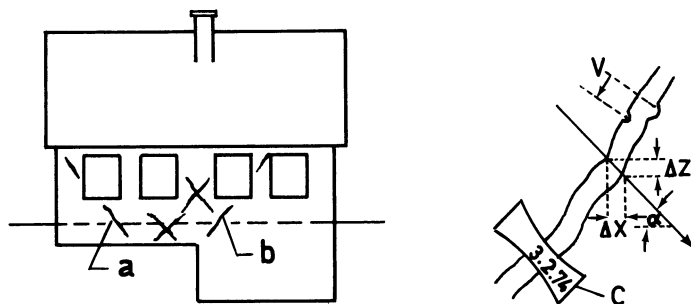


Fig. 326. Settlement cracks (a) and frost-heave cracks (b) in an only partly cellared house (left). Gypsum marker (c) for monitoring a wall crack (right)

A relatively heavy building within a row of buildings can create a base pressure in the ground which encroaches on the foundations area of neighbouring buildings (Fig. 327, left). Shear cracks and tilting are the consequence. If a gap in a row of houses is later filled, the consequence is likewise delayed settlement in neighbouring buildings, even if the new building is no heavier than the others (Fig. 327, right).

Faults in construction, which later find expression in cracks, frequently arise on the fitting together of bearing structural members, on the bonding of steel constructions with brick or concrete walls, or in the distribution of loads to the foundations. Faults in materials or in execution often become evident in peeling plaster or in sagging arches, window and door lintels and abutments. Timber members alter in length with humidity content, whereas structural elements in metal or concrete undergo temperature-dependent stresses and deformations. If the outer walls are in hard-burnt brick and the inner walls in sand-lime brick, which is prone to a high degree of shrinkage, differential reduction in wall dimensions can occur on drying out, and this can lead to strains, cracking and ceiling arching, especially in the upper floors.

A number of frequently encountered *faults in construction* or sources of damage requiring to be distinguished from mining damage can be mentioned in list form:

- movement joints omitted at structural and material transition points;
- natural stone which absorbs water, swells, and disintegrates;
- external plaster on an insulating base, or on gas concrete or lightweight tiles, peels off;

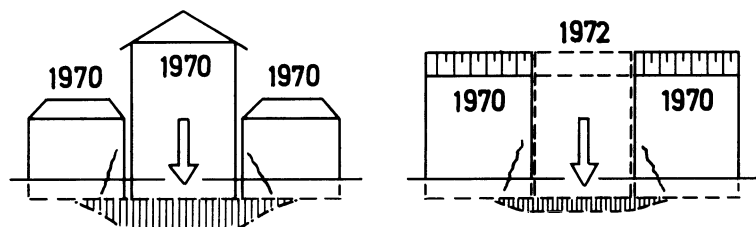


Fig. 327. Left: Simultaneous construction of buildings of differing weight. Right: Subsequent closing of a gap in buildings. (Settlement trough hatched)

cracks form and plaster peels on mixed masonry of varying brick types;
 plaster over water-absorbing mortar joints becomes burnt;
 one-sided exposure to the sun creates stresses in the structure;
 packing material in joints becomes brittle and shrinks;
 efflorescence behind non-watertight exposed masonry;
 jointing mortar porous and not binding properly because applied too dry;
 absence of edge-isolation strips in glass-block walls (heat expansion);
 floors in cellars rendered insufficiently watertight;
 steam pressure and freezing water-of-condensation acting destructively in walls and ceilings;
 gypseous outer plaster becoming detached from gypsum-free underplaster;
 external bricks must not be sensitive to frost;
 cold-bridging, or coats of black paint which absorb heat, lead to cracks;
 edge-pressure destruction in prefabricated concrete members because of absence of joints;
 abutment-pressure cracks in pillar and window and door openings.

If cast-iron piping is laid at less than the frost-free depth of 1.60 m, there is a risk that it will crack in winter. In *furnace installations*, the inside walls of the firing channel are 1000°C hotter than the outside surface during operation. At a temperature of +1200°C, chamotte brick will have expanded linearly by almost 1%, and so if there are insufficient expansion joints, compression damage will result. The longer a firing kiln is left in operation without interruption, the deeper will the heat penetrate into its base. The ground under the kiln will dry out and the clay beds will shrink, with the result that the kiln often subsides unevenly by several decimetres. Thermal diffusion indices enable heat flow in brickwork and floor to be calculated. In the case of *brick walling*, the annual, climatic temperature fluctuation of 50°C exerts a harmless linear deformation of 0.25 mm/m. Most bricks, however, contain soluble salts of sulphur, which attack the Portland cement mortar of free-standing, wet walls. The mortar consequently expands, and damage similar to a compressive mining influence results. Chimneys can by this process be set at a tilt away from the weather side. Walls with rusting iron inlays crack open. Floor slabs carried on iron girders may dilate by 2 cm over a width of 6 m as a result of rust expansion. In civil engineering works below ground (e.g., for an underground railway), intersected float-sands may run into the excavation and cause the ground on which neighbouring houses are built to subside unevenly.

Traffic and machinery *vibration*, arising from the impact of rolling-stock wheels on end-joints in railway lines and from unevenness in roads, as well as from rams, compressors, and other large machinery, damages structures up to 50 m distant if the oscillations of 10 to 50 Hz, of which the horizontal component is particularly dangerous, coincide with the innate frequency of the structures. A further circumstance in which damage is caused is if the product of amplitude and frequency number comes to more than 0.4 cm/s, or if the rate of vibration, as measured with electrodynamic instruments, exceeds 5 cm/s. In blasting operations, oscillations can be as much as 0.1 mm in amplitude. At depths of over 60 m and distances of over 100 m, however, mine blasting presents no risk to

objects at the surface. For the recognition of vibration damage it is noteworthy that the size of oscillations, and consequently the damage to structures, increases from the bottom upwards.

The tendency to *crack* formation under tensile, compressive and bending stresses is represented in Fig. 300. If it is desired to check whether the cause of the cracking corresponds to the effects of mining, attention must be paid to whether the course of the cracks is oriented towards openings in the brickwork (windows, doors) and follows the line of least resistance. In investigating the cause of damage it is necessary to imagine the force that would be required to close up the crack in the brickwork. Two identical points on either side of the crack are sought which lay together before the fracture. The purpose is to establish with a magnifying glass whether the crack in question is a shear fracture in which one crack edge has moved by the amount v in relation to the other (Fig. 326, right). From the shape of the crack contours and the direction of the fracture points, it can be seen which part has been displaced. In a separation fracture with a wide split under a tensile force acting perpendicularly to the line of crack, $v = 0$.

In addition, the displacements Δx and Δz are measured and the angle of cracking α determined from $\tan \alpha = \Delta z / \Delta x$. The angle of cracking generally points towards the focal point of the deformation which has produced the cracking. The further development of crack formation can be followed with the aid of gypsum markers fixed firmly to the plaster and brickwork in the shape of a swallow-tail and marked with the date.

The list of non-mining causes of damage enumerated above could be further extended, but in the event of a claim the mine owner will be able to find proofs to differentiate from true mining damage only the particularly conspicuous types of pseudo-subsidence mentioned. In the construction field there is need for more research than hitherto into the deformation behaviour of the parts of a structure and of the area built on, as well as into the bonding properties of the newer construction materials.

12.8.2 Frequency of Structurally Induced Damage

In recent years attempts have been made to distinguish between mining damage and damage caused by changes in the structure itself, through a reconstruction of the processes of structural movement or change based on manifestations of damage and analyses of causes (E. Hoffmann, 1973). It became evident that the scope for speaking of typical mining damage and typical structurally induced damage is limited. In distinguishing between the two kinds of damage, only the cause of damage is significant, and the nature of the damage is in many cases the same. Cracks in buildings caused by mining are found predominantly at cellar and ground-floor levels; because of the tension-flange effect of the floors, these cracks peter out in the upper storeys. By contrast, cracks due to faults in materials or bonding multiply in the upper storeys. Accordingly, in simple cases it is often possible to draw conclusions about the cause of damage from the location of the cracks. A further indication is the direction in which the cracks run.

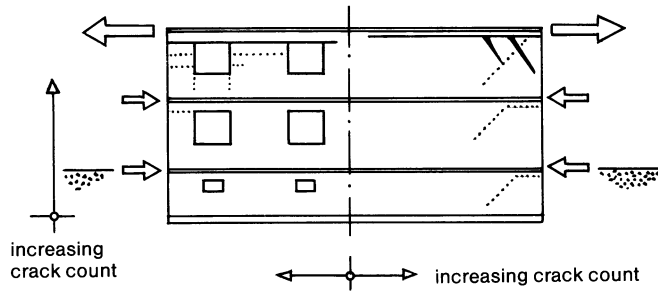


Fig. 328. Position and direction of structurally induced cracks (after E. Hoffmann, 1973). Increase in temperature in storey floors, and shrinkage in concrete. *Left:* house with window openings. *Right:* development of cracks without the influence of openings in walling

In the case of *cracks* associated with structural movement or change, the investigations made enable the following varieties to be distinguished:

- Shear cracks in the understructure of floor slabs and roofs, running horizontally in the first course of the brickwork in cases where concrete floor slab and brick wall are firmly interlocked, or passing diagonally through the brickwork along its vertical (butt) and horizontal (course) joints in cases where wall and floor slab are separated by an expansion joint. Cause: linear deformation of the concrete slab through thermal influences, contraction strain and creep strain (Fig. 328).
- Horizontal detachment cracks in the walls in the first course joint above the floor. Cause: floor slab sag as a result of effective load, contraction and creep bending (Fig. 329).
- Horizontal tilting cracks in the external walls at the level of the lower or upper floor girders, which become canted with the bend in the floor (Fig. 330). Cause: as under b).
- Twisting cracks at the corners of crosswise stressed floors if the lightly loaded slab corners bend upwards and become detached from the understructure (wall) (Fig. 331). Cause: as under b).
- Bending cracks in walls, which follow compressive stress trajectories or lines of weakness. Cause: as under b).

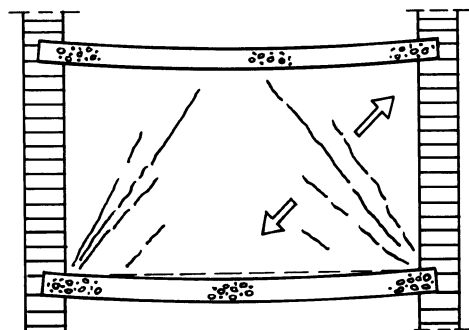


Fig. 329. Cracks in a wall over a flexed storey floor. (After Pieper, 1970)

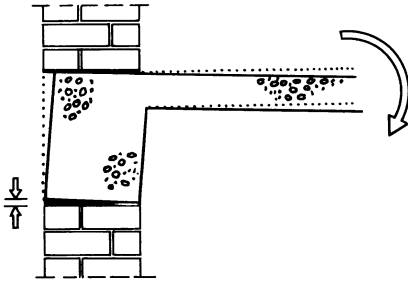


Fig. 330. Horizontal cracks on the canting of a floor. (After E. Hoffmann, 1973)

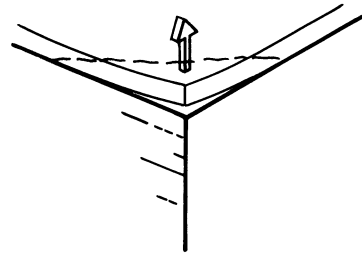


Fig. 331. Crack at the corner of an upward-bent floor. (After E. Hoffmann, 1973)

- f) Cracks from outward bowing of external walls, running diagonally through them. Cause: temperature gradient in the wall from irradiation by the sun or from external cooling and differential linear deformation on either side (Fig. 332).
- g) Vertical cracks in the structure. Cause: changes in strength, loosening in bonding.
- h) Plaster cracks on a grid pattern. Cause: varying thermal expansion in the wall layers.

The sagging of the floor slabs is the result, in part, of elastic sagging caused by the effective load, in part of creep strain under the long-term load, and in part of contraction strain from the drying-out of concrete. The effect of cracking is to produce constructive equilibrium in the component parts of the building: the polygon of forces constituted by the deformation forces and the load on the structure is completed at the point of cracking concerned. Openings in the walls and variations in strength (e. g., through differing types of brick) divert the distribution of stresses and the cracking. The damage-provoking forces, which arise with the temperature strain of contraction and creep, increase towards the gables and corners of the building.

The greatest degree of stress occurs in building components consisting of several layers when exposed to the *heat of the sun*. The difference in temperature

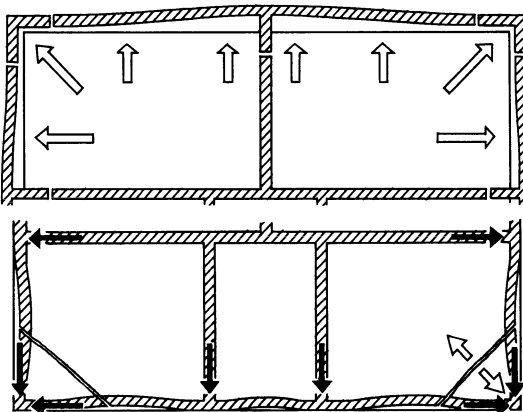


Fig. 332. *Top:* outward bowing of external walls in summer under the influence of the sun's rays, and (*bottom*) deformation of walls under a shrinking concrete floor. (After E. Hoffmann, 1973)

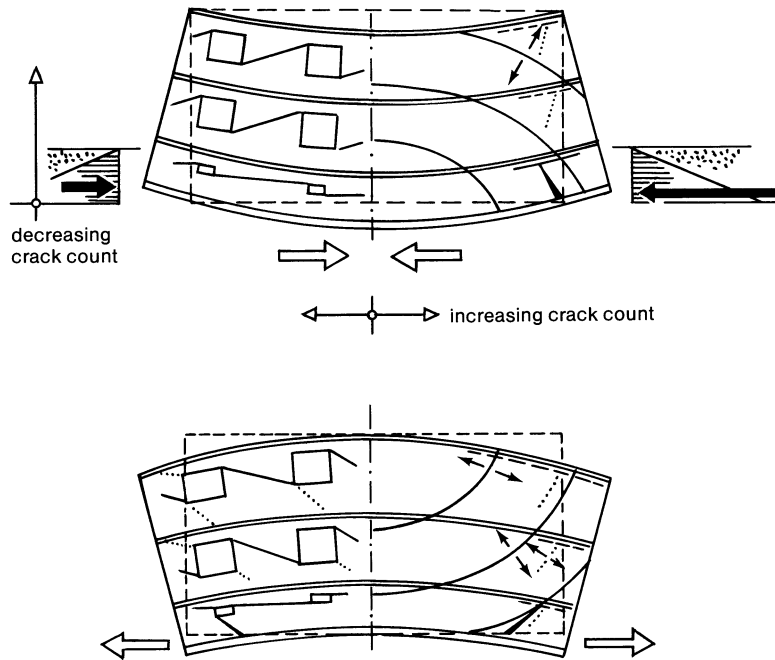


Fig. 333. Position and direction of cracks in a house affected by mining, with and without window openings (after E. Hoffmann, 1973). *Top:* bending and compression of the ground. *Bottom:* bending and extension of the ground

between the outside and the inside of a wall can amount to 10°C or more. Since furthermore the inner side of a wall reaches its maximum heat only 6 h later, in the evening, the effect is that on the outside there is a daily dilatation, and on the inside a contraction (Fig. 332). These linear deformations are of the order of 0.06 mm/m with a wall of gas concrete, or up to 0.15 mm/m in a wall of brick. The result is that in summer the wall bows outwards, and in winter inwards. Walls standing on the northern side receive only one-quarter of the amount of heat from the sun radiating maximally on the east and west walls (southern side 75%). These thermal stresses accumulate towards the gable end, as do the linear deformations (e.g., 0.01 mm or 14 N/cm^2 per 1 m and 1°C).

Many construction materials contract in the first 2 years as they give up moisture (drying-out) and can swell again later if they take up moisture. Concrete moreover also undergoes *creep* in the first 4 years if exposed to uniform tension or compression. A sag through creep¹² can attain four times the value of the elastic initial sag. As compared with thermal changes, however, it constitutes a once-only deformation. In brickwork, shrinkage contraction of mortar in joints is approximately cancelled out by moisture expansion in bricks. In hard-burnt brick the degree of shrinkage and swelling is small, but in sand-lime brick and in concrete can be as much as several tenths of a mm/m . The measure of creep in brickwork can be put at 0.03 mm/m . Attention should also be paid to the time of

¹² Allowable $L/200$ – e.g., $f = 5\text{ cm}$ on $L = 10\text{ m}$ (see Fig. 303 d)

construction: in floor slabs fabricated in autumn as in-situ concrete, tensile forces resulting from shrinkage and from falling climatic air temperatures (thermal strain) are cumulative. Contraction influences are greater in the upper part of a building than in the lower storeys, because the absence of overlying load there means that the principal tensile stresses are no longer counterbalanced by normal stresses.

If a comparison is made between manifestations of damage from mining and from structural causes (Fig. 333), it will be found that bending of the structure caused by mining begins in the foundations, whereas floor sag resulting from shrinkage and creep movements ends at the floor slab over the cellar. In the case of horizontal forces, ground forces resulting from mining likewise act on the foundations, whereas thermal forces principally affect the top storey. Whether it is possible to facilitate damage analysis with the aid of crack indices – defined as the product of crack length and crack width, as refined by classification according to position and direction of cracking – can only be judged after statistical evaluation of damage to a large number of buildings.

13. The Law Governing Mining Damage

13.1 Historical Background

The concept of mining damage was unknown in the ore-mining activities of former times, which tended to take place in wooded hills away from closed townships. In those times, in most countries, the right to win minerals rested with the sovereign alone, who in return for a fixed share of production – the “tithe” – accorded the deposit and the necessary land surface above it to the mining undertaker to exploit. In the nineteenth century, the increasing spread of coal-mining in flat country away from the hills was opposed in Germany from the first by many owners of land, who regarded the mining undertaker as an intruder into their land, which belonged to them, as they supposed, not only at the surface but also downwards. This was their attitude even though the principle of *freedom to mine* had prevailed in Central Europe since the Middle Ages – a principle which precluded any rights of disposal over minerals by the land-owner and accorded those rights, on request, to the finder (by concession). In addition there was the fact that coal-mining, in following dipping seams, was being extended over an ever greater area of land which, because of the relative instability of the underlying strata, was finding itself lowered by several metres. The damage caused thereby to buildings or to agriculture led land-owners to demand compensation from the mining undertakers.

In this connection the question arose whether the mining undertaker would be justified in knowingly damaging or even destroying the property of others, and whether the land-owner could require the mining undertaker to spare the land surface or, failing this, to halt his mining operations. Initially also the manner of making damage good remained uncertain. Should it in principle be compensated for in money, or should the original state be restored as nearly as possible? For the mining undertaker in turn it was important to know whether he would be acting culpably if a mine working, even with the usual technical precautions being observed, led to damage at the surface, whether there was any limit to the amount of prejudice from the mine working that the land-owner was required to accept, and whether the mining undertaker for his part had any right of veto against building on the area of land overlying the deposit once the state had granted him the right to win minerals from it.

In the administration of justice on the Continent there gradually developed in the years from 1830 to 1850 a practice whereby the mine owner, even without being at fault, was required to render full financial compensation for all damage occasioned by his mining activity to the land and property of another, and neither one could proceed against the other – neither the land-owner for cessation of mining activity, nor the mining undertaker to prevent building on the land

surface over his mine. This interpretation of the law was enshrined in the General Mining Law of Germany in 1865, which was applied until 1981. In the succeeding years, resort to direct settlement between the parties rather than to arbitration commissions become the general practice in Germany. Since that time the legal path entailing an action to establish damage or to fix compensation is seldom followed, and then only in grave cases or when a fundamental issue of principle is involved and attempts at an amicable settlement have failed.

The converse of the freedom to mine is *land-owner mining*, as was, for example, the practice in Britain up to 1938. The situation was that the land-owner also owned the ground beneath his land, including all minerals except gold and silver (under Common Law). For the working of minerals he was at liberty to sell or lease off vertical or horizontal sections of his landed property – i.e., to sever a coal-seam, for example, from the remainder of his land. The mining undertaker, as tenant of the coal-seam, was required to undertake to support the overlying strata in such a way that the land surface remained in its original position unaltered. This condition could seldom be fulfilled by the tenant, as he naturally wanted to leave as little coal as possible unmined as a pillar of support, and so it often came to a dispute between the mining undertaker and the land-owner over payment for mining damage. With the object of enabling economic working of coal-seams and assuring adequate production of coal, mining tenants were granted in the Mines Act of 1923 the right to complete extraction of the deposit, even against the wishes of the land-owner. In return the tenant was required to render full compensation, in money or by restoration, for all mining damage caused by him at the surface. This right to have the effects of mining as felt at the surface accepted in return for full compensation was finally consolidated in the Coal Mining (Subsidence) Acts of 1950 and 1957. The principles of the law governing mining damage dealt with below are in essence the same in all countries.

In settling for mining damage a distribution is drawn between *remediable damage* (e.g., cracks in walls, loss of production) which can be met by payment of repair or replacement costs, and irreparable, *permanent damage* (e.g., tilting, swamping) for which the compensation for decrease in value requires to be established in the light of technical and economic factors. A special case is the balancing of responsibility for mining damage attributable jointly to several mine owners in the vicinity of a mine boundary (Table 23). Examples of prejudice to landed property caused by mining stretch from immediate damage to consequential damage and ultimate loss in resale value. Although there may be differences in opinion over the scope and timing of repair work for remediable damage, in the matter of repair costs and consequential damage there are often local standards, or standards customary in the industry concerned, to which the parties can refer and on which they can agree. By contrast, the satisfying of permanent damage where a dwelling house has become tilted was the subject of dispute for many decades in Germany until a simple means of settlement was agreed between the mining industry and the representative association of house owners. Decrease in the resale value of parcels of land in an area affected by mining is assessed by the courts on the basis of the cost of insuring a new building against mining damage.

Table 23. Compensation for mining damage to landed property and appurtenances

Prejudice to landed assets through	Examples of damage to landed property	Manner of compensation by mine operator	Remarks
Direct physical effects of mining	Cracks in walls Drainage interference Fractured rails or Houses tilted Houses collapsed Flooding Loosening of joints	Reimbursement of cost of curing damage and of repairs or Making up loss in value where damage permanent	Complete and immediate compensation Restoration or compensation in money Limitation on claims, say 3 years after damage noticed
Consequential (indirect) damage	Loss of production Loss of rent or Removal costs Redecoration Survey costs Slow-running stretches of railway line	Reimbursement for proven losses or for ancillary work	Liability for endangerment Principle of causation Toleration of mining and of prejudice Balance of advantages Amicable settlement
Threat of future mining damage	Reduced value owing to impairment of suitability as building land	Making up loss in value (insurance costs or loss in market value)	Suit to establish damage Suit to fix amount of compensation

13.2 The Legal Relationship Between Mining and Land-Ownership

13.2.1 The Law on Mining Damage

As an example of legislation governing mining damage, Section 148 of the General Mining Law of Germany¹ may be cited:

1. "For all damage done to a landed property or its appurtenances by the underground or opencast operation of a mine, the owner of the mine is obliged to render full compensation, regardless of whether the mine operation took place beneath the damaged property or not, of whether the mine owner is to blame for the damage or not, and of whether the damage could have been foreseen or not".
2. "No special compensation is due to mortgagees on land or buildings or to rent creditors".

According to this, so long as there is a priority public interest in a mining activity, the land-owner must put up with the mining operation permitted by the

¹ In Section 117 of the new German mining law, compensation for mining damage will be regulated, from 1982 onwards, in accordance with the terms of the code of civil law as in the case of an illicit act, with compensation for material things limited however to their market value, and liability for personal injury limited to DM 500,000. For damage to parcels of land and their constituent parts and appurtenances there will be as before, in return for tolerating the mining, no limit on liability.

Mines Authority and the damage inevitably arising from it to his land and its appurtenances. For him it is a matter of “suffer and charge”. A preventive injunction, which could otherwise be obtained against damage to property, is reserved only to the state in respect of public utilities installations, in the security of which there is an indispensable public interest. For its part the mining industry must accept that areas of influence at the surface will be built on and that mining damage will consequently occur (for exceptions see Sect. 14.4).

By its nature, mining is tied to the place where the deposit is and where the mineral find is made, and it cannot avoid the land-owner's land. *Mine ownership* gives the mine owner the exclusive entitlement, on the basis of layout plans previously examined by the Mines Authority, to prospect for and win, within his concession, the mineral named in the concession document (and usually reserved to the state) and to take all necessary measures for this purpose both underground and at the surface. Prior approval of all planned workings by the Mines Authority ensures that the mine owner, in the interests of the land-owner, employs all economically and technically justifiable means to reduce the effects of his mining to a tolerable measure (the inspection principle). Besides imposing conditions on the layout plan, the Mines Authority has at its disposal, for the protection of life and limb, the issuing of mining decrees and the collaboration of other authorities in taking any necessary steps. Accordingly, in the event of mining damage, the mine owner is not held to have acted illegally, especially since a mine operator is usually quite unable to prevent some damage occurring despite taking the greatest care. The obligation to accept responsibility, without blame, for a dangerous but permitted activity is known as “liability for endangerment”, another example of which would be running a motor vehicle. So, without being held to blame for the mining damage, the mine owner must, in return for toleration of his mining activity and impairment to landed property, compensate in full for the reduction in asset values caused by his mining to the property of others (the *principle of causation*).

Typical of the concept of liability for endangerment is a situation where experience shows that certain lawful acts may give rise to damage without its being established whether and what tangible damage will in fact occur. If on the other hand it could be foreseen that the act would provoke a particular damage, it would have to be stopped in advance, otherwise blame would attach. With liability for endangerment therefore, although damage is caused without blame, it is nevertheless unlawful. By contrast, Section 148 quoted above constitutes an offset for permitted interventions, and so the concept of liability for endangerment is not strictly applicable to damage knowingly caused. Where damage can be tangibly foreseen, as in the case of mining damage with underground working, and where nevertheless it is permissible to proceed and therefore to cause the damage, it is now considered more appropriate to speak of “liability for sacrifice”. That is to say, because the one who is in the right (the land-owner) is obliged to suffer the intervention and to forgo his inherent right to protection in favour of the common good – i.e., the mining that the legislator has declared to have priority – he is entitled, in return for the individual sacrifice laid upon him, to receive compensation for his damage. The mine owner, in making economically efficient use of his own space, may nevertheless not infringe the invio-

liability of the individual. Granting mine ownership thus represents a not inconsiderable inroad into the rights of land ownership, but since all land-owners may be affected, this situation should not be seen as amounting to expropriation. It places an obligation on the favoured mine owner to shoulder responsibility for the damage and to ensure public safety.

On the other hand, the obligation placed on the land-owner to have regard to the needs of mining in the way he uses and builds on his land (the *duty to conform*) has proved unworkable in litigation concerning mining damage. The mining law states, it is true, that the mine owner is not obliged to compensate for damage if the damaged installations were erected negligently, without precautions against the effects of mining and at a time when the land-owner should have been aware, given ordinary attentiveness, of the danger of mining damage (advance warning). The plea of joint culpability is well founded, according to the interpretation of the courts, only when the specific danger of which the mine owner has in vain warned the building owner corresponds to the damage which actually develops later, for only if the prediction is correct can the structure be secured at the right place and by the right amount. The disadvantages or the expenditure associated with the measures to conform proposed by the mine must furthermore be in appropriate proportion to the reduction in mining damage attainable by those measures. Awareness of or drawing attention to a generalized risk from mining is not sufficient for prevention or reduction of the damage. The cost of securing a structure is borne by the mine in most countries. If it is mining that has triggered off the damage, and this has been aggravated by other, non-mining causes (e. g. poor ground for building – joint causation), the mine must nevertheless bear the whole cost of the damage: without the effects of mining, the building standing on poor ground might have remained undamaged.

13.2.2 The Priority Rights of Communications Installations

Even the legal remedies in relation to public communications installations which are reserved to the mining industry in many mining regulations have proved to be inadequate in practice². Although the mine operator is entitled to be heard before the trace of new public roads, railways, and canals is decided, and to give his views on whether and how the disadvantages for his mining operations could be reduced, he is accorded no right of veto, and the compensation payable to him – compensation for expenditure in the narrowest sense – is related only to mining measures taken which would otherwise not have been necessary. With any deviation from the planned route there would be, in the opinion of the legislators, a sacrifice of the public interest in having short communications and nearby interconnections to an individual mining operator's search for profits. Mining's privileged treatment in being permitted to do injury to the property of

² In Section 124 of the new German mining code a reciprocal duty for each side to take account of and fit in with the needs of the other has therefore been prescribed. The erecting or enlarging of a public communications installation can now be refused if the public's interest in the winning of minerals preponderates. By "public communications installations" are meant all stationary and fixed installations and equipment connected with the carrying of persons, goods, or news.

others comes up against its limit with communications installations through the need to safeguard the indispensable security of lines and thoroughfares. For mining to affect communications installations to a small extent is acceptable within the framework of the approved layout plan. The cost of measures taken by the mine operator directly and solely for the security of a communications installation and otherwise not required (e.g., stowing) must be borne by the communications organization, if the installation was approved after the grant of mine ownership. A claim for refund of costs is inapplicable, however, if the measures are taken only or simultaneously to avoid or reduce mining-damage costs for which there is in any case liability (i.e., a matter of balancing the advantages of mining-damage costs versus stowage costs).

Compensation is not granted for waste of minerals within a *safety pillar* required by the Mines Authority to be left under a communications installation, for the purpose of protecting the surface from mining influences harmful to the public. This is because mine ownership (the right to win minerals) is subject from the first to the limitations which spring from the need to safeguard the general good, public safety, the economic life of the community, and the superior rights of neighbours. Compensation is granted for an injury actually suffered (a loss of assets) but not for profit foregone. In the case of communications installations in existence before mining began, mining has no claim to compensation.

The only course open to the mine operator therefore, if he wishes to have a say in the utilization and building-over of the land surface, is to buy up as many parcels of land as possible. These he will then resell only under certain building conditions (e.g., insurance against mining damage) or in return for partial renunciation of compensation for mining damage.

13.2.3 Compensation for Subsidence Damage

Complete compensation for subsidence damage means restoration to the earlier, economically usable state (natural restoration) or – especially with irreparable permanent damage and with prejudice to asset values because of the expectation of future subsidence – full compensation in money. Compensation extends not only to the parcel of land itself and all that is attached to it, either by nature (plants, peat, ground water, soil) or otherwise (buildings, machinery), but also to appurtenances – i.e., those things and those rights which serve the economic aim of the main item (e.g., service mains, wells, losses of rent) or are associated with it as movable objects (e.g., seed-corn, railway rolling stock, farm tractors). Raw materials, supplies, and furniture do not count as among the appurtenances of the land estate, since damage to movables like these does not arise through damage to the estate itself. Entitlement to compensation begins from the moment that impairment of the asset becomes known, and it lapses if not asserted by an action in the courts within, for example as in Germany, 3 years of the person affected becoming aware of the damage.

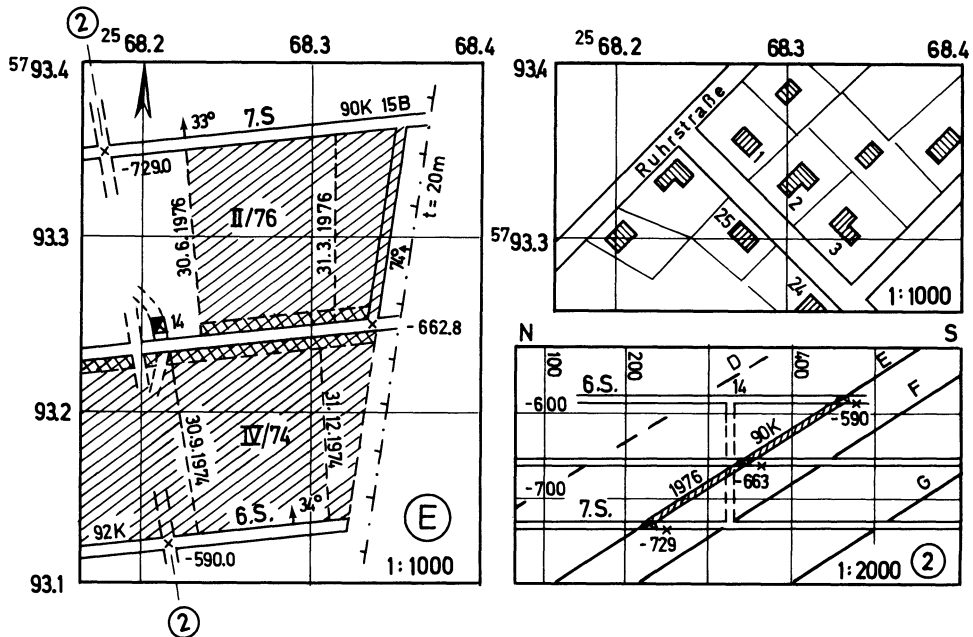
As a general principle of the law governing subsidence damage, entitlement to compensation applies only to whomever the article belonged to at the time of the damage, and not to someone who acquired it later. The mine owner is liable for

all subsidence damage arising during his time of ownership, regardless of the time when the operation causing the damage was carried out. His obligation to compensate for damage ends when he loses the capacity of being mine owner, unless it is a matter of a delayed subsidence effect causally connected with damage in his time. In the Ruhr District in Germany it has been possible since 1968 to make a claim for damages against an earlier mine owner; and from 1982 onwards an emergency fund is to be set up there at the expense of the mining companies, in order to ensure compensation for subsidence even on the failure of a mining undertaker. Against the state, however, no claim for compensation lies once its ownership of a mine has ceased. As for repairs, the mine itself may be called upon to effect them, or the injured party may have them carried out by another firm. The mine owner may offset for the advantages accruing from restoration (offsetting for betterment). Emergency repairs in the first instance are permissible in the case of continuing effects from mining.

In restoring the state that would have existed if the circumstance necessitating compensation had not occurred, it is sufficient if an essentially equivalent state is achieved. The land-owner cannot require "natural" restoration if that would bring a significant economic betterment, irrespective of whether he is prepared to make an equalization payment himself. On this basis, the mine owner may make restitution for prejudice to the value of, for example, a very old house by means of financial compensation.

In Germany the injured party was required until now to show that the effects complained of stemmed most probably from mining and that there was no other cause possible. He was for this reason permitted to inspect the mine plans held at the Mines Office (Fig. 334). In revising the mining code, provision has been made in Section 120 for placing the *burden of proof* on the mining industry as the economically stronger and more expert party, as has been the case in Britain since 1950. To the advantage of the land-owner, the assumption is then required to be made that every instance of damage attributable to ground movement is a case of mining subsidence. Evidence to the contrary must be adduced by the mining interest. In this way, acknowledgement of the existence of pseudo mining damage gains in importance (Sect. 12.8). As regards entitlement to compensation, the injured party can, if no amicable arrangement with the mine owner can be arrived at, pursue this by means of a suit to fix damages and have their level established by a judicial expert.

The obligation to provide compensation in full covers also indirect damage and permanent loss in value. *Indirect damage* to a landed estate arising in causal connection with mining influences is constituted by, for example, operational difficulties, loss of output, removal or suspension of business, losses in rent during house repairs, survey costs, and fees to consultants. General financial loss not arising on the estate itself — for example, loss in the turnover of a trading store on the evacuation of surrounding damaged dwelling-houses — is not made good, however. Only profit lost, or still being lost, on the estate itself is eligible for compensation, and not the return depending on the personal, non-localized ability of whoever is conducting the business. By "*loss in value*" is understood financially compensatable permanent damage remaining as an irreparable part of total damage, or which cannot be remedied by commercially justifiable means.



Key

14 = blind shaft No. 14

6.S. = level No. 6

33° = dip of seam 33°

-729 = point 729m
below sealevel

90K 15B = 90cm coal

15cm country rock

93.4 = coordinate in km

1 : 1000 = scale

31.3.1976 = 31-3-1976

Fig. 334. Inspection of mine plans for evidence of mining damage. Left: ground plan of two panels in seam E, up to the right-hand fault line (mined-out areas shaded). Top right: topographical map showing houses. Bottom right: section 2-2 through the underground workings and seams D to G from 6th to 7th levels

Under this heading is included a tilt in a house sufficient to constitute a loss in returns because of problems in utilization or of higher maintenance costs, a loosening of joints sufficient to constitute a reduction in the value of the building because of a shortening in its probable life, prejudice to the suitability of plots as building land through the effects, or expected effects, of mining, and changes in ground-water level bringing a diminution of yield in agricultural land. Among financial losses are to be counted, besides material damage, also depreciation losses, expenditure to prevent further damage, and loss of the benefits of use.

Reduction in the value of land ownership because of a *threat of mining damage*³ comes within the concept “for all types of damage” of Section 148.

³ Subsidence alone does not constitute damage to a structure. Only the uneven subsiding of foundations is, from a certain magnitude onwards, harmful.

Here it must be noted that the location of a piece of land within a mining area is not in itself a circumstance which adversely affects the value of that land. A mine working must already be affecting the land concerned or be planned for the near future in its area. A distinction is made between arable or pasture land, building land, and plots of land already built on. For arable and pasture land a risk of mining damage is of no significance, because damage is held to exist only when an involuntary diminution in value has actually occurred. A loss in resale value, on the other hand, cannot come into consideration, as every loss in produce caused later by mining is fully compensated. By contrast, in the case of building plots of which the use value or market value is diminished by the fear of mining damage, the occurrence of damage dates from the moment the danger becomes known. There is a diminution in use value when the building owner is obliged to make allowance for future deformation of the ground, restrict the area than can be built on, and take special steps to secure the foundations – all of which affects his costs. The suitability of the site for building can even be placed in jeopardy if cracks or steps in the ground or surface depressions are feared. If the land is already built on, it cannot be argued that it is unsuitable for building, nor even that building has been made difficult. Built-up plots of land have fulfilled their purpose as building land, and therefore all that their owner is entitled to is compensation for mining damage to the buildings. A piece of land suffers trading damage only when, for the first time, a damaging mining effect is expected which was not foreseeable at the time the land was bought and therefore did not influence the purchase price.

For the inexperienced property owner, adducing evidence that the trouble is mining damage and not a fault in construction is difficult, because without the necessary equipment and technical knowledge of surveying he cannot show that a movement of the land surface – apart from visible tilting of buildings – has taken place. In West Germany the mining undertaker is therefore required, from 1982 onwards, to hand the records of his measurements of ground movements to the

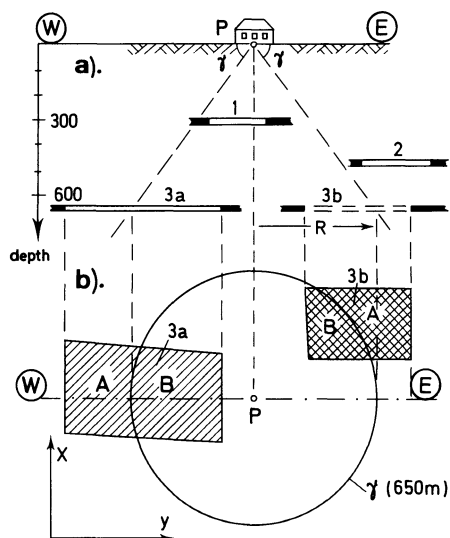


Fig. 335. Delimitation of the panels acting on the house at point P, using limit angles and the critical-area circle: **a** in section; **b** in plan

Mines Authority, in whose offices they can be inspected by the injured party (Section 125). However, in examining mine maps consisting of several hundred sketches and plans showing the progress of working from the beginning (without the planned panels), he cannot do without the advice of an expert.

From the plan of the estate the map square in which the damaged house stands is taken (Fig. 334, top right). In relation to this map section the ground-plan of the workings drawn up for each seam is examined, in order to establish whether and when any working has taken place in that area (Fig. 334, left). A look over all seams and workings together reveals the nearest profile drawing (Fig. 334, bottom right) – in this case profile No. 2. The meaning of the conventional signs in mine maps, which differs from province to province, is recorded on sample sheets. When all the workings in the area concerned have been located, the circular critical area for each mining depth is drawn on transparent paper (Fig. 335, bottom). The radius R of the circle is arrived at in advance in a profile drawing on the appropriate scale for the plan of the working and at the limit angle γ applicable (Fig. 335, top). It can be seen that, of the panels 3a and 3b, the areas marked B will have affected the house at the point P.

13.3 The Calculation of Diminished Value

Compensating for diminished value by a money payment has to substitute for the elimination of damage if complete restoration is for technical reasons not feasible and some permanent damage remains. In Germany, mining damage of this kind consists chiefly of tilt in buildings through uneven subsidence⁴, loss of produce in agricultural land through a disadvantageous change in its distance above ground water, and loss of value in building land through the initial effects of mining.

13.3.1 Tilt as a Basis of Measurement

If a house becomes tilted, it is difficult to say by how much its value is affected by a tilt of, say, 2 to 4 mm/m. In general, a tilted house's *reduction in value* is the result of

- difficulties in use
- unusual appearance
- loosened framework
- impaired resale possibilities
- diminished rentability
- curtailed returns.

Difficulties in use become noticeable in the opening and closing of doors and windows, the aligning of furniture against inclined walls, and the canting of table

⁴ Restoring a tilted house to its former condition by raising it up is in many cases technically possible today but very expensive, costing on average some 25% of house value.

surfaces and consequent rolling-off of round objects. They are just as difficult to express in terms of value as is the disadvantage of appearance. There may be no mathematical relationship between loosening of the framework and tilt, but it is advanced as an aspect of loss in value on the assumption that, besides tilting of the building, very often a slight flexing of the ground has taken place, which will have loosened or at least strained the bonding of members, unless the structure is particularly bend-resistant. Even if all visible cracks in the walls are repaired at the expense of the mining undertaker, the possibility cannot be excluded that hair cracks and minute splitting planes may have formed within the mortar joints and bricks. Such unnoticed breaks in the plastering and brickwork facilitate the entry of damp and cold. This loosening of the structural framework remains, or may even increase, if the tilt in the house disappears under the contrary influence of a second working. The house owner is therefore not required to refund compensation (Fig. 347). An example of reduced returns is the enforced remission of rent for apartments in a tilted building.

The reduced value of a damaged article is given basically by a comparison between its sale prices as realized in ordinary transactions in an undamaged and a damaged state (difference in market value). In the case of mining subsidence damage this comparison procedure has the disadvantage that in the sales contract the value of the building, which is all that matters in the calculation of reduced value, is not specified separately and only the total price — value of building plus value of ground — is stated. Since moreover a sale is usually not intended, and a price comparison of like objects in a damaged and undamaged condition in equivalent locations is rarely possible, a procedure unconnected with selling has to be found which not only makes possible a rapid evaluation but also rests on verifiable bases of calculation. Of the enumerated factors affecting reduced value, only tilt is capable of being ascertained objectively. The value put on other aspects of damage will depend, as has been shown, very greatly on the personal views and experience of the expert concerned. For this reason the tilt in a building, which is easy to measure, has for decades served as a basis for establishing its overall reduction in value. In this it is accepted that an increasing degree of tilt has a progressively unfavourable effect on the building's use, substance (structural loosening), resale value, and financial return. A tilt of 2 mm/m and upwards is regarded as value-reducing.

In Germany, just after the first world war, a formula for the relationship between tilt and reduction in value was proposed (Leyendecker). In it the reduction in value increases progressively as a percentage of the building value, up to a reduction of 75% at a tilt of 50 mm per metre length of building (Fig. 336). For loosening of structural bonds a further 25% is added. This high reduction in value was not recognized by the courts in the immediately succeeding years. Instead, they declared for a scale providing for a linear reduction in value of 1% for each 2 mm/m of tilt (the Vennhofen curve). As a departure from this, by an agreement reached in 1962 between the Ruhr mining industry and the representative association of property owners, for tilts of over 20 mm/m a rate of 2% for each 2 mm/m is used as the basis, or some other relationship to be agreed in individual cases. The reduction in value derived in this way includes structural loosening and other losses in value. In special cases additional compensation can

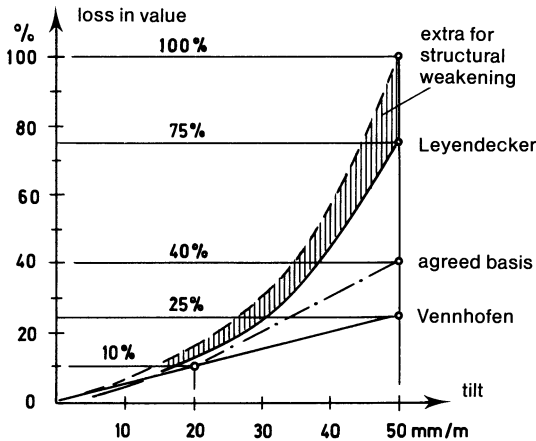


Fig. 336. Loss in value of a building as a function of tilt

be fixed for exceptional loosening of joints in the load-bearing structure, insofar as it is not remediable.

As a measure of tilt the value of the maximum inclination was taken formerly, and later either the mean of the slope as measured in three directions from the highest (lowest) corner of the building, or the mean of the values for the four external sides and the two diagonals (Fig. 337). What is left out of account in this is the usually unknown original tilt in the footings and floors of a house, which often amounts to 4 to 8 mm/m on its completion, and which the total tilting brought about by mining influences will intensify or reduce according to the direction of slope in the ground.

In calculating the reduction in value, it has recently been proposed to start from the average tilt, which is arrived at by multiplying the principal tilt

$$v'_{zm} = 0.64 v'_{zh} \quad (389)$$

by a factor of 0.64. It is defined as the arithmetical mean of the angles of all tilt lines in an inclined surface. The principal tilt vector intersects the contour lines of

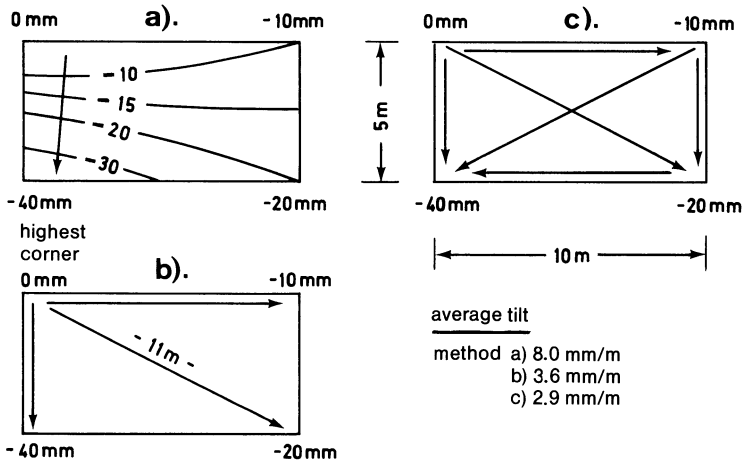


Fig. 337. Methods of finding the average tilt in a house as measured in plan

an inclined and at the same time convex surface at the position where the distance between them, as seen in plan, is smallest (Figs. 133 and 135). In the Ruhr it is the general practice, since the agreement of 1962 mentioned above, to calculate tilt from three angles of inclination.

Instead of measuring tilt at each individual house, it is also possible to base the calculation of reduction in value on tilt in the terrain caused by mining, provided subsidence in all streets of the locality is measured periodically, at permanently marked points, from the moment the first effects of mining are experienced. This procedure has the advantage that the original tilt in houses, developing during their construction, does not enter into the calculation.

13.3.2 Determining a Building's Value

If a comparison of purchase prices is out of the question, the value of a building has to be ascertained, separately from the value of the ground, by either the material-value method or the yield method (Table 24). Over the course of time, the courts have sanctioned reference to material value as the more reliable basis of valuation. The material value of a building is arrived at from the cost of constructing a replacement for it, with account being taken of a "new for old" discount and of other circumstances affecting the value. For extras (architects', engineers' and official fees) a rate of 8% of house value is used for houses built after 1 January 1938. The cost of external works (paving of courtyards, fencing, and laying-out of gardens) does not come within the valuation.

Against the yield method it is argued that, in times of controlled accommodation, rents — the returns — are held down, and the yield (e.g., an innkeeper's rent per hectolitre) depends on the ability of the individual. Since the freeing of rents, however, the *yield method* can once again be used for establishing building values. In this method the effectively attainable annual net yield, which is arrived at from gross yield (i.e., from renting and leasing) less management costs (20%–35%) and interest payments on the ground value, is capitalized using a multiplier to obtain the yield value of the building (Table 24). The value of this multiplier depends on the residual economic life of the building and the rate of interest taken. For example, for a residual economic life of 50 years, it will be 18.26 with a rate of interest of 5% (dwelling-houses) and 15.76 with a rate of interest of 6% (industrial estates).

In estimating a building's value in the event of mining subsidence damage, the starting point in the Ruhr is usually the normal *construction cost* of an equivalent replacement building at 100% of the 1913 price basis as applied to the enclosed volume. This reference value for the building is then adjusted to give the time-related value on the day of damage by applying an inflation supplement based on the official building-price index and an age-depreciation allowance corresponding to the time the building has stood:

$$\text{Value of building} = \left(\text{construction cost 1913} \times \frac{\text{building-price index}}{100} \right) \cdot (390) \\ \text{minus age-depreciation allowance}$$

Table 24. Ascertaining the market value of landed property

Method of evaluation	Ascertainment of values (procedure)	Remarks
Comparison method e.g., for building land, the ground value	1) Comparison of the purchase prices of suitable plots of land in sufficient number, taking account of material, legal and commercial circumstances (e.g., index values) Unusual and personal circumstances are ignored in the price comparison 2) Market value = comparison price $\pm$ situation on the market for landed property	Reference value – e.g., 1913 Time-related value on the target date for ascertainment – e.g., in 1972 Value of building derived from construction costs alone, incl. extras
Material-value method e.g. for single family houses, factory and hotel sites, public buildings	1) Cost of construction = cubic volume $\times$ normal construction costs per m³ in 1913 incl. extras $\times$ building price index for buildings, external items, industrial equipment, etc. 2) Value of building = total construction value minus technical and commercial reduction in value plus/minus circumstances affecting value 3) Material value = value of building + value of ground 4) Market value = material value $\pm$ situation on market for land	Cubic volume price based on historical rates of 1913 Building-costs index for converting prices from 1913 to target date for evaluation – e.g., 1972 New-building value = normal construction cost of a replacement building on the target date – e.g., 1972
Yield method e.g., for plots for apartment blocks, commercial businesses, etc.	1) Net yield = gross yield minus management costs 2) Yield value of building = (net yield minus interest on ground value) $\times$ capitalisation factor according to rate of interest and balance of useful life 3) Yield value = yield value of building + value of ground $\pm$ circumstances affecting value 4) Market value = yield value of plot $\pm$ situation on the market for land	Value of building = value of new building less age-depreciation allowance, etc.

Prices per cubic metre of enclosed volume in the year 1913 can be obtained from the relevant price tables. If construction extras are not included in these empirical rates, they must to be estimated for separately. The volume enclosed by building is the space enclosed within the farthestmost surfaces of the lowest-storey floor and the attic-storey floor or ceiling. Light outbuildings, verandahs, passageways, and balconies are ignored. An undeveloped attic space is counted as to one-third. The economic life of dwelling-houses can be put at 100 years, and that of industrial buildings (sheds, warehouses, multi-storey buildings) at 30 to 50 years in the case of timber construction and 50 to 100 years in the case of solid construction.

In the calculation of reductions in value, the official building-cost index is used as a multiplier to adjust the construction cost for the year 1913 to the present-day price level (new-building value).

The drop in value of old houses must be assessed predominantly according to commercial considerations, because in towns the majority of houses are demolished as being outdated and unprofitable (commercial loss in value) even before they have reached their technical life expectancy, which depends essentially on the durability of their irreplaceable external finish. This *commercial superannuation* is accounted for by flat-rate deductions of

- 30% for older buildings erected before 1924,
- 15% for pre-war buildings constructed between 1925 and 1937,
- 20% for houses built between 1938 and 1953, and
- 10% for the older new-buildings occupied between 1954 and 1961.

The ever greater decay of houses (technical loss of value) with advancing age can be traced as a parabolic curve (curve A in Fig. 338). By constructional improvements the fall in value can be interrupted for a period of varying duration, but this can be judged only by inspection or assessment on the spot. Up to two-thirds of the cost of exceptional renewals and repairs may be deducted from the ascertained fall in value. For mining subsidence damage the line C – the frequently applied 1% deduction per year – was formerly used; in view of the desire to simplify the calculation of reduction in value and the flat-rate evaluation of tilt, this procedure was useful and justified. Now that flat-rate deductions have been introduced for commercial loss in value, technical loss in value is calculated according to curve B. A noticeable reduction in the yield value as a result of age and wear in the building occurs only at a late stage (curve D).

Examples: A dwelling-house dating from 1930, with an enclosed volume of 1500 m^3 , has – at a price of 10 (gold-standard) marks per cu metre – a constructed value related to the year 1913 of 15,000 marks (the reference value). Constructing a replacement would have cost in November 1972, at the time when damage was established (determination of tilt), $15,000 \times 8.138 = 122,070 \text{ D-}$

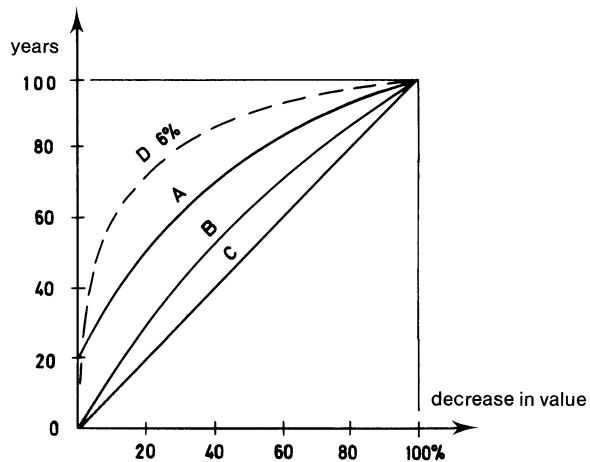


Fig. 338. Curves of reduction in value in a house with increasing age

marks (post-1948 marks), from which an age-depreciation allowance of a linear $42\% = 51,269$ DM or, by the new agreement, of 15% (flat-rate) + 30% (curve B) = $45\% = 54,932$ DM is to be deducted. On this basis, the fall in value calculated from a tilt of, say, 5% will represent a damage amounting to $3,540$ DM or $3,357$ DM to a replacement building worth $70,801$ DM or $67,138$ DM (time-related value). The factor $q = 8.138$ is obtained from the quotient of 813.8 (index for 1972) divided by 100 (index of normal construction costs for the basis year 1913).

If the construction costs, including extras (e.g., architects' fees), of a building erected in 1938 are known from the construction documents to be, for example $65,000$ DM and to correspond to the normal construction cost excluding externals, then the cost today can be calculated, from the rise in the building-costs index between 1958 and 1972, as being $65,000 \times 2.342 = 152,230$ DM. Deducting age depreciation of a linear $14\% = 21,312$ DM, or the agreement-based figure of $10\% + 8\% = 27,400$ DM, gives the time-related value of the replacement building on the target date (November 1972) as $130,918$ DM or $124,830$ DM, to which the rate for calculating fall in value is applied.

13.3.3 Reduction in Value for Landed Property

Reduction in value resulting from the tilting of a house is related to the damage to the building and not to the value of the ground. The land-owner has no claim to compensation for reduction in the value of the grounds of a building caused by mining, at least so long as the building stands. In most cases the life of the building is longer than the duration of mining influences⁵. While the building is standing the ground is fulfilling its function as building land. Mining-related prejudice to the grounds can enter into the question only if the building is demolished, or if an intended reconstruction or fresh construction is placed in doubt by the effects of mining.

A piece of land within a zone affected by mining can be displaced horizontally, along with neighbouring pieces of land, by a metre or more. It is generally held that legal ownership moves with the ground. Putting the boundaries back into their old geographical position would not only be too expensive and, where there is building right up to the boundary, make no sense but is also not desired by the land-owners themselves. Reduction in the size of a piece of land in the compression zone overlying the extraction area, on the other hand, can amount to a reduction in assets if valuable building land is concerned. For example, the area of a plot measuring 50 m by 50 m decreases by 15 m², given compression of 3 mm/m from all sides. Over the zone in front of the face, the plot of land is shortened parallel to the edge of the working but lengthened at right angles to it, with the result that the area can in fact become greater there.

In the case of areas used for agriculture or forestry, compensation for damage to vegetation caused by mining-induced changes in the ground-water level is calculated taking account of normal yields and the percentage loss in

⁵ An exception is salt-mining with permanent pillars, where ground subsidence often continues over a period of 100 years (Fig. 251).

produce. Damage to growth and differences in yield can be either estimated by field inspection or positively established by weighing the produce yielded by proportional areas and comparing with normal yields. Whether and to what extent mining subsidence, withdrawal of ground water, inadequate fertilizing, penetration by smoke or gases, and plant diseases have played a part in the damage to fields, only soil-science or microscope investigations can, in many cases, make clear. In the case of a single occurrence, the level of damages is determined by the average harvest of the year in question. In calculating the sum for damages, prices at the nearest produce exchange are employed, together with an allowance for the cost of freight, merchandising and, where appropriate, difficulties in running the estate. In the case of annually recurring impairment of agricultural output, normal yields for the type of soil in question, available from statistics grouped by type of terrain, are taken as the basis. Compensation is usually paid in cash.

Compensatable prejudice to crop growth also arises, however, if impassability of sodden fields in the subsidence zone prevents timely sowing, fertilizing, and weed-killing in the spring. Fields with loamy soil in particular, if standing under water in the winter, do not dry out in good time in the spring. Delayed working of fields makes necessary an additional application of fertilizer and weed-killer in the early summer, the cost of which (in working time, chemicals, diesel oil, etc.) must be made good by the mine. The deep imprints made by farm vehicles in soggy fields hinder working in summer and lead to strips of differential ripeness in crops. The additional labour and fuel costs, as well as the reduction in yields, also have to be made good. Soaking of fields over a period of years leads to compaction of the soil and a permanent reduction in crop yield. A shortfall in harvest can make it necessary to purchase expensive substitute feed for cattle, the additional cost of which has to be borne by the mine.

Discharging damages by means of a capitalized amount based on an annuity formula is, in the opinion of the mining industry, not appropriate, because with mining continuing for a decade at a time, the ground-water conditions can alter from year to year, and a management policy which will increase yields, given expert advice and through the payment of bonuses, is entirely possible even in a district affected by mining.

With *building land*, liability for compensation exists in Germany not only when there are physical effects of mining (subsidence) but also when a well grounded fear of future mining damage, not previously experienced, reduces the utility value and sale value of a piece of building land. Consequently, even if the intention to build is not definite, the land-owner must be indemnified for a temporary or permanent loss of building-site capability (major loss of value). If the land's ability to be built on is only prejudiced, the so-called minor loss in value is measured by the cost of steps necessary to secure future structures of average characteristics against the risk of mining damage, such as the provision of slab or strip foundations or of special anchoring. The land-owner entitled to compensation is permitted to base the calculation of his damages on his own particular circumstances (calculation of concrete damages). He may also cite as damage an advantage foregone – for example, extensions to his business premises pre-

vented by mining – which in the normal course of events could in all probability have been expected (calculation of abstract damages). The claim to “immediate” compensation which exists in Germany is not invalidated by the fact that ground movement will cease again in a few years when mining is finished, and that the terrain will then be suitable for building on. This circumstance can, however, be of importance to the level of compensation in the meantime.

A direct claim to undertake measures to secure a structure against mining damage or to compensation for the cost of such measures is not open to the landowner, because the mining code provides for a claim to compensation but not to complete upholding of normal conditions. The cost of measures to secure a structure are not mining damage as such but only a means of justifying the loss in value of the piece of land, since a purchaser of the land will deduct from the purchase price the cost of any constructional measures necessary on account of mining.

When the insurance value of a mortgage is reduced by mining damage, the mortgagee has the same rights as in the event of a compulsory auction. In the loan contract it is often agreed to charge any payments for loss in value against capital.

In many countries the building undertaker must bear precaution costs in full or in part, and prejudice to building land through mining is not recognized. In some countries the tilting of a building does not require to be compensated for by the mining industry, because the increase in values which land-ownership experiences through the industrialization introduced by mining is regarded as offsetting such damage. In France and Japan the mining companies are required to deposit in advance a sum to compensate for mining damage. In the United Kingdom there is a duty to notify repeated mining damage to a building to the building authorities. In the USA the Surface Mining Control and Reclamation Act (1977) enabled the Federal government to assist the states in controlling future subsidence. Many countries prescribe areas in which building is permissible only with protection against mining damage. Almost everywhere, personal injury falls within liability for mining damage.

13.4 Sharing the Costs for Damage from Two or More Mines

Where damage is caused by the operation of two or more mines, the owners of these mines are under an obligation to provide compensation as joint debtors. As between themselves they are liable in equal proportions, unless they have agreed some other division of responsibility. Joint liability presupposes uniform mining damage to a locally continuous property. In this respect it is immaterial that the workings have exerted an influence at different times and that damage has occurred only when the last influence was experienced. Joint liability applies even when the older mining operation has been discontinued before the occurrence of mining damage, so long as the mining concession is still in force. The claim as between mine owners to share the costs lapses in Germany only after 30 years.

With the growth of jointly caused mining damage towards the end of last century, the desire was expressed in the Ruhr District for a method of calculation

which would take into account the degree of influence of individual workings on either side of a mine boundary. To begin with, the so-called simplified and quantitative methods were developed. In the *simple method*, workings lying within the angle-of-break cone were evaluated according to an index of measurement:

$$Z = Maz \quad (391)$$

i. e., only on the basis of thickness of seam M , manner of working a , and age z (up to 30 years), but not according to the horizontal relationship to the object of damage or the extent and depth of the working (influence factor e). The angle-of-break cone was traced in strata section from the object of damage to the working seams. This basis for settling up was later expanded into the so-called *quantitative method* of calculation by introducing the area of extraction A and the depth h :

$$Z = \frac{AMaz}{h} \quad (392)$$

The majority of agreements between mining companies on sharing damages were concluded on this basis.

In the German coal-mining industry the following values are used for the time factor z :

In the 1st year, $z = 7.5$ (corresponding to 75% of total subsidence);
in the 2nd year, $z = 1.5$; 3rd year, $z = 0.5$; 4th year, $z = 0.3$;
5th year, $z = 0.2$; 6th to 10th years, $z = 0.1$.

Workings over 10 years old are left out of the accounting exercise. The factor a is given a value of 5.0 for pneumatic stowing and of 9.0 for caving procedures. The agreements between neighbouring mining companies provide for each pit to settle for mining damage and precautionary measures notified in its area, and the proportionate amounts due are settled between the companies half-yearly. Respective shares of the cost of damages or precautions can be rapidly determined with the aid of zonal tables or graphs according to the distance of the object of damage from the boundary between mines.

The methods of apportioning described, in which no distinction is made according to the nature of the cause of damage, are very simple owing to the way that subsidence damage and displacement damage are treated together. They are, however, unsatisfactory if mining damage attributable clearly to subsidence, tilt, bending, extension or compression are apportioned, by virtue of an agreement, on the basis of these "subsidence formulae". A large amount of local subsidence does not necessarily imply also a large amount of tilt or linear deformation in that place. Damage which in the case of communications installations, water-courses, and agricultural land is conditioned predominantly by subsidence, and in the case of building structures by tilt, bending, or linear deformation, can be apportioned in cost terms on the basis of the appropriate ground movements produced by each pit at the object of damage with very little expenditure of time, now that data-processing systems are available.

A working in pit A can exert on a structure lying within the area of pit B and near the mine boundary – so far as the horizontal component of ground move-

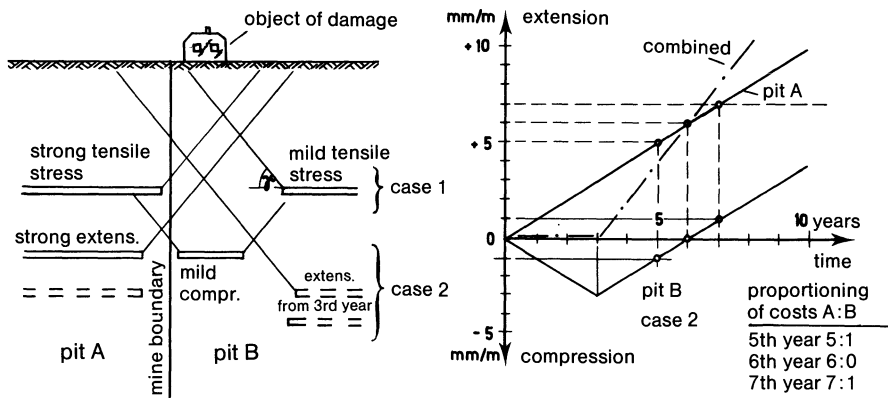


Fig. 339. The effects of two mines on a building near the mine boundary. *Left:* section showing extraction panels. *Right:* calculated linear deformation of the ground at the object of damage

ment is concerned – only a tensile stress (Fig. 339). It can thus either reinforce a tensile stress, or weaken a compressive stress, produced by a working in the neighbouring pit B. If the working in pit A has had a tensile effect on the building which reaches the limit of its rupture strength, a small amount of extraction effected in the following year in the neighbouring pit B, if it also had a tensile effect, would trigger off mining damage in the structure (case 1). It would obviously be wrong to lay the whole cost of this damage on mine owner B, since the structural strain produced by the first working through frictional forces or earth forces, and which still persisted latently at the start of the second working, was responsible for the greater part of the rupture deformation. If tensile and compressive stresses act together – for example, of workings in pit A produce great tensile strains in a building and those in pit B mild compressive strains at the same time (case 2) – one might be inclined to charge only pit A with the cost of the damage, since the compressive stresses emanating from pit B are overlain by the greater tensile stresses from pit A and thus cannot exert any harmful influence. The compressive stresses in fact prevent greater damage such as might occur if pit A were worked alone. On the other hand, the compressive stresses set off by the workings in pit B might lead to mining damage if they were pursued alone.

From these two examples it can be concluded that the mining influences of each pit should be considered separately and without regard to the age of the workings concerned; also, that it is immaterial whether their effect was to set the mining damage in train or to trigger it off, to intensify it or moderate it. The effects, ascertained in accordance with procedures for the calculation of ground movement, of workings up to, say, 15 years old – subsidence in the case of ground-water and gradient damage, tilt for loss of value in houses, and bending or linear deformation for cracks in buildings – are treated as being of equal value in time and added together for each pit, with the correct plus-or-minus sign, in order to arrive at the amount of ground movement manifested at the object at the time of damage, and to register a reversal of movement such as is represented in Fig. 339, right, as a compression/extension transition. In accordance with the *apportionment formula*

$$Z = \frac{\text{total effects of pit A}}{\text{total effects of pit B}} \quad (393)$$

the costs of mining damage arising and notified in the 5th year are apportioned in this example in the ratio 5 to 1, with the mathematical sign of the total effect ignored. In the 6th and 7th years the ratios are 6 to 0 and 7 to 1, respectively. The method proposed can also be followed for apportioning the costs of subsidence and bending damage and the loss in value on tilting (Hilbig and Kratzsch, 1965).

Later workings in a neighbouring pit can only increase subsidence, and decrease tilt and curvature, in the border area. Whereas damage from subsidence and tensional strain is frequently triggered off only by the subsequent extraction activities of a neighbour, the latter is actually at an advantage in the matter of damage from compression, bending and tilt, because the earlier ground deformation was always in the opposite direction to the new influences. If the workings in pit A have tilted a building by 4 mm/m and mine owner A has compensated for the consequent loss in value, mine owner B does not need to pay anything for an equally great but opposite (and restorative) amount of tilt produced by his own, later working – unless there is an agreement to equalize costs retrospectively. Similarly, a mine owner could, with the first boundary workings, lower a canal bridge by, say, 90 cm in relation to an unaltered water level, thereby reducing the free headroom from 5.50 m to 4.60 m. This would cause no damage, but it would leave his neighbour's mine a reserve of only 10 cm subsidence down to the minimum permissible headroom of 4.50 m.

A equitable settlement of costs for jointly caused mining damage should therefore ensure each mine equality of rights in extraction – in terms of quantity and location of extraction as well as of share of the costs of damage – at all times, in the context of long-term planning for boundary working. To this end it is necessary to assess the degree of participation on the basis of that component of ground movement which has chiefly caused the damage, and to leave the age of the working out of consideration. With the aid of evaluation tables, the progressive increase in damage costs in relation to the amount of ground movement, and the greater damage to concrete structures caused by tensional stresses, can be taken into account. These considerations show how difficult it is to achieve an equitable solution to the apportionment of costs.

13.5 Contingency Funds for Future Mining Damage

In conclusion, a brief look may be taken at the tax regime covering the costs of mining damage. On the liabilities side of his trading account, and in his tax account, the mining undertaker can show transfers to reserve in respect of the costs of mining damage which, in terms of nature and amount, is still not exactly defined because of the lengthy time that usually elapses between the causation, occurrence, recognition, and substantiation of mining damage. These withholdings are charged against the accounts of the years in which the working causing the damage took place, so that financial results can be correctly delimited in

terms of time. In making provision for mining-damage liabilities, it is thus not a matter of when the damage arose in law – i.e., when it was noticed by the injured party and notified – but only of the time of causation – i.e., the period when the relevant mining took place.

In transfers to reserves for still undefined liabilities for mining damage, a distinction is drawn in Germany between the following categories:

1. Withholdings for damage “caused” but not yet “arisen”: accounting for such damage is made either as a lump sum, or for individual objects according to the nature and locality of the damage. As an annual lump-sum provision in the *coal-mining industry*, a figure of five to seven times the annual cost found to be necessary on average over the previous 10 years is accepted by the tax authorities, including an allowance for the trend of prices. In *salt-mining*, where ground subsidence over chamber workings may continue for centuries and damage may appear only after a long time, damage and costs to be expected in townships, in agriculture, and at individual objects in 10, 20, 30 or 50 years have to be estimated on the basis of pre-calculated subsidence and displacement movements. Capital values becoming due only after decades have to be reduced to their present values, taking account of interest yields (e.g., 5.5% p.a.), and adjusted annually for the trend in prices. In calculating withholdings on account of a loss in value some decades hence for houses suffering a tilt of over 2 mm/m, account must also be taken of the physical and commercial deterioration in their value which will occur in the meantime. In estimating repair costs it can be assumed that repairs to buildings become necessary when the principal pre-calculated tensile and compressive strains rise by 1 mm/m, at which rate the degree of damage, for example, increases linearly. Withholdings not needed are to be liquidated at a profit.
2. Withholdings for mining damage which has already occurred but the exact cost of which is not yet known. Instances of this kind of damage are assembled in a table and shown at the amounts which, on the basis of earlier costs for similar objects, will in all probability be required to settle for them.
3. Withholdings for permanent damage. To this category belong all disbursements which have to be repeated annually over a long period for individual cases of damage, regardless of whether further extraction takes place or not. Examples are the costs of keeping subsided areas dry (pumping installations, drainage works), of compensating for loss of agricultural yields in temporarily swamped areas, and for providing water where wells have been made dry. For permanent damage of this type, 20 times the annual sum is placed to reserve.

For *tax accounts*, only withholdings for mining damage in the narrowest sense of the term are acknowledged. Consequently, expenditure on eliminating mining damage to an individual's or company's own land and buildings on buying building land to avoid future claims for compensation, or on fill material or precautions to structures to reduce damage, are not recognized for tax purposes. By contrast, the purchase of land is effective for tax if it is to be exchanged for inundated land which can no longer be used for agriculture. In this case it is the difference in value between arable land and waste land that is taken.

14. Measures to Reduce Mining Damage

14.1 Types of Preventive Measure

Where there is underground mining over a wide area, ground movement and mining damage cannot be avoided in densely populated districts. For example, the cost of repairs, compensation for loss of value, and structural precautions in the Ruhr District alone amounted in 1981 to some 770 million DM (say US \$ 320 million). Of this total, around 50% was for the control of drainage (Fig. 340). In addition, the Ruhrkohle AG group set aside three times the annual requirement in a reserve fund for mining damage. In the U. K., the cost of mining damage in the year 1964 amounted to some £ 4 mn (\$ 8 mn), of which almost 50% went on minor damage to houses. In potash mining and ore mining, expenditure for mining damage is somewhat less.

It was not only this high cost burden of 3 to 4% of sales price but also the requirement by the state to provide adequate protection for important communications and public-utilities installations which gave the impetus for the development of preventive measures in the field of mining subsidence engineering. The basic principle today is that “prevention is better than cure”. These measures relate to –

a conduct of *mining operations* such that the structure to be protected is either exposed to a less damaging type of deformation (extension or compression) or is stressed only to an acceptable degree of movement or deformation;

in new construction, *precautions* against mining damage, either by stiffening foundations to withstand the forces of ground deformation, or by enabling structures to escape from these forces through the provision of expansion and slip joints; and

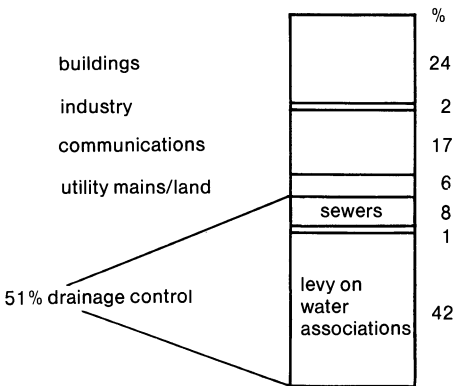


Fig. 340. The Ruhrkohle AG group’s costs for mining damage in the year 1972, broken down in percentage terms. (After Rürup, 1972). 1% = the cost of pumping installations of Ruhrkohle AG

regional planning to select, for large new structures, locations subject to a minimum of future mining influences.

In addition, planning models have been programmed to enable the cost burden of mining damage in proposed extraction panels to be assessed as early as the planning phase and be taken into account when deciding their economic workability. In these models, the costs of earlier workings affecting a similar area of surface are taken as the starting point, and it is assumed for simplicity that mining damage increases in proportion to the “average amount of subsidence”, which can be calculated, given a uniform drop in the area of influence (depth, area of extraction, limit angle), from the subsidence volume of the planned working (thickness of seam, method of working, area) as a quotient:

$$\text{average amount of subsidence} = \frac{\text{underground subsidence volume}}{\text{area of influence at surface}}. \quad (394)$$

The cost burden of mining damage found for each department of mining is assigned, together with other costs (face work, development, haulage), to the seam areas being evaluated. In a comparison between parts of a deposit evaluated in this way, a high proportion of mining-damage costs can on occasion be decisive for the conclusion that a particular panel of seam should not be worked.

For commercial and technical reasons, regard can be had only to significant and sensitive structures at the stage of detailed mine planning, because even preventive measures cost money underground and at the surface. Other buildings, if they are of large area, must therefore be protected, by special constructional measures at joints and foundations, against every likely linear deformation or flexure of the ground. Even so, the mining-damage costs saved thereby should be at least equal to the cost of the precautions. Only dwelling-houses standing separately can be left unsecured, because the minimal absolute deformation in a small area of surface can usually be absorbed by the foundations without too much damage, and the cracks in walls are generally easy to put right. It is by a combination of all three measures – planning of operations, securing of structures, and regional planning – that mining-damage costs are held to no more than the sums mentioned above. Without them costs would be several times higher, and it would be impossible either to mine economically under canal locks, motorways, railway goods yards, or industrial installations, or to contemplate the building of large towns, with all their communications and energy-supply installations, in mining areas. These measures to reduce mining damage leave untouched, because unavoidable, the many instances of loss in value affecting small objects as a result of tilt, shortfall in produce yields, or prejudice to building-land properties, as well as repair costs arising for unprotected older structures.

Observations made in the following pages refer exclusively to a flat-lying stratification, where operational planning is not so restrictive in the arrangement and size of workings as in a steep-lying stratification, and where the curves of ground movement on either side of the panel are approximately symmetrical.

14.2 Layout Planning Having Regard to Mining Damage

To protect an object at the surface, the mine engineer must so plan the layout and conduct operations that the effects of extraction shall be harmless, or that any harm caused shall be easy to rectify and, in the case of industrial installations, shall not affect their functioning or production (indirect mining damage). Examples of structures that are worth protecting, or are costly to repair, are historical buildings, town centres, industrial installations, power stations, large railway freight yards, major roads, and canal locks.

The simplest way to guard against the effects of mining is to leave an unmined, pyramid-shaped zone of protection under the structure (Fig. 341), defined by the limit angle γ or, if the effects of working at the margin can be accepted, by the angle of break β . Within this *protective zone* only partial extraction with permanent pillars, or only working with solid stowing, is then permitted by the mining authorities, unless extraction is prohibited altogether. With pillar support, subsidence at the protected object amounts to between 30% and 50% of the thickness of seam mined M , depending on the areal extent of the pillar, and in the case of total extraction with solid stowing to between 15% M (hydraulic stowing) and 50% M (pneumatic stowing). Outside the protective zone, working may proceed by caving methods (subsidence up to 95% M). One disadvantage for communications networks, sewers, and the ground-water level is the way the object appears to “grow out of” the ground between the adjoining subsidence troughs (as also happens with shaft safety-pillars). For this reason, a systematic

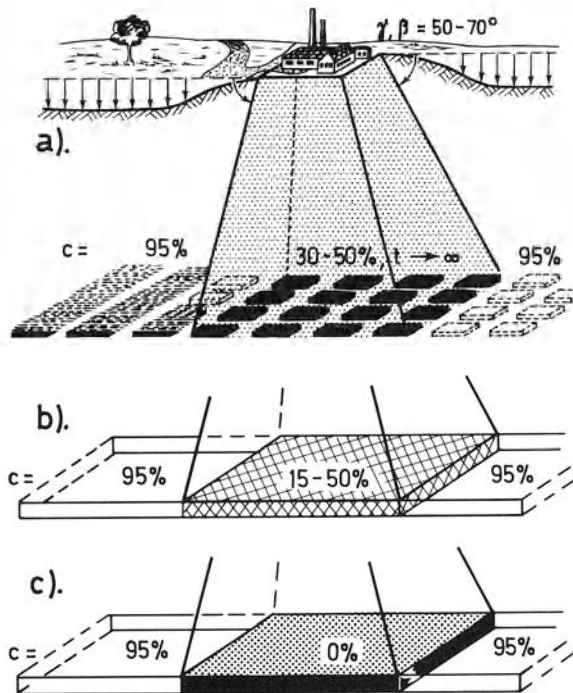


Fig. 341. Protective zones for surface installations. **a** Permanent pillars, giving convergence over a long period; **b** stowage working, with convergence completed in a few years; **c** deposit left unmined in the protection zone. (c = convergence)

lowering of the object is often preferred to complete protection, and a layout plan for the safety zone drawn up with the object of stressing the structure as little as possible and for as short a time as possible. In the Upper Silesian coalfield, 40% of all coal production has to be won from safety zones on this basis.

In *planning the layout* of a mine, decisions have to be taken on the shape of the extraction area (position, extent), the method of mining (caving, stowing, rate of advance, etc.) and the sequence of operations (direction, face advance, overlapping).

As a rule, not only the mining depth and the limits of workable deposit are given in advance but also the mining method and technique (Table 2). If it is advisable, and technically and commercially justifiable, in the interests of minimizing mining damage, the engineer will depart somewhat from this rule and pose the following questions in the elaboration of the layout plan:

stowing or caving working?
increased size for the pillars?
slower or faster rate of face advance?
concentration of working in one area?
extract nothing in a part of the area?
direction and sequence of individual operations?

To answer these questions it is necessary to consider whether it is chiefly

ground movement (subsidence, tilt, displacement) or
ground deformation (linear change, flexure)

which has to be kept to a minimum in the area of the object to be protected, or whether only

a particular kind of ground deformation (extension, compression, convex or concave flexure)

is to be permitted at the object.

The *harmful effects* of these ground movements and deformations are essentially the following (Chap. 12):

subsidence causes ground-water damage and difficult drainage problems in flat country, canal structures are “lowered”, rivers overflow to form lakes;

tilting sets structures and machinery out of true and alters the gradient of communications installations;

ground displacement leads to a horizontal change of position in buildings: which is in general harmless;

linear deformation creates frictional forces affecting elongated buildings, railway lines, and buried pipelines and, in the case of compression, increased earth pressure on deep foundations;

ground flexure causes bending stresses in long and high structures and sagging in elongated machinery.

To counter these influences and to avoid high repair and drainage costs or adverse public opinion, the mine engineer can take the following measures into consideration when planning the mine layout:

Subsidence can be reduced by

employing solid stowing,

leaving pillars of adequate size standing in the deposit (high sacrifice of mineral), and

reducing the area of extraction in the object's zone of influence (forgoing some extraction).

Tilting and horizontal *displacement* of the object can be avoided by effecting extraction which is at all times symmetrical to the object and is advanced either from the edge of the critical area, on both sides, towards the centre (edge working), or

in two wings from the centre outwards (centre working).

Ground deformation (flexure, linear change) at the object can be kept to a minimum by

the simultaneous influence of an edge working and a centre working, or

aligning the longer side of a panel parallel to the longitudinal axis of the structure.

The structure is exposed to ground deformation which is always of the same kind if the object

lies centrally over a two-winged panel (only compression, only concave bending), or is not undermined – i. e., remains in the zone ahead of the face (only extension, only convex bending, as in Fig. 142, field II).

These measures can be divided into static and dynamic measures (Table 25). *Static measures* include determining the position of the working in relation to the structure, limiting the panel in the protective zone, and choosing between caving,

Table 25. Layout measures to minimize mining damage, and their influence on stresses in a structure

Static measures		Dynamic measures	
The working	Influence on ground movement	The working	Influence on ground movement
Position	Nature and amount of influence	Direction	Nature of influence
Limitation	Amount of influence	Rate of advance	Amount of influence
Method of working (stowing, caving, chamber-working)	Amount of influence	Sequence of working	Amount of influence

Table 26. Measures to minimize mining damage to structures

Ground deformation	Nature of stress dependent on		Intensity of stress dependent on		Structural precautions (apart from regional planning)
	Position of structure in trough	Direction of mining	Structure lying over (minimal stress)	Layout: damage minimized by (maximal stress)	
Extension	Margin zone Corner zone	Towards structure	Edge of working	Centre of margin zone	1) Reinforced foundation slab, strip foundations 2) Slip joints on underside 3) Separation gaps 4) Small, light, separate buildings
Compression	Centre zone	From centre, away from structure	Edge of working and centre of critical area	Centre of subcritical area	1) See under Extension 2) Little depth of foundation 3) Stiffened cellar walls
Convex flexure	Margin zone Corner zone	Towards structure	Edge of working	Centre of margin zone	1) Stiffening 2) One, two or three supports 3) Flexible construction 4) Small plan area
Concave flexure	Centre zone	From centre, away from structure	Centre of critical area	Centre of subcritical area	1) See under Convex Flexure 2) Stiffen floor slabs
Tilt	Anywhere except trough centre	Each working within zone of influence	Centre of working	Edge of working	1) Gap from neighbouring structures 2) Jacking arrangements
Structure's sensitivity (limit):	Extension 0.2 mm/m unreinforced concrete 1 mm/m masonry		Compression 1.5 mm/m concrete 2 mm/m masonry	Flexure 2 to 0.5 km radius	Tilt Railway lines 1 : 100 Houses 2 mm/m and over

stowing, and chamber working. The *dynamic measures* have an influence on interim ground movements and concern the direction and rate of face advance, as well as the chronological sequence of working and the overlapping of the influence of workings. In planning the layout it should moreover be borne in mind that concrete structures can absorb tensile stress without harm less well than compressive stress (0.2 mm/m against 1.5 mm/m). With a knowledge of the sensitivity of a structure, the mine layout can in many cases be so planned that, for example, an elongated concrete structure is tilted around its longitudinal axis and only compressed to an acceptable extent (Table 26).

14.2.1 Static Measures

If the position and outline of a future extraction panel have been fixed in the vicinity of structures worth protecting (A and B in Fig. 342), a check is made with a profile drawing as to which structures, if any, lie within the *range of influence* of that panel. Through the centre Z of the extraction area, two profile lines are laid out parallel to the edges of the working. On to the profile line W – E down the dip of the seam (20 gr) the right-angled lines of projection from the panel edges and the corners of the building are traced. The altitude of the panel edges noted on the ground plan (–480 m, –640 m) are drawn in below the profile line at the scale of the mine plan, taking account of the altitude of the land surface (+110 m). The arms of the limit angles pointing upwards from the panel edges delimit the zone of influence of the extraction area at the surface, down the dip,

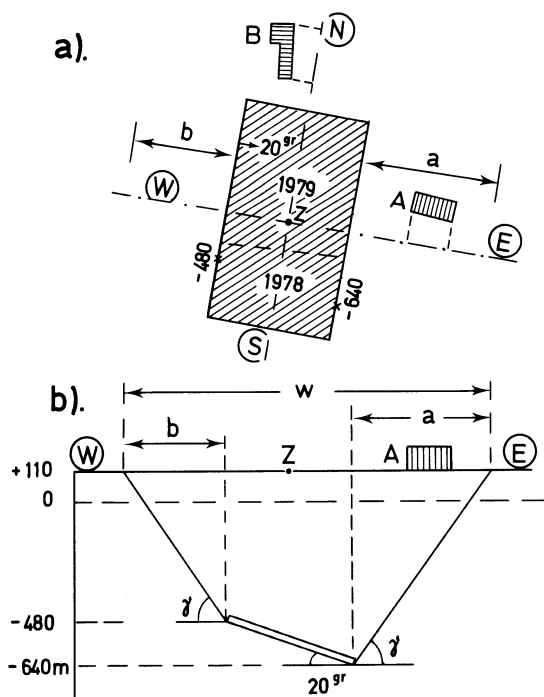


Fig. 342. The extent of ground movement over an extraction panel. a) in plan; b) in section

with the measures a and b. Structure A lies within this zone. The profile N – S along the strike of the seam is constructed in the same way with an eye to the situation at structure B.

The *manner* in which a structure within the area of influence of a mine working becomes stressed – for example, through tensile or compressive stress, saddle-shaped or trough-shaped bending, or tilting around the longitudinal or transverse axis – depends on its horizontal alignment in relation to the extraction area. A structure directly over the extraction area experiences compression from every direction. In the area of the trough margin, outside the limits of the extraction area as seen in plan, a structure will be compressed or extended depending on the direction considered (Fig. 343). It will be extended along its axis which points “radially” to the working, whereas along its axis lying parallel to the edge of the working it will be compressed. Tilting of the structure takes place around the “tangential” axis, which should therefore be the longer axis if possible. The direction of the manner of bending corresponds to that for linear deformation: trough-shaped, concave bending (compression) directly over the working and also in a tangential direction at the trough margin; and saddle-shaped, convex bending (extension) in a radial direction in the trough-margin area and the corner zones. In general therefore, the rule should be to undertake extraction, or to align the structure, so that its longer axis is parallel to the lines of equal subsidence – i.e., tangential to the extraction panel (giving compression in the longitudinal axis and tilting around the same axis).

As shown earlier in Sect. 7.2, the *amount* of ground movement and deformation is also dependent on positioning (Fig. 344). For example, maximum tilt occurs directly over the edge of the working (point 2 in Fig. 343), where linear deformation (neutral zone) is slight. The maximum values for compression, and for trough-shaped bending, lie approximately at point 3 over the working (sub-critical area). In the middle of the trough edge, at point 1, extension and saddle-shaped bending in a radial direction are at a maximum. Maximum possible, or critical, compression occurs in the end-stage of ground movement, centrally at point 4, over an extraction area of 1 R diameter (approx. 500 m at a depth of 800 m). In deciding the length of working along the strike when mining under the structure to be protected is proceeding symmetrically, every endeavour should

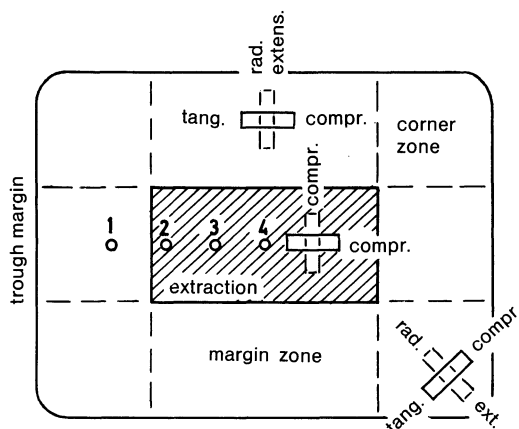


Fig. 343. Tensile or compressive stressing of a structure, depending on its alignment to the area of extraction

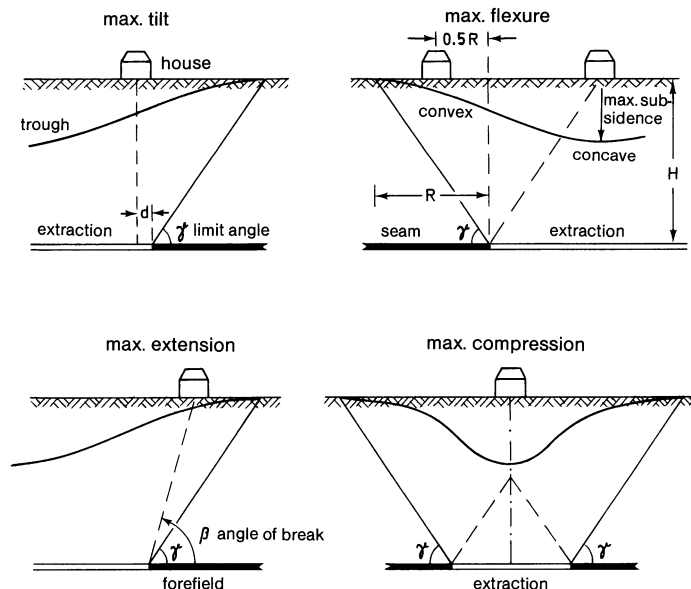


Fig. 344. Positions showing maximum ground movement or deformation over an extraction panel at depth H (in section)

therefore be made to halt the working either as soon as it reaches a length of $0.5 R$ or only after its length has reached $1.5 R$. Compression, extension, tilt, and curvature are, however, greater across than along a rectangular extraction area – a fact which can be explained by the greater steepness of the trough side transversely (Figs. 123 and 124).

The layout engineer will therefore delimit the extraction area in such a way that the working will assume in its final stage the size and alignment to the structure that pre-calculation shows will be the most favourable in terms of the nature and magnitude of ground movements stressing it. In this connection, consideration must be given to whether, on approaching the object of protection, working should proceed on a wide or a narrow front in relation to the structure's longitudinal axis, whether the structure can tolerate the increasing amplitude of ground movements which goes with increasing size of extraction area, and whether extraction of any part of the seam should be forgone (Fig. 345, left). In chamber working a widening of the pillars in the centre of the panel can be considered if excessive deepening of the trough is to be avoided – for example, in low-lying terrain where the ground water is not far below the surface (see Fig. 359d). If the choice lies between stowing and caving methods, it will be found that the magnitude of ground movements can be reduced by half through the support provided by *stowing*. In badly shattered strata that have been worked through several times, stowing reduces tilt and compression by even more than half. With hydraulic stowing and cemented back-fill, ground movements and deformation are reduced to around 20% as compared with caving. By graduating convergence in the working area through hydraulic stowing at the outside and pneumatic stowing and caving in the interior, the trough side in the vicinity of a

structure requiring protection can be flattened out (minimal ground deformation and tilt). In the Polish coal-mining industry, stowage zones are made $7\sqrt{H}$ wide. When mining under old panels, the delayed subsidence originating from them should be borne in mind. A further advantage of stowage working is that dynamic tilt, flexure, and linear deformation over the advancing extraction face are also less than with caving, because the dynamic trough profile becomes flatter (Fig. 244, broken line).

In flat country, the use of stowage working mitigates especially those forms of subsidence damage which lead to the flooding of wide areas¹ or to the damming up of waters flowing in rivers and drains. Moreover, the subsiding of railway lines (changes in gradient) and canals is lessened by the use of stowing. In the Ruhr, as is known, water damage alone constitutes 50% of all mining damage costs. Under rivers and other waterways serving to drain an area, and under depressions, railways and canals, recourse should therefore be had to as “low-subsidence” a method of mining as possible, such as stowing or room-and-pillar working. Other measures for reducing mining damage in the interests of protecting structures may take up the largest amount of space in the literature, but it should not be forgotten that introducing stowage material or leaving bearing pillars standing in room-and-pillar mining, chamber working, or auger mining are among the most successful measures in combating mining damage, since by reducing ground movement around 50% they not only conserve drainage flow but also help to lessen the impact of ground-deformation forces. Nevertheless, stowage working does increase extraction costs. On the other hand, the cost of waste tips is spared, and there are no environmental problems from acidic water in tips and tip fires or in the transport of wastes. In addition there is a more advantageous distribution of strata pressure at the mining horizon, less fissuring of the roof (meaning less emission of gas), and a cooling of mine air by damp stowage material.

If it is further considered that subsidence of the land surface cannot be reversed and lasts “for ever”, with all the harmful consequences for the ground-water level and the flow in waterways, stowing should be the rule today in flat land lying little above the water-table in ground and rivers, and a legal requirement in any area that has already been artificially drained. The advantages of caving methods in terms of cost and extraction technique are outweighed in the course of time by the cost of permanently maintaining pumping and waste-water installations, unless the water so raised can be used for water supply (see also Sect. 12.5).

Another “low-subsidence” method of mining is *chamber or room working* with yielding pillars (see Figs. 1, 2 and 252). This is well suited to the mining of massive, horizontal deposits under sensitive industrial and communications installations, or under low terrain lying close to the level of ground water and streams. Depending on the degree of partial extraction, maximum subsidence in

¹ In the Ruhr, the ratio of mine wastes to coal produced is of the order of 9:10, and 38,000,000 Mg of waste (= 67%) has to be dumped in tips. For this reason there is a proposal for “evening out subsidence above ground” by filling in subsidence troughs with mine wastes in thinly populated districts. The costs are for tips 3–6 DM/Mg waste and for pneumatic stowing 15 DM/Mg waste (for comparison sales price 1981: 260 DM/Mg coal).

chamber working can show a considerable reduction as compared with total extraction (100%) of a deposit. Assuming equal seam thicknesses M , it can be brought down to a value of $0.5 M$ of longwall mining with stowing, given chambering with 50% excavation, or even to a value of $0.2 M$ of hydraulic stowing, if chambers are back-filled with solid material. Leaving 50% of a deposit standing as pillars thus produces the same diminution in subsidence as bringing in wastes for stowing in longwall total extraction.

Subsidence in chamber working is not only less than in total extraction without stowing but also slower. This is an advantage as regards mining damage, because

when ground deformation is slow, preventive measures can be taken in good time;

if flexure of the ground is slow, the structure can settle gradually, with the result that peaks in the distribution of pressure at its base are evened out (Figs. 289 and 293);

a slow linear deformation of the ground can be partly compensated by an internal rearrangement of the grains of soil (Fig. 279);

during phase II of delayed subsidence, which in salt-mining accounts for about 80% of all subsidence and lasts many decades (Fig. 251, bottom), the zones of extension and compression over the halted working are stationary, with the result that an alternation between tensile and compressive stressing of structures is avoided;

structures erected only for a short time are not stressed up to the long-term value of ground movement, and new structures experience only a part of delayed subsidence.

On the other hand, the long duration of ground movement and deformation is also a disadvantage, because repairs to buildings, communications installations, and waterways cannot wait until all ground movement has died out. Damage of more than a certain degree must therefore be put right several times: which is likely to prove more expensive than a single restoration. On the first repair of communications or drainage structures, a reserve in terms of extra falls or head is usually created, which should last several decades but which commits in advance and often uses up the funds earmarked for mining damages.

Instead of by chamber working, mining damage can also be reduced by only *partially mining* the deposit, up to 70%, with longwall faces between which wide pillars are left standing, in long strips (Fig. 345, right). The width of the pillars and panels must be such that the individual troughs they cause overlap to form a single, flat trough which does not exceed a given depth of subsidence. The profile curve of the combined trough should not show any humps over the pillars or any zones of ground extension. In this form of partial extraction, which is frequently employed in the U.K., the amount of the maximum subsidence for an individual panel is read from the nomogram in Fig. 162 and the combined trough traced by summation of the individual troughs. Special nomograms for fixing pillar and panel widths are issued by the National Coal Board in London (Fig. 345,

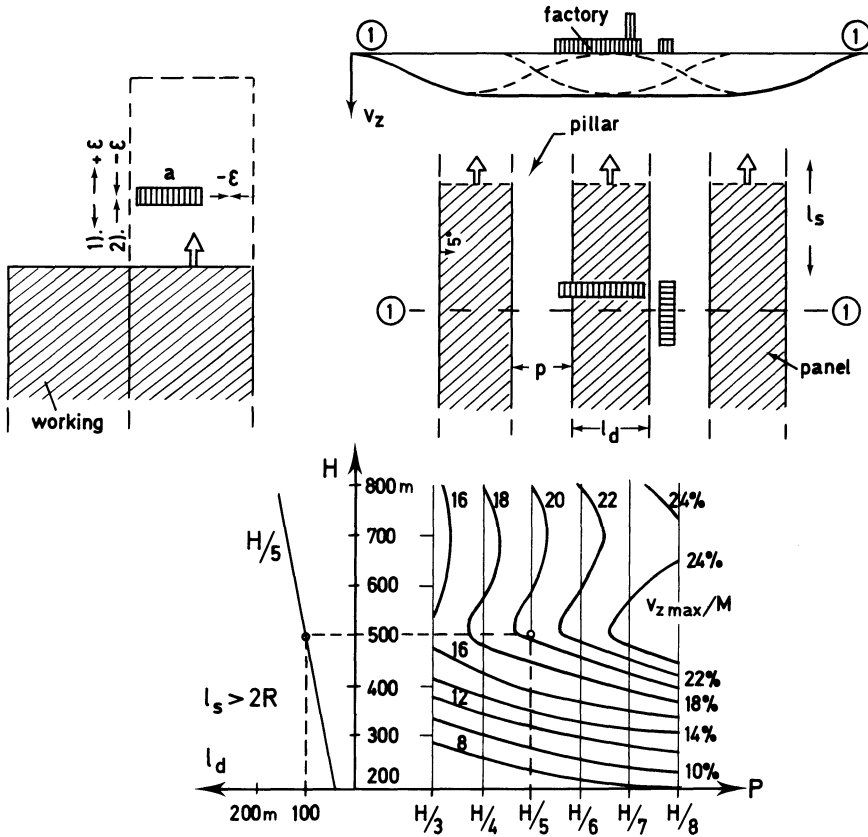


Fig. 345. Left: mining under a sensitive structure (a) on a narrow front. Right: typical layout for longwall partial extraction under an industrial area. Section 1-1 showing individual troughs and combined trough, also ground plan of panels and pillars. Bottom: partial-extraction layout nomogram for width/depth ratio 0.20

bottom). For a 50% excavation at a depth of $H = 500$ m, a maximum subsidence of 20.5% M , given a panel and pillar width $H/5 = 100$ m, can be read off such a nomogram. If this subsidence is too great, either narrower panels or wider pillars can be selected, though the excavation rate will then alter, of course. If, for example, pillar width is increased to $H/3 = 166$ m, a maximum subsidence of only 15% M will result, assuming extraction traverses the whole critical area ($l_s > 2R$). Otherwise maximum subsidence must be reduced (Fig. 162). According to the NCB's data, continuous pillars of coal, to be permanently stable, should have a width, given a seam thickness of $M = 3$ m, of at least $0.12 H$ – i.e., 36 m at a depth of 300 m, and 72 m at a depth of 600 m – and the panel should not be more than $0.6 H$ wide. Square pillars in pillar-and-stall working, with stalls 5 to 8 m wide, should measure approximately 0.04 times $0.04 H$ to be stable (Fig. 341 a). By comparison with complete extraction of a seam, however, there is always the danger in partial extraction without stowing that pillars which are too narrow and liable to give may suddenly, after initial convergence, break up and penetrate a soft floor or roof, perhaps creating unwanted pressure zones in a

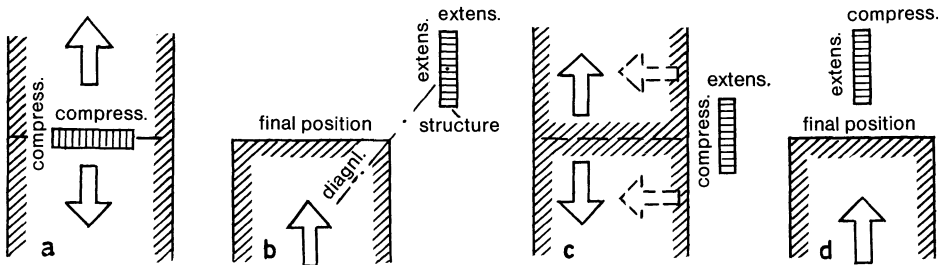


Fig. 346. The four basic examples of a layout designed to produce an acceptable extension or compression of the structure

lower, workable seam (Fig. 110). Whenever economically possible therefore, chambers should be back-filled.

Stowage working as a protective measure is so problem-free from the mining-engineering standpoint that there is no need to go into it further. By contrast, the implications of the size and position of an extraction panel for the structure that is to be protected are not so easy to sum up, and they are examined below.

The possibilities of *mine layout* in terms of positioning and areal extent can be summarized in four basic schemes (Fig. 346). These provide for working under the structure concerned in such a way that

- along both axes it is only compressed at all times (important if, for example, it is a concrete structure): achieved by central working under the structure, in two wings starting from its centre line as projected down to the ground plan of the mine;
- along both axes it is only extended (e. g., for a long row of terrace buildings): achieved by diagonally lateral extraction up to just before the downward projection of the building;
- it is compressed only longitudinally and extended only transversely: achieved by working down the long side in two wings, starting from the transverse axis of the structure as projected, or by working on a broad front away from the structure;
- it is extended only longitudinally and compressed transversely: achieved by working on the transverse side of the structure up to just before its downwards projection.

These layouts apply equally to saddleform flexure (extension) and troughlike flexure (compression) of the ground under the building. With layout a), tilting is avoided. With layout b), a long structure can be twisted around its diagonal axis. Because working is halted before reaching the structure, layouts b) and d) mean forgoing some extraction and consequently sacrificing some mineral. Nevertheless, at shallow depths of working, leaving an unmined protective zone under a valuable structure can be an economic solution to the mining-damage problem, entailing little sacrifice of mineral.

14.2.2 Dynamic Measures

Within the boundaries fixed for mining, the cut can begin in various places and be pursued in a variety of directions, in more than one seam, with a choice of

chronological order. Whereas the mining layout must consider the amount and kind of ground movement in the long term, in determining the *sequence of working* it is the interim, active ground movement which has to be borne in mind. For example, the tilt in a building will alter continuously as the working front traverses the critical area from I to V (Fig. 347). During direct undermining, it is inclined most strongly at stage III in the direction of the focal point of the extraction area affecting it, and then its tilt decreases again until it resumes a horizontal position. Similarly, displacement v_x at first increases up to the focal point of extraction and then decreases after the passage of the face. The building thus undergoes both a swing movement and a tilting movement around the point A. As it does so, subsidence v_z increases continuously and is the only trough element that is not reversible.

In *mining centrally* under the building, if the cut begins not at the middle line a but at the edge b, the structure will first undergo extension and then compression (Fig. 348). This alternation is stress, particularly from compression to extension (for example, with the first cut centrally from a outwards, and the second later in another seam, from b inwards), is more damaging to the framework of the building than a mining influence which remains uniform. The positioning of the initial cut (the opening-out face) and the direction of working are also decisive for the amount of ground movement in the long term, since a steeper trough flank forms over an opening-out face than over an advancing or halted face. It will therefore usually be an advantage to have the working run up to the structure and not start directly under it. That applies also to central undermining, since a dangerous peak of compression (see Fig. 262) can be kept away from the structure only by having the working start to one side and advance up to the structure. Where working is in two wings from the axis of the structure outwards, a constant peak of compression forms in the sole of the foundations, which in the long run develops into the critical peak of compression (1 R) or into the almost compression-free state (2 R), depending on the length of the panel along the strike.

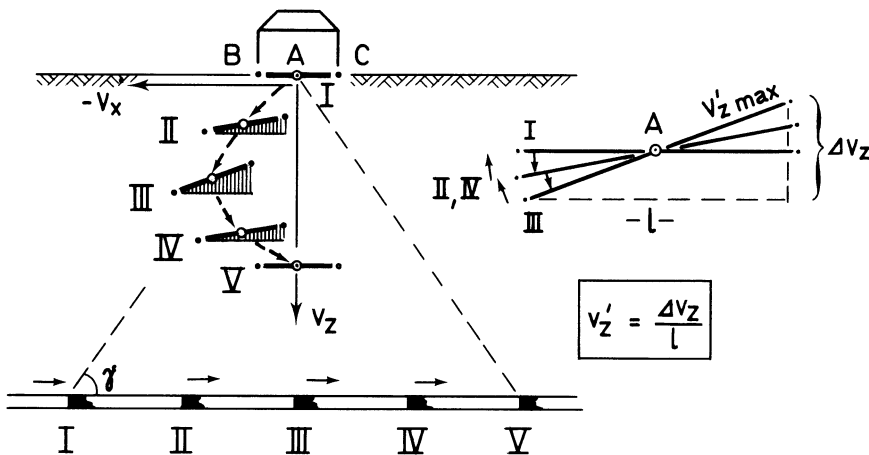
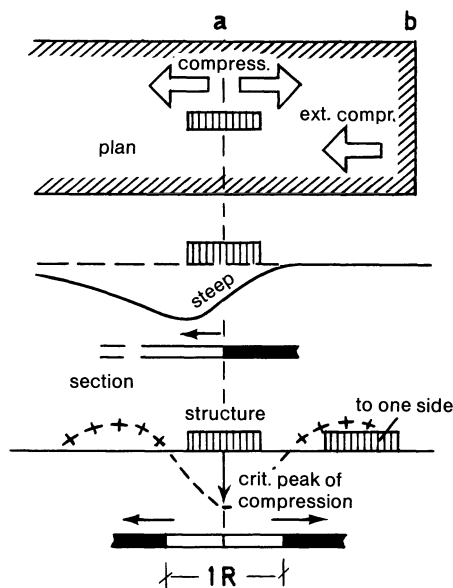


Fig. 347. Alteration in the position and tilt of a house as the face passes under it

Fig. 348. Interim stressing of a structure according to the siting **a** or **b** of the opening-out face: transition from extension to compression (*top*), steep edge of trough over the opening-out face (*centre*), and critical peak of compression on undermining centrally (*bottom*)



Rapid extraction reduces dynamic or “active” extension, compression and flexure above the advancing face in a still intact rock mass showing predominantly visco-elastic reactions, because in such conditions the subsidence trough widens faster than it deepens and consequently flattens temporarily (Fig. 254, top). Rapid extraction as a measure to reduce mining damage is not, however, of great importance, since the rate of advance is largely determined by the nature of the deposit and the extraction technique and therefore cannot be freely selected. A further factor is that, from the latest observations, a high rate of face advance in combination with caving is conducive to fissuring in strata already worked through several times, and the resultant steeper trough sides and greater ground deformation act unfavourably on building structures (Fig. 254, bottom). The rapid increase in tensile stresses and the rapid change from extension to compression are also more damaging to structures than would be a slower transition (Fig. 304). In short, in a fractured, clastic rock mass, dynamic strains increase with faster rates of extraction, and the same applies also to convex and concave flexure of the ground.

For an unfractured rock mass showing a high degree of delayed subsidence, the dynamic extension and compression developing at the point A (Fig. 349) if the rate of face advance is increased from v_{t1} to v_{t2} and v_{t3} can be calculated using Eq. (322). The exponent t in Eq. (322) is substituted by the quotients $(R - b) \div v$, with the result that the temporary peak of extension decreases according to an e function

$$\frac{\epsilon_{\text{dyn}}}{\epsilon_{\text{end}}} = 1 - e^{-\frac{c(R-b)}{v_t}} \quad (395)$$

with an increasing rate of face advance in relation to peak extension ϵ_{end} in the long term. This is naturally only a working hypothesis. (“ b ” represents the horizontal distance between the extraction front and the point A.)

Fig. 349. Dependence of the peak values of dynamic tensile strain on the rate of advance v_t as the face passes under the surface point A, given visco-elastic strata (top right), and comparison with the long-term state of linear deformation for each stage of mining

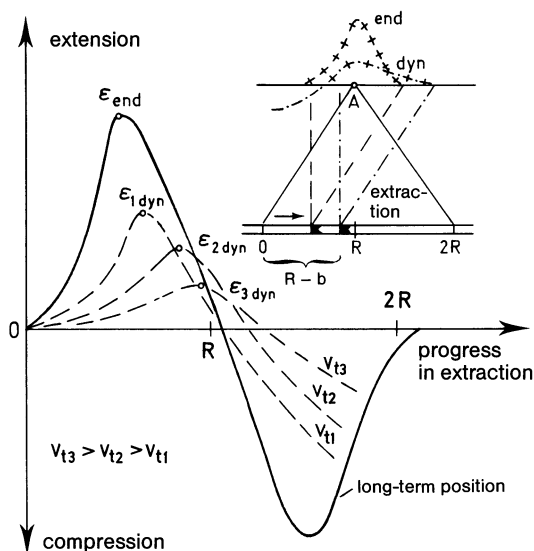


Figure 349 shows schematically how, with increasing speed of extraction, the peak of dynamic extension in an unfractured rock mass becomes smaller on the one hand and, on the other, moves more and more towards the working face. Thus with a very rapid advance it appears directly before the passage of the face under the structure A, whereas on the halting of working it lies at about $0.5 R$ in front of the face.

More reliable than this assumption from Eq. (395) is to interpolate from observed values the appropriate rate of advance in m/d for an acceptable degree of extension or compression. In doing this, the influence of depth on the amount of ground deformation and its increase over time must be allowed for (Fig. 247).

An effective measure for reducing the stress on a structure consists in *overlapping*, at the right time and at the right intensities, opposing ground movements and deformations, such as tilt to the north and tilt to the south, concave flexure and convex flexure, extension and compression, produced simultaneously at the object of protection by two or more workings. If, for example, a structure must not at any time be tilted, one solution is to mine symmetrically outwards from or inwards to the centre line under it (Fig. 350). In the first case the building is always at the centre of the trough (compression, concave flexure) and in the second the edges of the two approaching troughs overlap (extension, convex flexure). Since, after passage of the face, linear deformation and flexure are smaller in the direction of working than parallel to the working front (see Zone I in Fig. 142), the longer axis of the structure in Fig. 350 should lie in the working direction or – quite commonly – always tangentially² to the panel (see Figs. 132

² For the edge zones II to IV of the trough in Fig. 142, a diagonal lie for the structural axis in the direction of the tangents 1–1 and 2–2 of the polar deformation figure (Fig. 141) would be the most favourable from the point of view of minimal extension and compression; but with such an alignment the structure would be more intensely bent and tilted in its longitudinal axis than with a tangential alignment.

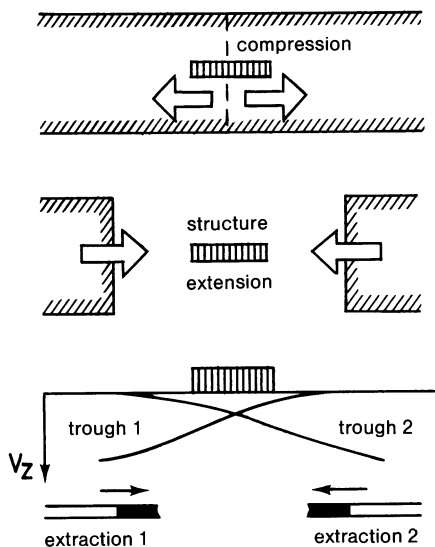


Fig. 350. Mining symmetrically under a structure while avoiding tilt

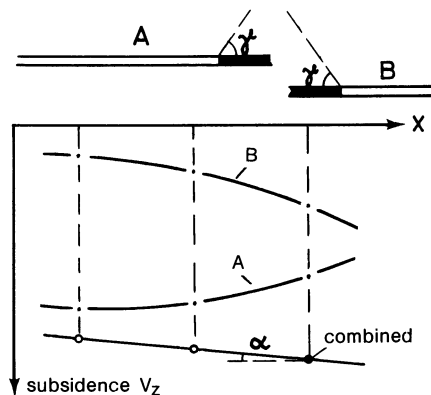


Fig. 351. Superimposing a trough-shaped flexure (central working in seam A) on a saddleform flexure (edge working in seam B). Residual tilt α at the surface

and 142). Nevertheless, tilt cancels itself out completely only at the line of symmetry between the two workings, and the parts of the building lying on either side of it on the trough or saddle flank show a slight tilt, which is at its greatest transversely to the working, constituting the principal tilt (Fig. 133). Similarly, the trough-shaped flexure of the building site caused by working seam A centrally can be evened out by overlapping it with the saddleform flexure from an edge working in seam B, but a certain amount of tilt usually remains (Fig. 351).

By contrast, a cancelling-out of extension and compression for all time sections can be realized only as a rough approximation, even in favourable circumstances in flat, undisturbed stratification, as shown by addition of the displacement curves for two workings offset by the distance R (Fig. 352). Compression of the structure resulting from undermining in seam 1 is completely cancelled out by extension resulting from an edge working in seam 2 only if the sum of the two displacement curves gives a horizontal straight line ($\pm \varepsilon = 0$). That happens only with truly symmetrical and congruent displacement curves – i.e., the thickness of seam, the stowage material, the influence factor, and the widths of the extension and compression zones must be identical for the two seams. (For precise precalculation of the ground displacements being added together, the time factors for the separate stages of working must also be known from earlier observations.) Furthermore, the point of intersection of the two curves must lie at half the height of the displacement maxima $v_{x\max}$, because if the displacement maxima are too great ($v'_{x\max}$), the line of their sum (shown dotted in Fig. 352) will be curved: which would indicate some continued extension and compression. The partial cancelling-out of extension and compression through combining layouts a) and d) in Fig. 346 presupposes one central and one edge working in two seams at the same time, and this implies forgoing extraction under the structure in one of the seams.

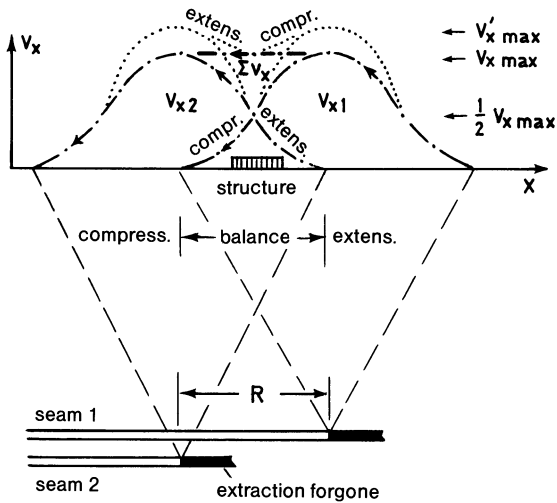


Fig. 352. Extension and compression cancelling out over two workings separated by the distance R . In the vicinity of the structure the sum of the two displacement curves produces equivalent point displacements

14.2.3 Some Examples of Mining Procedures to Minimize Damage

If the edges of workings in the various seams are not offset laterally (Fig. 353), as is often the case at company or pit boundaries, tensile strains emanating from the individual workings can combine to produce cracking of the ground at approximately 7 mm/m. *Ground cracking* is promoted if extraction also starts in a neighbouring concession on the far side of an unworked boundary zone (working No. 4 in Fig. 353), because saddleform ground flexure over the residual pillar is thereby reinforced. If there is already a crack in the ground, tensile strains come together more especially in the zone of fracture, even if working faces are offset

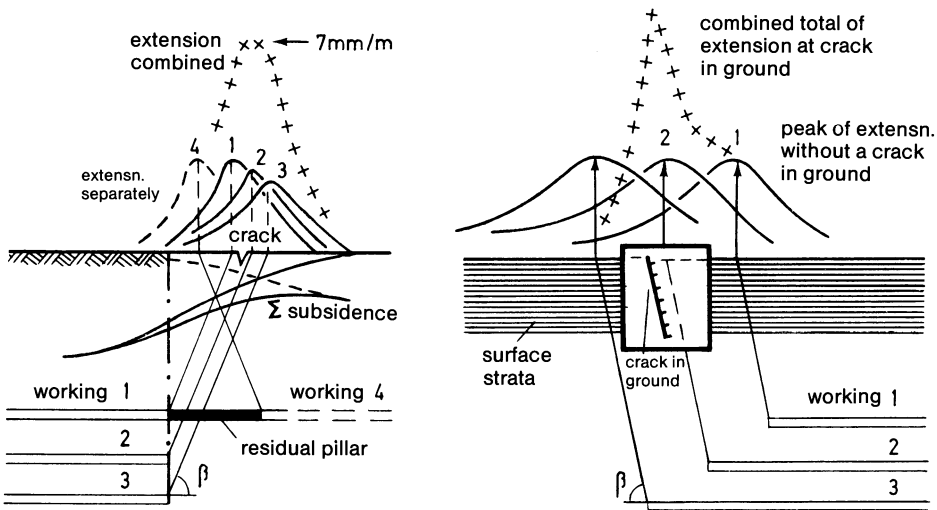


Fig. 353. The overlapping of tensile strains from several panels. Crack in ground on the working of panel 4 because of strong ground flexure over the residual pillar. Cumulative tensile strains at an already existing crack in the ground (right)

laterally (Fig. 151). Ground cracking may also, however, be intentionally induced as a protective measure some 30 to 50 m ahead of the structure, to stop ground movement at the crack and keep the structure outside the then shortened influence trough (Fig. 143e). To achieve this, several seams are worked in close succession (retreat working), starting at the arm of the angle of break β (Fig. 150b).

A mining procedure which temporarily evens out extension and compression at the structure's foundations – especially one in which several seams of one section are worked under surface objects at the same time – is known as “*rolling extraction*”. In panels lying one over the other, workings in the individual seams follow each other at a distance of 0.5 to 1 R, and those in each pair of seams combine to produce the cancelling-out of extension and compression.

Figure 354 shows two variants of rolling extraction in two seams and workings. If both seams, Nos. 1 and 2, are worked simultaneously from the right-hand panel boundary towards the left at a distance of 1 R apart (case a), the building finds itself at first in an extension zone of both seams. If both seams are worked from left to right (case b), seam 1 produces a temporary amount of extension at the structure, which is less than in case a). With the wrong layout therefore, the structure can suffer damage before the desired equalization of deformations has been achieved. As the workings advance, cracks in the foundations are closed up again by the following-on compression effect of undermining in seam 1. However, extension and compression stresses at the structure cancel out at the final stage of mining only if working directly under the structure is forgone in every second seam, or if working reaches the critical area in all seams (Fig. 355). Otherwise there remains an excess of compressive stress.

In Fig. 356, seams A and B beneath a building which is sensitive to linear deformation are to be mined. If they are mined one after the other from left to right in the x direction, the following will happen. At mining stage I in seam A the structure is extended to the maximum, and at mining stage II compressed (diagram top left). This change-over from extension to compression is repeated

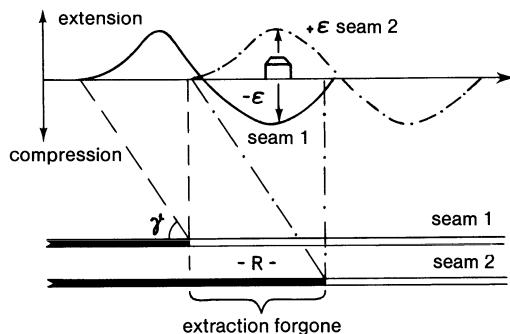
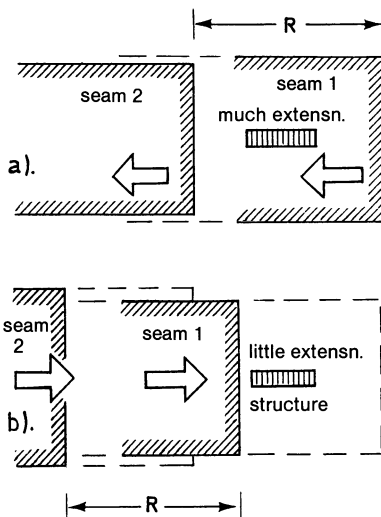


Fig. 354. Rolling extraction of two seams under a structure

Fig. 355. Cancelling out tensile and compressive stresses at a structure by offsetting two workings laterally by the distance R

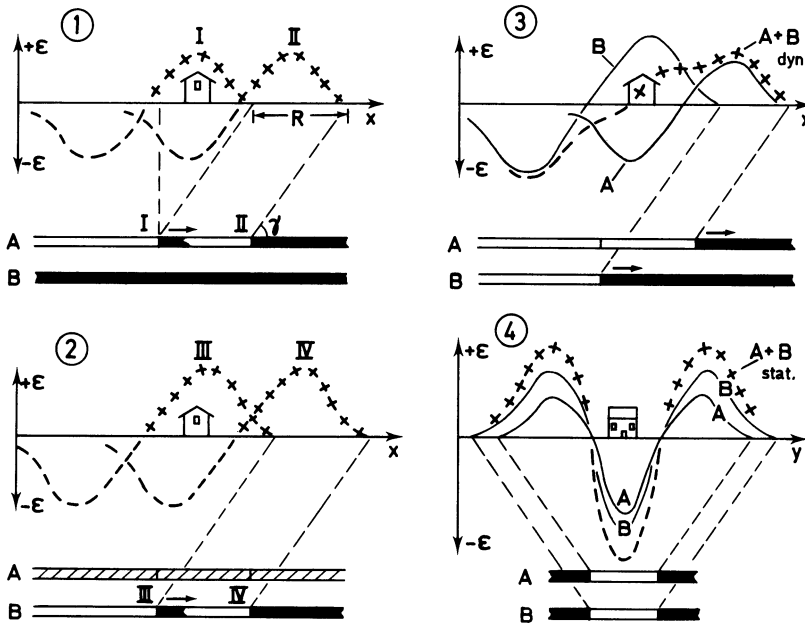


Fig. 356. A structure under tensile and compressive stress on the extraction of seams A and B. (1), (2), and (3) Longitudinal sections; (4) transverse section parallel to the extraction front

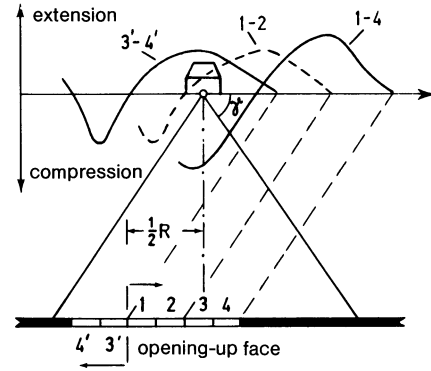
during the later working of seam B (diagram bottom left). This triple alternation of linear deformation ($+\epsilon/-\epsilon/+ \epsilon/-\epsilon$) and also of ground flexure can be reduced to a single change-over if both seams are mined simultaneously at a distance apart of $1 R$. The resultant linear deformation $A+B$ then passes only once from extension into compression (diagram top right). In the transverse direction y the structure undergoes intense compression both before and after being undermined. The greater the interval between the extraction fronts, the slower the rise in compression: which is an advantage for the building. In fractured strata, ground deformation over the second working is often greater than over the first (Sect. 6.5, Fig. 182b).

Extraction of the critical area in chequerboard sections, as foreseen for harmonic mining (Fig. 91c) is rarely possible, either because of geologically disturbed stratification or because parts of seams have already been mined.

To a certain extent the deformation effects from several seams can be reduced by adopting an appropriate sequence in mining them. One example would be if in Fig. 105 a thin seam were mined first, thereby creating a de-stressed zone for the thicker seams above and below it. In the shadow of the resultant goaf, convergence in the working area in those seams is then, contrary to the superposition principle, less strong than without anticipatory removal of the thin seam.

Mining under a structure in a single seam begins, so far as geological circumstances permit, to one side of it at a horizontal distance of approximately $0.5 R$ (Fig. 357). The aim of this is that only a small wave of tensile strain shall form over fields 1 and 2 as working moves towards the structure. The compressive strain which sets in at the structure after it has been undermined in fields 3 and 4

Fig. 357. Mining under a house in a single seam, starting from an opening-out face set to one side. Slight tensile strain at mining stage 2; cancelling-out of tensile and compressive strains $4-4'$



is attenuated by an overlapping tensile strain caused by working fields $3'$ and $4'$ in an outwards direction. The effect of opening up the face to one side of the object of protection is therefore that neither tensile nor compressive strains can develop fully at the structure in the interim period.

The uncertainty in the pre-calculation of interim values of ground movement permits of only a qualitative determination of the overlapping components of this movement, because the influences emanate as much from areas of the panel near the opening-up face mined several months previously as from sections behind the advancing face and only just mined (Fig. 250). Furthermore, the linear superposition of mining influences assumed in the calculations describes only very inadequately the reciprocal influence of neighbouring workings, both in the rock mass and at the surface.

In inclined to steep stratification (Fig. 154), because of the unsymmetrical displacement and subsidence curves in the dip direction, an evening-out of tensile and compressive strains, and also of tilt, will be achieved only in the strike direction, where ground movement develops just as uniformly as in flat stratification. Furthermore, in steep stratification the length of face, the direction of working along the strike, the method of stowing, and the alignment within the various

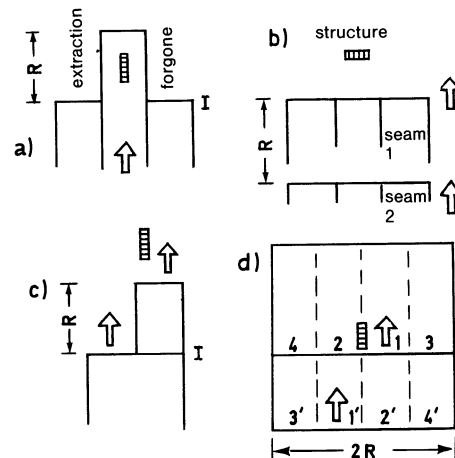


Fig. 358. Examples of the conduct of operations to minimize stress on a structure

levels are all predetermined, with the result that no variation in the mine layout is possible, especially in a transverse direction.

Figure 358 gives several examples of mine layouts designed to reduce mining damage. By forgoing mining on either side of the building it is possible to reduce the compressive strain caused by the centre face-working on passing under the building by superimposing a tensile strain (a). Where a structure lies parallel (tangentially) to the face, working should pass under it on a wide front (if possible $1 R$ wide), to keep compressive stresses in the longitudinal axis within acceptable bounds (b). In example c), a working which continued at the full width of the line I would provoke strong tensile stresses in the longitudinal axis of the building. The right-hand face, continuing at only half width, generates a weak tensile stress at the structure, which turns into compression after the face has passed. To have this compressive stress rise less strongly in both axes, the left-hand face, producing tensile stress, follows at a distance of approximately $1 R$ (the *stepped-face method*). In example d), the compressive strain of working 1, starting just before the building, is reduced by the tensile influence of a simultaneous working 1' advancing from the edge of the critical area. For the same reason, workings 2 and 2' are "rolled" either away from or towards the structure. Once these four fields have been worked to the full length of the critical area, the structure remains compressed only in its transverse axis. It is only when the border strips, and with them the whole critical area, have been mined that the building is more or less compression-free in its transverse axis also.

In the example in Fig. 359, a tension-sensitive coking plant is stressed in tension in its longitudinal and transverse axes by face 1 (diagram a). Faces 2 and

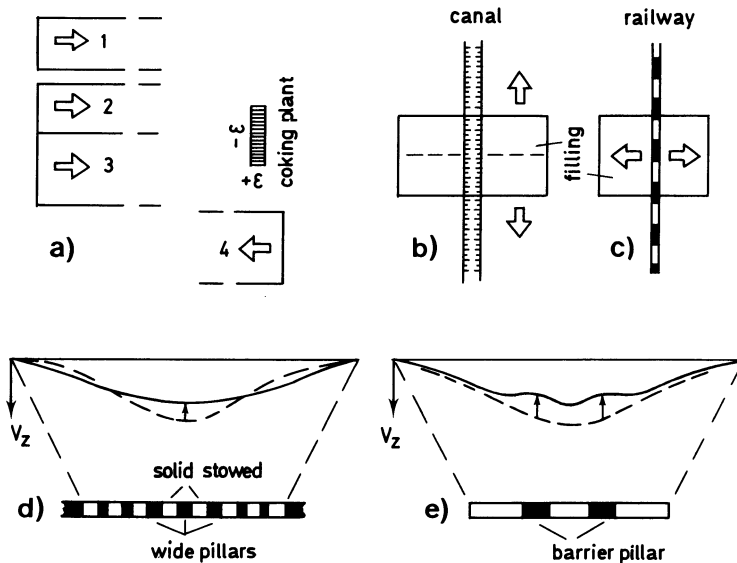


Fig. 359. Examples of procedures for mining under a coking plant a), a canal b), and a railway line c); also for reducing maximum subsidence in chamber working by widening pillars and filling in chambers at the centre of the panel d), or by leaving barrier pillars between smaller panels e)

3, which then advance with a common front, produce compression in the plant's longitudinal axis and reinforce the tensile stress in the transverse axis. With a fourth face advancing in the opposite direction, from right to left, it is possible to weaken the tensile stress in the transverse axis, rising dangerously in the first year of working, by superimposing a compressive stress at the right moment and, in addition, to reduce the compression in the longitudinal axis by means of a tensile effect. When mining under a canal, it should be lowered, if possible along the whole length of a level including both locks, by working in two wings parallel to the waterway, in order to be able later to lower the water-table deliberately and so avoid a comprehensive raising of banks and reconstruction of locks (b). Under railway installations, on the other hand, working at right angles to the rail axis is appropriate, to facilitate surveillance of the tracks in cases where an extension and compression section persists locally for a lengthy period (c). If working were carried out in the direction of the rail axis, the zone of tensile stress would move along with the extraction front.

If the vertical distance between the terrain and the ground-water level (annual average) is small, then with *chamber working* and yielding permanent pillars there are the following possibilities for reducing subsidence at the trough centre:

- to make the pillars in the middle of the panel wider (Fig. 359d);
- to fill the central chambers with stowage material;
- to divide a large panel up into smaller fields with barrier pillars between them (Fig. 359e).

With the first two measures the bottom of the trough is "lifted up". As an offset to the widening of the central pillars, the border pillars in the low-pressure zone 2 (Fig. 14) can be made smaller, until they bear the whole strata pressure according to Eq. (10) and in addition are stable according to Eq. (306). This admittedly increases subsidence over the border zone (Fig. 359d). With the third measure, the profile line of the dish-shaped trough is altered by the barrier pillars into a wavy line, in which the bottoms of the waves over the individual fields of working are higher than the trough centre over the large, undivided panel would have been (Fig. 359e). The zone of compression over the single, uninterrupted panel is broken up by the barrier pillars into several small zones of compression and extension.

The advantages of rolling extraction (Figs. 354, 356, and 358b, d) disappear in chamber working if active subsidence over the extraction front is very small and the main subsidence sets in only after abandonment of the panel (phase II in Fig. 251, bottom). The troughs which then form, and their zones of extension and compression, are stationary, as in longwall mining in transverse profile (Fig. 123). That applies also to the other dynamic measures affecting the siting of the opening-up face (Figs. 348 and 357) or the direction of working (Figs. 350, 354, and 359). By contrast, mining damage can be reduced by the static measures depicted in Figs. 346, 351, and 355, even in chamber working.

In *steep stratification*, because of the minimal strata pressure perpendicular to the strata [Eq. (7)], the pillars can be made narrower, with the result that the volume of subsidence is greater than in flat stratification. The subsidence maximum lies, as in longwall mining, over the lower half of the inclined working

(Figs. 46, 154, and 165). In fractured strata this maximum can migrate to the upper half (Fig. 164), as is also the case on the extraction of a steep critical area (Fig. 158, top).

14.3 Structural Precautions

In order to prevent mining damage to building structures, or at least to reduce the scale of damage, their builder is in many countries required by law to ensure, on the demand of a mining undertaker, that they are *adapted*, in terms of location, alignment, and construction, to the mining influences foreseen. For example, in the siting of new buildings, areas of earth cracking are avoided, swimming baths are not sited in zones of maximum future tilt, dwelling-houses and factories are aligned with their longitudinal axes parallel to lines of equal subsidence; in construction, care is taken to ensure that cellars are of small and uniform depth, and the keying of individual elements of foundations (e.g., supporting pillars or piles) into the ground is avoided.

Where these measures of adaptation are insufficient, the mining undertaker can require the builder to incorporate *safeguards*, the cost of which falls on the mine. In extreme cases, where neither adaptation nor precautionary measures will provide adequate protection against future mining damage, the mining undertaker will issue a *warning against building* and will compensate for the consequent reduction in the value of the land (see Sect. 13.3.3). In Germany, if the builder fails to comply with the call for precautionary measures or to forgo building, he loses his right to claim compensation for mining damage.

The constructional measures designed to render surface structures in a mining district insusceptible to mining damage, to a greater or lesser degree, were first developed as so-called total precautions (Mautner, 1920) and later as partial precautions (Luetkens, 1941). Such precautionary measures are intended to protect the structure against flexure and linear deformation of the ground it stands on, either completely (total precautions) or only in respect of major damage, so that its utility and stability are not affected (partial precautions). The precautionary measures relate therefore to deformation of the ground in the vicinity of the structure and to the resultant extraneous forces transmitted to the structure, but not to changes in the lie of the whole foundation soil (subsidence, tilt, lateral displacement), from which an earthbound structure cannot escape.

Obviously, vertical changes in position such as subsidence and gradient changes (tilt) are of importance for communications installations and public utilities such as roads, railways, canals, oil pipelines, sewers, etc. For this reason, the subsidence to be expected in a mining area is taken into account as early as the execution stage for such installations, with the result that costs for later conversion, raising, or demolition are reduced. Such precautionary measures cannot, however, be seen as safeguards against mining damage in the narrowest sense, as they are concerned more with adjusting to the presence of mining and making provision for rapid and economic elimination of damage than with preventing the occurrence of damage. Among these precautionary measures are also

to be counted such constructional arrangements, mentioned earlier, as recesses and bearings for presses intended to raise a tilted building or a sunken bridge. As a displacement, a change in the horizontal alignment of the whole structure is of importance only in relation to neighbouring structures with which it may be connected by railway lines, conveyor installations, or pipelines.

In designing safeguards for a structure, the civil engineer or architect concerned needs to bear in mind that

tilt and ground flexure alter during mining not only in magnitude (Fig. 347) but also in direction (Fig. 238);

a structure under which mining passes is exposed alternately to tensile and compressive stress, convex and concave flexure (Fig. 298);

a structure standing over the zone ahead of the face is compressed in one axis and extended in the other (Fig. 142).

It is against these alternating stresses that the structure has to be secured.

In Fig. 360, as the extraction front advances from position 1 to 2 and then 3, the trough lengthens and its centre migrates from Z_1 to Z_3 . Whilst the left-hand and lateral areas of extension over the zone around the working and the area of compression over the working itself remain stationary, the right-hand area of extension moves farther rightwards at the pace of the extraction front (in coal-mining, 3 to 7 m per day). The shape of the trough profile line indicates the position and direction of tilt and curvature at any given moment.

The construction engineer can make a rapid *estimate of the maximum values* of ground movement and deformation to which the structure would be exposed in the worst conditions by applying the simple formula:

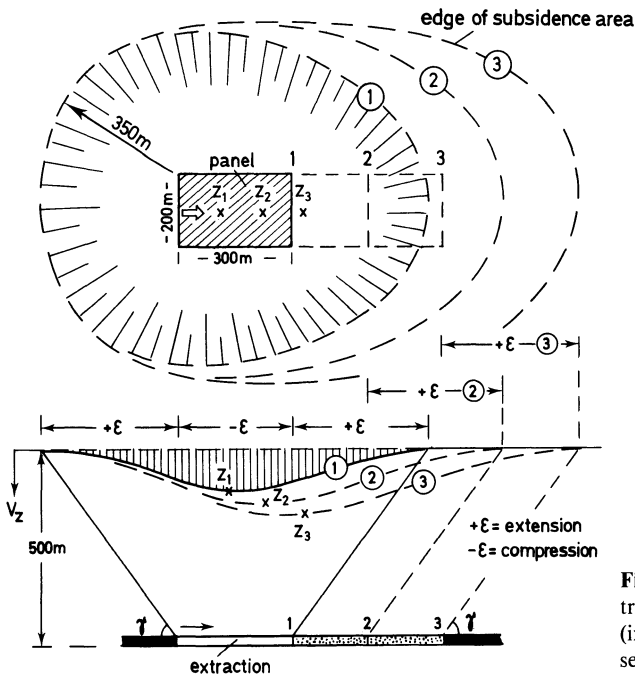


Fig. 360. Enlargement of the trough in the direction of mining (in plan above, in longitudinal section below)

$$\text{Tilt, extension, compression (mm/m)} = k_i \frac{\text{maximum subsidence (mm)}}{\text{depth of working (m)}} \quad (396)$$

In the literature the coefficient k_i is given the value, for tilt, of 1.0 (Ruhr), 1.6 (Poland), and 3.4 (U.K.); for extension, of 0.25 (Ruhr), 0.45 (Poland), and 0.8 (U.K.); and for compression, of 0.5 (Ruhr), 1.0 (Poland), and 2.2 (U.K.).

For maximum subsidence, the figure used is 50% or 90% of the thickness of seam mined when stowing or caving respectively. For chamber working with permanent pillars, maximum subsidence in the long term c_r can be derived from the formulae in Fig. 252. In addition –

$$\text{minimum radius of curvature (km)} = k \frac{\text{working depth}^2}{\text{maximum subsidence}} \quad (397)$$

where $k = 50$ to 300 .

The diagram in Fig. 361 enables peak values of ground movement and deformation for a 1 m thick working to be read off according to depth and method of working, using coefficients of average value. For other seam thicknesses and coefficients the results need only to be altered linearly – for example, for $M = 1.8$ m, they should be multiplied by 1.8 or, in the case of radius of curvature, divided by 1.8.

With increasing depth of working H , deformation of the ground becomes less severe, as Fig. 362 makes clear. Given the same thickness of seam ($M = 2$ m) and the same method of working (caving) at both depths, extracting the critical area at a depth of 1000 m produces the same amount of subsidence (0.9×2 m = 1.8 m) at the central point P as extracting the critical area at a depth of 500 m but a wider trough at the surface – from A' to B' as compared with A to B . The trough from the deeper working is therefore flatter and the deformation less (see also Fig. 127). At great depth, however, extracted areas seldom reach the size of a critical area (which at 1000 m and a limit angle of 55° would be 1,400 m long and wide), particularly if there are any geological faults. For subcritical areas therefore, Eqs. (396) and (397) contain an additional element for safety. On the other hand, in intensely worked and fractured strata, double the intensity of deformation and tilt must be expected.

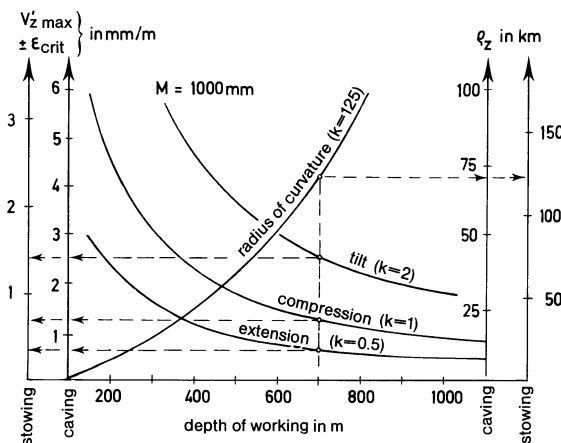
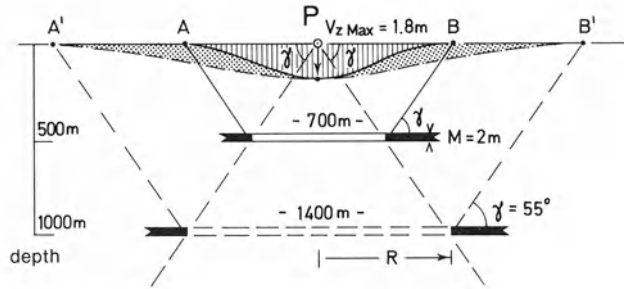


Fig. 361. Peak values of ground movement and deformation over a mine working, as a function of depth, thickness, and method of working (caving, stowing)

Fig. 362. Influence of working depth on the degree of ground deformation



A very important precaution, finally, is keeping the structure under *observation* during the period of mining influence. This means repeated measurement of tilt and curvature by levelling, and of extension and compression by linear measurements. With communications installations, smelting furnaces, and oil or gas pipelines, the monitoring of deformation is unceasing. The data yielded by these measurements enable conclusions to be drawn about the degree of stability and functional effectiveness of the structure and the timing of repairs, raising, or vacating of the building. Particularly suitable for constant monitoring are stationary electric measuring devices such as electronic levels and bonded resistance strain-gauges, coupled to recording devices.

14.3.1 Total and Partial Precautions

The structure can react to deformations of the ground in one of two ways: it can resist them, or it can accommodate them. In the first case we speak of mining-damage precautions on the resistance or rigidity principle, in the second case of precautions on the avoidance or flexibility (yieldingness) principle³. Each of these approaches can be applied as much to ground flexure as to linear deformation of the ground.

The *resistance principle* demands a rigid structure which is not susceptible to bending and distortion or to changes in length, and this can be achieved only with large cross-sections for walls, beams, and floor slabs, entailing heavy expenditure. The costs of a bend-resistant construction can be reduced, however, if the structure is divided up into stiff independent units standing on foundations of small area, which consequently exert an increased base pressure (area support). This is because, on the one hand, the costs of stiffening a structure increase with the length of the free-lying area to be bridged over, and because, on the other hand, the geometrical free height between the ground and the underside of the foundation (at its centre if the ground sags, and at its ends if the ground forms a saddle) decreases with the structure's load and becomes even smaller if the ground yields plastically under high local loading, until the whole area of the foundation is bearing (Fig. 289). A weak subsoil such as silt is therefore an

³ The concepts "total" and "partial" applied to precautions refer to the degree of securing against mining damage, whereas the principle of "resistance" or "avoidance" refers to the nature of the measures taken and the way they act.

advantage over coarse sand in mining areas, because given a thin and weak cushion, the reaction force of the seating is more uniform and is distributed to all areas of the foundation. To secure the structure on the resistance principle, the design engineer has a choice between thick foundation slabs, one- or two-point support systems, and four-point support systems to deal with ground flexure, and to deal with linear deformation, a choice between strip foundations and unstiffened foundation slabs, the function of which is to maintain the distance apart between neighbouring foundations when the ground is deformed horizontally.

For cost reasons, the majority of structures are provided with only a small degree of intrinsic stiffness, with the result that any uneven movement of the ground will deform them (Fig. 363); but these flexible structures will fail only when a certain degree of deformation is exceeded. Consequently, on the *avoidance principle*, minor deformations can be tolerated, provided the cracks so formed are finely divided and pose no threat to the safety of the structure. This limit of ground deformation lies in general at around 1 mm/m linear change and 500 m radius of curvature. Precautions against mining damage on the avoidance principle therefore need to be effective only after the acceptable limit of deformation has been exceeded, and can thus be tailored to the sensitivity of the structure in question. This makes them significantly cheaper than strengthening the structure. Parts of the construction which, because of inadequate bending, buckling, tensile, compressive, or shear strength, cannot absorb without damage the additional forces expected should offer those forces as little resistance as possible. They should, however, have as much “give” as other parts of the structure, so that stresses and strains are distributed uniformly over the whole structure – unless the intention is to concentrate the forces on to an accessible and easily repairable member and induce the failure there. Otherwise each insufficient stiffening only increases the damage, as the strengthened structural member absorbs more force than the yielding members and in the end undergoes coarse fracture at its weakest spot – e.g., at a wall opening, where the work output from the stress (force) and deformation (travel) is smallest. Damage increases then with the degree and lack of uniformity in stiffness. If individual components such as silo cells and machinery beds require to have a degree of “give” different from the rest of the structure by reason of their function, the possibility of separating them from the structural entity by means of joints should be examined.

Ideally, a structure secured against mining damage should allow free play to ground deformation without any forces or changes in shape being set off in the

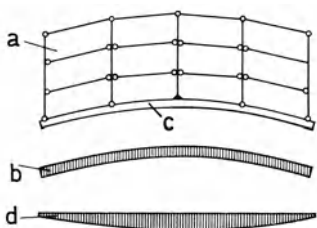


Fig. 363. Deformation of a flexible structure (a) standing on a flexible slab (c), showing uniform distribution of base pressure (b) even on ground flexure. For comparison, uneven base (soil) pressure (d) in a stiff, indeed rigid structure

Table 27. Structural precautions against mining damage on the resistance principle and the avoidance principle

Resistance principle	Against flexure: high rigidity, and also single or double surface and four-point support Against linear change: horizontal stiffening by means of strip foundations or a flexible foundation slab to maintain the distance between foundation walls
Avoidance principle	Against flexure: increased flexibility of structural elements; less frequently, three-point support (only where sufficient rigidity) Against linear change: increased play or travel for movement by means of expansion joints or slip joints, and increased horizontal flexibility; less frequently, rocking pillars (socketed stanchions), stem footings, and roller bearings (only where sufficient rigidity)

structure itself (Luetkens, 1957). On the avoidance principle therefore, structural measures are not confined to merely increasing the flexibility of foundations and walls by using small bricks and weak mortar (lime), leaving room for movement between walls and struts in half-timbered construction, and so on, but also include the provision of open expansion joints, slip planes, rocking pillars, roller bearings to accommodate changes in length, and three-point support systems to cope with bending – all of which allow the part of the structure concerned an almost unimpeded escape from horizontal and vertical ground forces. Table 27 summarizes the most common precautions against the risk of mining damage, grouped according to these two principles. A third principle increasingly followed today is that of *restoration*: buildings are so constructed that they are easily repaired and returned to horizontal in the event of later damage.

On the basis of the details given above, a totally secured structure is intended to withstand ground deformation arising during the working of a seam of mineral without suffering the slightest damage. With *total precautions* therefore, bending and distortion in the structure, but also length changes, must be avoided (Fig. 364). Securing a structure against sagging and distortion can be achieved only by having bearing members of large dimensions. Total precautions therefore presuppose a rigid, bend-resistant structure, supported most suitably as a tower or bridge on a small area, or on two reinforced raft foundations, on four

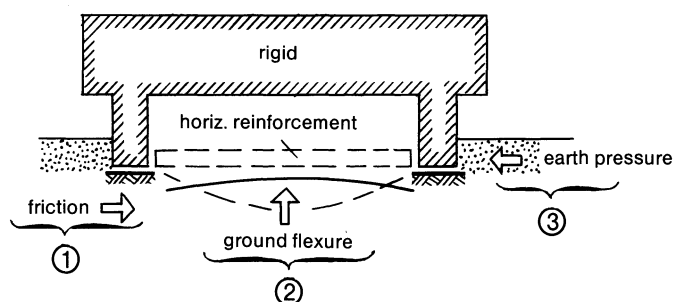


Fig. 364. The principle of total precautions in a rigid structure. (1) Slip joints, rocking pillars, roller bearings; (2) one-, two- or three-point support; (3) horizontal stiffening with a slab or strip foundation

points (resistance principle) or three points (avoidance principle), in order to reduce the effects of ground deformation by having a small foundation area and a correspondingly high pressure on the subsoil. Horizontal ground forces, on the other hand, do not necessarily have to be rendered harmless by horizontal reinforcement (resistance principle) when the aim is total precautions: they can also be neutralized with the aid of slip joints, roller bearings, and rocking pillars, on the avoidance principle.

A flexible structure can be protected only in part against mining influences, because it cannot be supported as a system of points or on small areas and the whole body of the structure has to follow the curvature of the ground. In *partial precautions* therefore, a small amount of crack formation is acceptable, provided the functional efficiency of the structure is maintained. Partial precautions are used in structures of low rigidity, the walls and foundations of which are made flexible (avoidance principle; Fig. 365), in view of the unavoidable sag when ground movement occurs (the resistance of walls to bending increases with the third power of their height). Horizontal displacement of foundation walls is limited either by horizontally reinforced foundation units and floor slabs (resistance principle) or by providing a slip plane between the subsoil and the foundation constructed on it (avoidance principle), as shown in Table 28. The National Coal Board has reported that modifications may be made to *existing structures* before ground movement occurs if it is believed that the amount of damage avoided will justify the expense or if there are other circumstances which make it imperative to minimise the damage without weighing the cost too closely. Such circumstances may, for example, include: schools and hospitals, where alternative accommodation may not be available if repairs become necessary of such a degree that the building has to be evacuated; or churches, houses, etc. which are scheduled as ancient monuments. In all cases where the fabric is of such a construction or age that even moderate movement might involve danger, it may be necessary to provide temporary supports and/or strengthening before mining takes place. Several types of support have been used to preserve the shape of arches when the building undergoes tensile strain.

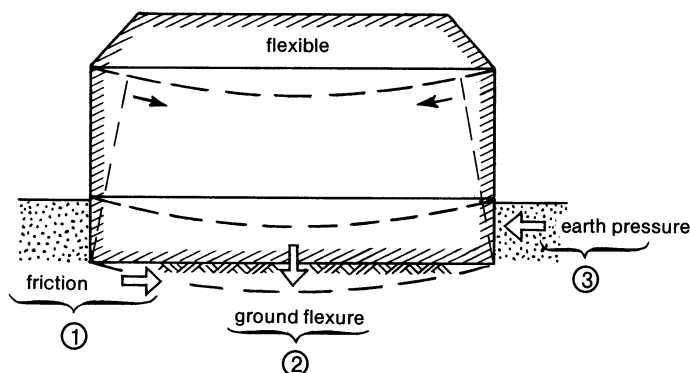


Fig. 365. The principle of partial precautions in a flexible structure (in this case on concave flexure). (1) Slip joints or flexible slab with reinforcing; (2) flexible construction, with floor slabs on each floor stiffened; (3) stiffening of cellar walls, peat-filled trench in front of house

Table 28. Structural precautions, total or partial, against mining damage

Total precautions in a rigid structure	Against flexure: resistance principle (stiffening); less frequently avoidance principle (three-point support) Against linear change: resistance principle or avoidance principle (e. g., roller bearings)
Partial precautions in a flexible structure	Against flexure: only avoidance principle (flexibility) possible Against linear change: resistance principle (maintaining distance apart); avoidance principle (skeleton construction, slip joints)

The use of the traditional tie rod is only justified if the predicted extension of the ground is sufficiently severe to make it likely that roof trusses will be pulled off their bearing when the supporting walls move apart. Injudicious use of tie rods can lead to needless disfigurement of buildings by the rods and star plates themselves and by concentrating damage at the points where the plates tend to pull through the walls. In some cases extra bearing has been given to roof trusses by means of temporary corbels.

By means of a steel mesh which is fastened and pre-stressed to the face of exterior walls, and which can later be dismantled like scaffolding and re-used at other masonry buildings, it is even possible to stiffen the entire structure of a building during the extension phase (see Fig. 374.2).

The effects of compression may be reduced by removing part of the rigid paving, or even the superstructure, to provide room for compression. This can be done by cutting out slots in floors and paved areas joining adjacent buildings, or at intervals over large areas of flooring to provide adequate compression space. Concrete floors are difficult subjects, but wood block floors and brick and stone paving are easily dealt with. In some cases it has been thought a reasonable precaution to remove and store stained glass windows: partitions, screens and ornaments can generally be freed from walls and floor and independently supported. In extreme cases part of a house in a long terrace has been cut out, or even a complete house removed, to allow for compression space.

The excavation of trenches around buildings has been successful in reducing the damage due to horizontal strains, particularly compression (see Fig. 372).

It is difficult to predict if and where slip planes will occur in old buildings. However, the introduction of an artificially created slip plane in the correct position can be beneficial as a structural precaution in a new building (after NCB 1975).

14.3.2 Some Examples of Structural Precautions

As a rule therefore, costly industrial installations and machinery beds are fully secured against ground curvature and linear changes by reinforcement, whilst flexible structures are partially secured with strip foundations or unreinforced slabs.

14.3.2.1 Total Precautions

Examples of *total precautions* are given in Fig. 366. Founding on a single surface is well suited to structures with a circular base, such as chimneys, cooling towers, and machinery beds, and it is even possible to make them smaller at the base by cantilevering out during construction (a). For elongated structures, such as coking plants, total precautions with founding on two surfaces on the lines of a rigid beam resting on two supports, is preferred (b, left). The centre portion hangs low, in order to keep the moments of force arising from bearing friction as small as possible. In high foundations, either one support takes the form of a rocking pillar (avoidance principle – b, right), or a horizontal, flexible plate (resistance principle) is introduced over the two surfaces or four points providing the support and under which, separated by an artificial slip plane, the actual foundation pedestals stand (see d). The plate absorbs the horizontal earth and friction forces without conducting them to the construction above as moments or shear forces. The three-point support system with roller bearings (c) as encountered in kiln foundations or winding towers, or with plates as distance pieces, can be regarded as the ideal form of total precautions: with anything more than three supports the evolution of forces in the structure follows the type of ground flexure. With a four-point support system, however, it is possible to correct an uneven settlement by jacking or by inserting an adapter piece and thus restore the load uptake of all four supports (d). Examples of measures for total precautions can be found at, among other places, power stations in mining districts (Savelsberg, 1972).

In the case of *tanks* constructed in mining districts, the National Coal Board draws attention to alternative methods of allowing for movement, including the separation of large tanks into smaller units with flexible connections. – In South Wales a 2.25 million litres rigid tank was built on three concrete pads and made capable of being jacked level if necessary (NCB, 1975).

The essential design principles in this example include the following:

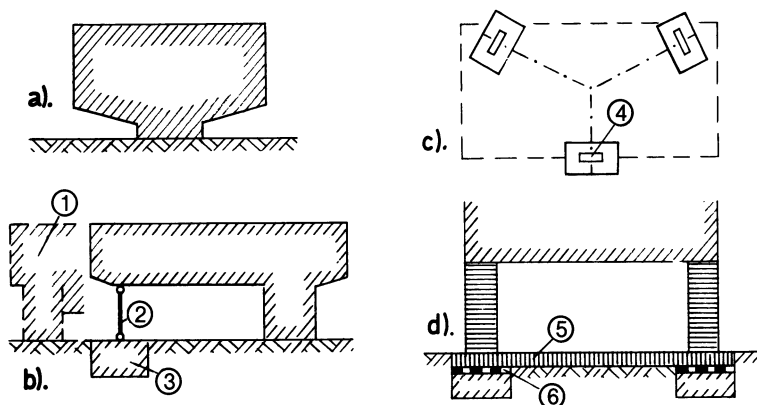


Fig. 366. Examples of total structural precautions against mining damage (after Mautner among others). (1) Horizontal stiffening; (2) rocking pillar (socketed stanchion); (3) pedestal; (4) roller bearing; (5) slab; (6) slip joint

Foundations are as small and stiff as possible to reduce and cope with bending, extension and compression of the ground during subsidence.

Ground pressures are kept as high as possible to enable pressure redistribution to take place under the tank.

The structure sits on a sliding layer and on granular material to reduce friction.

The conical shape is more rigid than a cylindrical shape.

Three jacking pockets are provided.

Twelve of these tanks are planned to be constructed and will be subjected to total subsidence of up to 7 m (over 20 years), the shallowest seam being 353 m deep.

The more normal type of service reservoir should be kept small, i.e., units of from 1 million litres to $2\frac{1}{4}$ million litres capacity, with flexible jointed pipe connections between units. Construction is generally in reinforced concrete, kept watertight by copper or rubber seals.

In this connection an example of total precautions in the sphere of *house building* may be mentioned. In the southern Ruhr district, because of earlier workings near the surface the location of which is mostly unknown today, dwelling-houses have been aligned in the strike direction and their basements built as rigid, reinforced-concrete boxes. The latter measure entails a 5% increase in cost, but it does mean that, in the event of a collapse of the ground, a house can bridge a cavity of 6 m diameter without damage. Some buildings of the University of the Ruhr in Bochum have been secured against the risk of surface collapse in an area of outcropping coal-seams, either by transmitting the structural load via deep foundations of heavy-duty piling, up to 30 m long and 1.5 m thick (which “nail together” the upper strata) to the stable strata beneath, or alternatively by distributing the load to a as large grid-shaped foundation as possible. In addition, broken up or loosened ground (Fig. 367) was stabilized by injecting a suspension of cement. To enable surface collapses to be recognized and cavities to be filled in good time – if necessary with a rapidly sealing barrier of stowage material to guard against even deeper workings – the floors of cellars

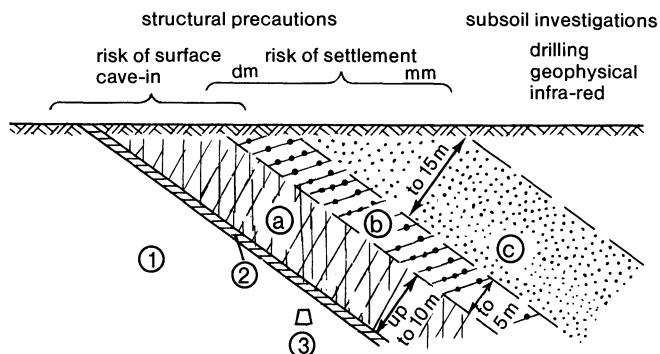


Fig. 367. Risk to structures over near-surface mining (after Klose and Hollmann, 1971). The width of the zones a, b and c depends on the dip in the beds. (1) Solid footwall; (2) old working; (3) mine roadway; a, b, c loosening, fracturing, breaking-up of the rock structure

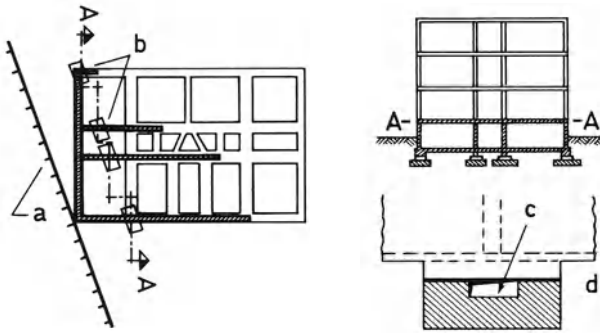


Fig. 368. Securing a building over a subsided step in the ground. (After Mehrhoff, 1974)

and basements are not fixed to the foundation grid. To increase structural rigidity, separation joints were avoided as far as possible in the body of the buildings. In open spaces and sport grounds a safety net was stretched immediately under the turf from foundation to foundation.

Similar total precautions have to be undertaken where there is a risk of subsidence cracks or steps in the ground. In the building in Fig. 368 the risk is that the left-hand side, beneath the foundation, will be exposed as a result of a step-like subsidence (a) of the ground. This part of the building is therefore stiffened against bending with walls of reinforced concrete and supported on foundation plates (b). Beneath the walls these foundation plates have recesses or pockets (c) for the later incorporation of hydraulic jacks. A slip membrane (d) on the foundation plate reduces the frictional forces.

The National Coal Board recommends that, for isolating foundations of rigid structures from the subsoil, the foundations themselves should be strongly reinforced concrete rafts or beams capable of supporting the superstructure over ground subject to both change of length and curvature. The superstructure should also be as rigid as possible, with damp-proof courses, for example, of Staffordshire blue bricks to obviate sliding.

In the experience of the NCB, the principle of keeping buildings to independent units of small plan area is most important and allows higher construction, even – with suitable rigidity – to the extent of 15 or 16-storey blocks of flats. It is desirable that such tower blocks be capable of being jacked to new levels should permanent tilting occur, and that they should be built only over areas of deep mining, where the maximum gradients which mining can produce are small. These tower blocks have plan dimensions of about 25 m × 16 m and are supported on raft foundations about 2 m thick, suitably reinforced and imposing a maximum increased load of 22 Mg per square metre. The excavations below the raft are carefully inspected at formation level, and if the stratum is weak it is excavated and back-filled with a 1 : 10 mix of mass concrete. The mass concrete is brought to formation level by a 50 mm thick protective layer of high-quality concrete, which in turn is covered by a layer of polythene sheeting. A 150 mm thick layer of compacted sand is laid on top of the polythene and a further layer of polythene sheeting is laid on the sand; then another layer of blinding concrete 100 mm thick is laid, upon which the raft is constructed. The soffit of the raft is horizontal with no projections. The working space immedia-

tely around the raft is filled with ash or some other suitable crushable material to minimize the effect of horizontal stresses upon the structure. The finished surface immediately around the block is not paved with flagstones but is finished as a strip of garden or paved with macadam. The lift walls are constructed with an additional 75 mm clearance on all sides over the normal requirement of the lift engineers. Main piped services have flexible joints where they enter the building and observation sockets are built in just above ground level (NCB, 1975).

14.3.2.2 Partial Precautions

According to a point of view frequently put forward in the literature (e.g., Luetkens), linear deformation of the ground has a far greater share or responsibility than ground flexure for the whole range of mining damage⁴. Accordingly, if this opinion is correct, it is possible to forgo costly total precautions against ground flexure and protect the structure only against horizontal deformation forces in the ground: in the vertical direction, a suitable degree of flexibility in the structure will suffice (avoidance principle) – i.e., partial precautions. The more flexibly the structure can adjust to ground flexure, the less will bending forces be able to reach the superstructure.

On this basis, the structure must be secured (Fig. 369) against

the frictional forces F_i (exhibiting a triangular pattern of increase and decrease) on the base and sides of the foundation, which exert a tensile or compressive action according to the way the structure is aligned to the mining area (Fig. 142); the forces of earth pressure E_i on the sides of the foundation lying at right angles to the direction of ground compression (Fig. 343);

the reciprocal displacement of supports, columns, and load-bearing walls of the foundation.

Against these forces and displacements the most frequent precautions effected are on the avoidance principle (Fig. 369):

artificially created horizontal slip planes (b) beneath the foundation;

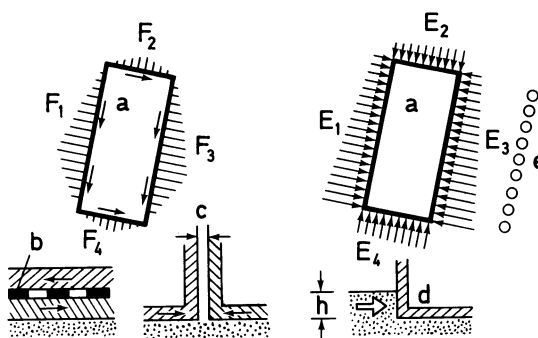


Fig. 369. The frictional forces F_i and earth-pressure forces E_i acting on the structure. **a** The cellar in ground plan; **b** to **e** precautionary measures

⁴ When it is considered how much cracking in walls is caused by uneven settlement in newly built structures in areas free from mining, the inclination is to ascribe the greater share of damage to ground flexure.

vertical separation gaps and movement joints between individual structural units (c);
 minimum depth of foundation h (d);
 a cushion of peat in front of the foundation to counter earth pressure (e);
 and on the resistance principle:

maintenance of the distance between load-bearing elements of the foundation by means of horizontally stiffening reinforced-concrete strips, or by a base slab, strong in tension, underneath the load-bearing parts of the foundation (Fig. 370a and b).

Various possibilities for *partial precautions* are shown in Fig. 370. In one, the longitudinal and transverse walls of the cellar are erected on a horizontal grid consisting of reinforced-concrete strips. A strip foundation of this kind absorbs ground forces in its longitudinal direction, but the strips and ribs lying across or diagonally to ground displacement are exposed to horizontal bending and twisting, which they can withstand only if stoutly reinforced laterally and braced diagonally (a). The cheapest way to provide horizontal stiffening such as this is by means of a continuous slab placed either above or below the point and strip foundations (b). A slip joint or slip surface between foundation soil and foundation, or between foundation and superstructure, limits the transmission of forces during linear changes in the ground to friction alone, the intensity of which can be estimated. To prevent any significant bending moments developing in the foundation slab, which is loaded only in strips, it should be so finely proportioned to the tensile forces in the direction of both axes (only 10 to 25 cm thick) that it yields vertically and acts merely as a distancing layer. It is known as "flexible slab" and is as thin as possible but is strong in tension by virtue of non-prestressed steel reinforcement. For heavy structures standing on soft ground, however, a thick foundation slab can be more suitable because of its ability to spread the base pressure uniformly. In residential buildings the floor slab is usually directly next to the foundations, without a slip layer.

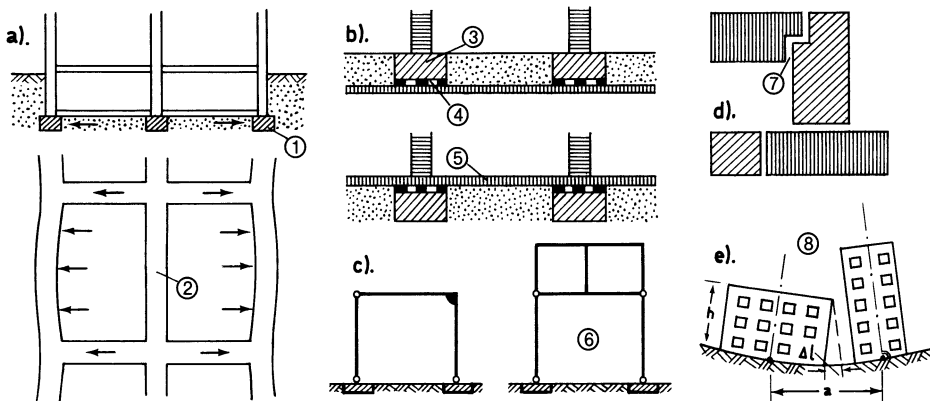


Fig. 370. Examples of partial precautions. (1) Reinforced concrete strips; (2) strip foundations; (3) pedestal; (4) slip joint; (5) flexible slab; (6) rocking pillars (socketed stanchions); (7) open separation gap; (8) maintaining a distance from the neighbouring house

Of the two solutions in Fig. 370b, the *flexible slab* lying underneath the foundation, on the subsoil, has the advantage that no turning moments or bending stresses are transmitted to the foundation, as would be the case with a slab lying above it. In addition, the earth pressure forces on the inner sides of the concrete strips are eliminated (a). The low-positioned slab does, however, make it necessary to excavate down to the bottom of the foundation over the whole ground area of the building and to fill in again up to the base of the cellar. In both cases it should not be overlooked that, even before the effects of mining are felt, bending and tensile reactions developing in the slab from uneven settlement of the foundation will consume part of the tensile forces intended to be absorbed by the steel reinforcing in the slab.

The role of a stiffening foundation-plate in entranceways, or under other weak places in the structure, in the event of compressive ground deformation can be demonstrated by means of a model (Fig. 371, left). On a rubber belt, which is stretched at one end by attached weights, a building consisting of several parts is assembled, leaving in the centre an opening (1) of width a . If the tension in the rubber belt is reduced by removing some of the weights, the width of the opening will narrow to b , simulating ground compression. If on the other hand the opening is stiffened with a horizontal plate (2), the width of the entrance "a" is maintained, even under ground compression (see Fig. 302).

In sheds erected in skeleton construction (c), the foundations under the legs and rocking pillars should be able to shift freely in the transverse direction, especially if the walling in the ground floor has not been filled in and the supports of the portal frames or cruciform bracing are not needed to provide stability and absorb wind pressure, because of stiffening through a support system at the

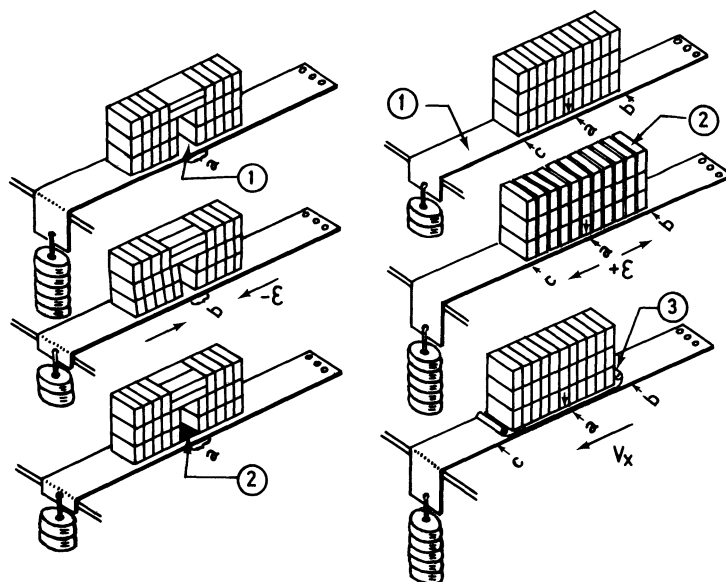


Fig. 371. Left: narrowing of an entranceway on ground compression (1). Horizontal plate (2) for stiffening. Right: the effect of a foundation slab (3) on extension of the ground (1). Tension cracks (2) in an unsecured building. (After Neuhaus, 1954)

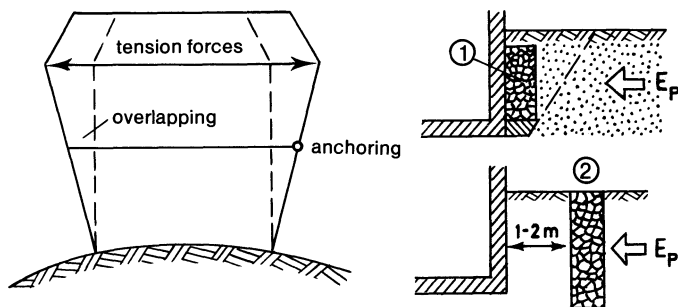


Fig. 372. *Left:* tension stresses in floor slabs on convex ground flexure. *Right:* reducing earth pressure E_p on the cellar wall with (1) a cushioning layer, (2) a peat-filled trench

centre of the shed. Estate walls and railing pillars incorporate vertical expansion joints against linear change, which are filled with a plastic compound. Movement in the railings can be provided by means of slotted fishplates.

Where a traditional building is to be built without special foundations or stiffening, all parts of the superstructure should be as flexible as possible. Lime mortar should be used in brickwork, damp-proof courses should be of the bitumastic type; window frames should be of wood with oversize rebates, or if steel frames are used they should be mounted in wood. Fibreboard ceilings should be used instead of plaster, and drywork generally is beneficial, including on walls. Arches of brick or stone should be avoided (NCB, 1975).

Concave curvature suits a flexible structure better than convex curvature, because the pressures arising in the upper fabric can be absorbed by the side walls and floor slabs (Fig. 365). In the basement the customary reinforcing against tension provides adequate resistance to tensile bending forces and compressive frictional forces acting against them. Under convex curvature, on the other hand, the floors in the upper part of the building are stressed in tension (Fig. 372). Floors must therefore be given adequate steel reinforcing throughout in both directions, overlapped or anchored in the joist bearings (ring anchoring). Reinforced-concrete skeleton structures for dwelling-houses and office buildings are not sensitive to flexure (Fig. 363); and with the increasing strength of floor beams the distance between supports is also increasing, with the result that the overall stiffness remains the same. It is only with industrial structures having useful floor loads of 1 to 5 Mg/m² that significant differences in deformation cannot be accepted.

According to investigations by the NCB, there are today over a hundred types of industrialized building systems (pre-fabricated buildings) for dwelling-houses alone, including both rigid and flexible methods of construction. But very few are either sufficiently rigid or sufficiently flexible to be recommended for mining areas.

In general, many of the wooden-framed houses are flexible – or could be so with some attention to the method of applying cladding – and some of the steel-framed houses are rigid, especially where the cladding itself is rigid. Those types which consist of precast concrete slabs are not likely to withstand any movement without suffering damage, and certain prefabricated wooden components are

fitted to such close tolerances that quite small ground movement would cause distortion (NCB, 1975).

Long terraces of buildings must be interrupted by vertical *gaps* to limit the increase in frictional forces (Fig. 286) and avoid bending damage in the upper storeys (Fig. 370d and e; Fig. 303c). The width of the separation and movement joint “a” between neighbouring buildings is the sum of the two distances a_1 and a_2 (Fig. 373).

The distance

$$a_1 = 10 \text{ cm} + \frac{L_1 + L_2}{2} \frac{1}{200} \quad (398)$$

between the ground-plates (3) is to enable displacement of the at most $L = 30 \text{ m}$ long building under ground compression of up to 5 mm/m (safety factor 10 cm). The distance

$$a_2 = \frac{L_1 + L_2}{2} \frac{h}{5000} \quad (399)$$

is to allow for tilting of the houses given concave ground flexure down to a minimum radius of curvature of 5000 m , so that houses (4) of height h shall not touch.

Protection against strong *earth-pressure forces* on the sides of the foundation can be provided by placing in the excavation a cushion of compressible material such as boilerhouse cinders, foamed plastics, peat ($E_s \sim 50 \text{ N/cm}^2 = 0.5 \text{ MN/m}^2$) or the like. Further relief of the cellar wall in a zone of compression can be sought by providing a concreted foundation-base flush against the natural soil, with the aim of conducting earth-pressure forces into the floor slab (Fig. 372, right). In the U.K., trenches filled with cinders, excavated mechanically at a distance of 1 to 2 m from the house at right angles to the direction of compres-

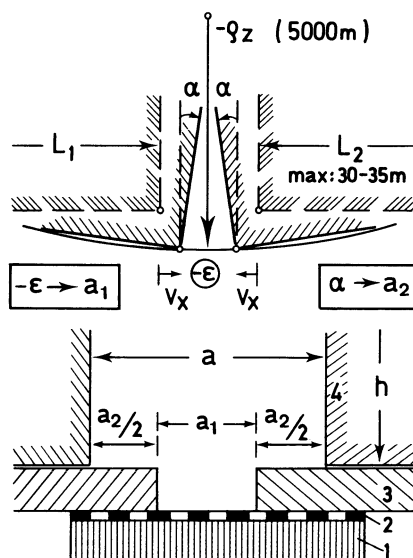


Fig. 373. Width of the gap a between two neighbouring buildings. (1) Strip foundation; (2) slip joint; (3) floor slab; (4) external wall; α = vertical tilt

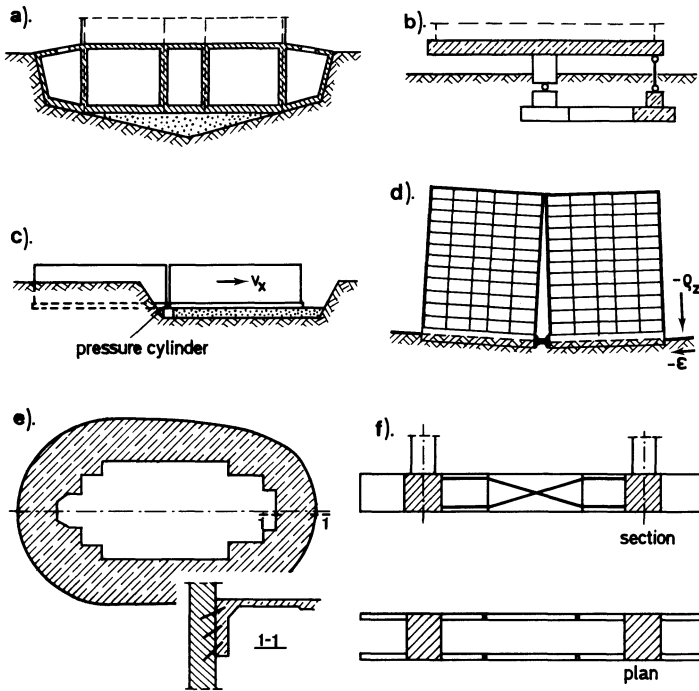


Fig. 374. Examples of structural precautions in the Polish coalfields. (After Ledwoń, 1981)

sion, have proved to be of value. In the Ruhr, pressure in the ground in front of the cellar wall is relieved by a closely spaced row of vertical boreholes, filled with peat and covered with a rubber belt.

In a Polish coal-mining district, a 12-storey block of flats was protected against tilt by laying out the floor of the foundation excavation in the shape of a wedge (as seen in section) and resting only the edge of the rigid box-foundation on firm ground (Fig. 374a). The object of this was that, when the ground later tilted, the foundation would press into it on the higher side, thereby producing a tilt of the foundation and building only half as great as the tilt in the ground. This one-sided settlement can be artificially encouraged, and the house thereby kept vertical during subsidence, by filling the outer space on the higher side with sand to weigh it down.

Machinery which is sensitive to going out of true can be mounted on a tippable plate, capable of being levelled on the principle of three-point support in the event of later ground subsidence (Fig. 374b).

If the gap between two buildings is too narrow in relation to predicted compression and concave flexure of the ground, then it is possible to shift one of the buildings along the ground with presses positioned horizontally, even without special tracks (Fig. 374c). The structure to be moved is excavated down to its slab foundation, and the other structure serves as a counter-seating for the hydraulic presses introduced into the gap. Another solution makes use of the natural separating of adjacent parts of a building in the zone of extension induced ahead of an advancing extraction front. At the moment of greatest extension, the widened

gap opening up between the two buildings in their foundations is jammed open with a distance piece to ensure that, when the face has passed and the ground undergoes concave flexure, the increased gap will be wide enough even in the upper storeys (Fig. 374d). To withstand the doubly intense forces exerting a pressure up to the gap jammed open, the foundation must be provided with adequate stability.

Historical buildings, which are sensitive even to average deformation of the ground, can be protected by means of a rigid reinforced-concrete plate constructed around them (Fig. 374e) and anchored to the sides of the building's foundations. Its large area gives the plate a high degree of stiffness against horizontal deformations of the ground.

Where there is a possibility of a funnel-shaped collapse of the ground, perhaps several metres in diameter, large industrial buildings of open construction can be protected against the sudden settlement of a supporting pillar by means of crossed struts (Fig. 374f). Such struts, pre-stressed in tension, link the foundations of a row of supports.

14.3.2.3 Foundation-slab

Whereas the concrete of a foundation-slab can withstand compressive forces of up to some 3000 N/cm^2 (30 MPa) without breaking, *tensile forces* have to be absorbed almost entirely by the steel reinforcing inlays. The specifications for reinforcing a foundation against ground tensile forces by means of steel inlays in concrete (rods or steel netting) take as their starting point the maximum possible tensile force $F_{t\max}$ arising, by Eq. (338)

$$F_{t\max} = 0.667 F'_n l/2 ,$$

in the section at the centre of the foundation when, along the whole length l of the structure, a relative displacement $v_{x\text{rel}}$ takes place between the ground and the underside of the foundation (see Appendix and Fig. 286). The coefficient of friction $\mu = 2/3 = 0.667$ applies to all types of foundation (slab or strip) incorporating no special slip layer of bitumen or polythene sheeting. Since F'_n signifies the average structural load for a 1 m wide strip of the foundation slab at right angles to the direction of ground displacement and principal tension (Fig. 387, left), the maximum tensile force in the middle of the *foundation-slab* must be equal to two-thirds of half the structural load F_n (in MN) $= F'_n l - \text{i.e., by Eq. (337)}$

$$F_{t\max} = 0.667 F_n/2 .$$

In view of the high price of reinforcing steel, and in the light of new knowledge, this maximum value for tensile force is now considered to be too high. The following considerations are relevant:

The maximum tensile force $F_{t\max}$ in the middle of the foundation depends on the amount of relative displacement and of ground extension that takes place, as shown in Fig. 295.

As set out at the end of Sect. 11.4, linear change in the ground beneath the foundation is less than in ground carrying no building development, because the

structural load compacts the subsoil and counteracts its loosening up through extension (Fig. 288, top).

The bond between the ground and the foundation-slab is maintained in the central zone even in the event of extension, with the result that only shear stresses, determined by the amount of cohesion $c = 10 - 30 \text{ kN/m}^2$, are transmitted to the foundation in that area.

Relative displacement and frictional forces thus develop under the foundation-slab only in the marginal zones, at the left-hand and right-hand ends; and since these slip zones are narrow, the force of tension in the foundation, which increases linearly with the amount of frictional travel, remains small (Fig. 288, bottom).

Calculations of a foundation/ground system by the finite-elements method and on the above assumptions have given the values at which maximum tensile force is reached in the middle of an infinitely long and 20 m wide foundation slab (loading 100 kN/m^2 , corresponding to ten storeys, $F'_n = 2 \text{ MN/m}$). These values, for calculated ground extension produced in unbuilt land by mining, are 2–3 mm/m in the case of stony foundation soil, 3–5 mm/m in sandy, cohesionless soils (angle of inner friction $\rho = 30^\circ$), and 6–8 mm/m in cohesive soils ($\rho = 25^\circ$, cohesion $c = 15 \text{ kN/m}^2$; Zerna and Schultz, 1979). The effective coefficient of friction $\mu_{\text{effective}} = F_{t\text{max}}/0.5 F_n$, arrived at by dividing the maximum tensile force corresponding to the limit values of ground extension mentioned above by half the structural load, is reduced, in the light of the latest knowledge, from 0.667 for stony subsoils to 0.35 for cohesive soils, and 0.25 for sandy soils. This means that in cohesive soils 52.5%, and in sandy soils 37.5%, of the quantities of steel required by the specifications will be sufficient to render the foundation-slab secure against tension in a mining district. The value $\mu = 0.667$ therefore applies only to the extreme case of a stony subsoil in which the horizontal linear deformation caused by mining is not reduced by the load from a structure, as in the case of a loose subsoil.

The functioning of a foundation-slab exposed to tension can be further demonstrated with a model (Fig. 371, right). If the rubber underlayer (1) is stretched, uniform cracks appear along the whole length of the unsecured structure (2). The end sections and the centre of the loosely assembled structure have become displaced conformably with the rubber strip at the positions a, b, and c. If, however, a sheet of paper (3) is laid between the structure and the rubber strip, to represent a foundation-plate secured against extension, the

Table 29. Guidelines for reinforcing foundations against extensional forces

Type of steel	Permissible stress σ N/cm ²	Permissible stress for mining damage precautions: N/cm ²	E modulus N/cm ²	Elongation at rupture ϵ : in %	Permissible strain for mining damage mm/m
I	14,000	22,000	2.1×10^7	18	1.0
II	18,000	34,000		14–20	1.6
III	20,000	40,000		8–18	1.9
IV		50,000		8–16	2.4

rubber strip slips under the paper on expanding without straining the structure. The thin tension plate sags with the ground and isolates the superstructure from frictional forces (see points b and c: relative displacement).

In accordance with German specifications, *reinforcing steel* used in a mining district may be stressed in tension up to the stretch limit of 1 to 2 mm/m – a value which should seldom be reached in the structure, even with ground strain of several mm/m, as the calculation examples in Sect. 11.4 show. The maximum frictional tension $F_{t\max}$ of 10,500 kN in the middle of the structure, calculated there with $\mu = 0.65$, is capable of being absorbed without fracture by the reinforcement if the steel cross-section S

$$S = F_{t\max} \div \sigma_{\text{permissible}} \quad (400)$$

amounts to $10,500 \div 22 = 500 \text{ cm}^2$ for Type I steel, or 210 cm^2 for Type IV (Table 29). This high expenditure of steel – distributed over 15 m width in the example mentioned, i. e. $14 \text{ cm}^2/\text{m}$ width, or 18 Type IV rods of 1 cm diameter per metre width at the centre of the structure – can be reduced by the use of a slip layer between foundation and superstructure.

There are many proposals for creating artificial slip planes, by means of which the friction index of 0.65 for concrete/sand stated in the specifications can be reduced by more than half, with the result that the maximum frictional force is also reduced (e. g., from 10,500 to 5,250 kN) and only half the amount of reinforcing against tension is necessary.

In the simplest case a bituminous cardboard ($\mu \sim 0.3$) is used to divide the structure horizontally. Expensive graphite *slip-layers* of 2 mm thickness or polythene sheets ($\mu \sim 0.15$) are suitable for small supporting areas and for high pressures of 5 to 10 MN/m². Asphalt slip-layers of 3 cm thickness have to be sealed with edge strips to prevent outflow at the sides. Laboratory trials with slip-layers of cardboard incorporating bitumen or tar showed that slip resistance (shear stress τ) is only slightly affected by the contact pressure F_n but all the more noticeably by the rate of displacement $v_t = \Delta v_x / \Delta t$ and the temperature. Thus, slip resistance doubles on a decrease in temperature from 12° to 9°C. No initial strength (see Fig. 294) could be established. For a rate of displacement of $v_t = 0$, the slip resistance is also $\tau = 0$. In mining districts a maximum relative displacement at the end of the structure of 0.02 mm/h can be assumed (see Sect. 11.6), for which a shear stress of under 1 N/cm², or 10 kN/m², can be ascertained from Fig. 375. If a value of 10 kN/m² is taken, then the figure obtained for a foundation area measuring 15 by 50 m, and quite independently of normal force, is a frictional force in the middle of the structure of 150 kN per running metre multiplied by 25 m = 3,750 kN (3.75 MN), as against 10,500 kN without a bituminous slip-layer. Since the slip resistance of the bitumen, as with a viscous material, is almost independent of the structural load, the reduction in frictional force achieved by the use of a slip-layer increases with increasing structural load, unlike the friction with a sandy soil. On the other hand, the amount of friction transmitted to a light structure (under $1.5 \text{ Mg/m}^2 = 15 \text{ kN/m}^2$) by the viscous, bituminous slip-layer is greater than that transmitted by the sandy soil. In addition, the frictional force for a proposed structure can be diminished by reducing the amount of bituminous slip-layer loaded, through opting either for an increas-

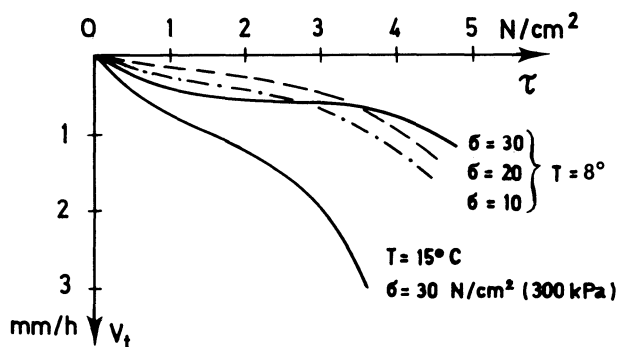


Fig. 375. Slip trials with bituminous cardboard (two layers of Semperfix). The slip resistance τ depends on the rate of slide v_t , the temperature T , and the contact pressure σ . (After Schmidbauer, 1961)

ed number of storeys and a smaller plan area for the same volume of construction, or for multi-surface support. In the vicinity of the focal point of all loads, a fixed point without slip-layer should be created, to ensure that a tilted structure does not slide of its own accord on the bituminous layer.

If a linear relationship is established between slip resistance and rate of displacement, then slip resistance, corresponding to relative displacement (Fig. 286) will fall linearly from 10 kN/m² to zero between the structure end and the focal point. With individual loads as in Fig. 376, this assumption leads to a simple calculation for the values of slip resistance between the separate foundations (area A) and the foundation slab

$$\tau_i = \tau_{\max} - \frac{l_i}{L} \tau_{\max} \quad (401)$$

as also for the frictional forces:

$$F_{ti} = \tau_i A \quad (402)$$

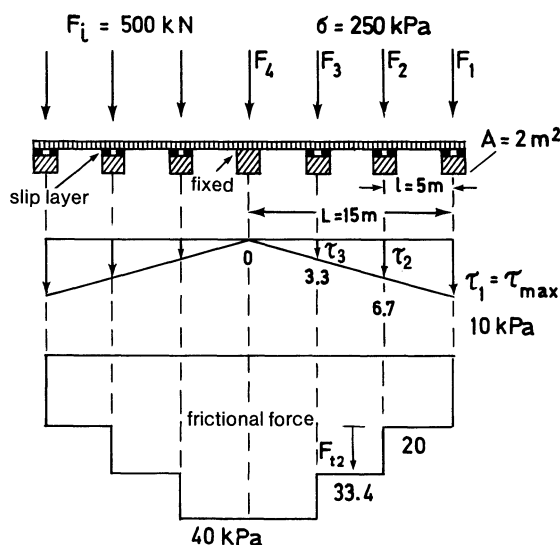
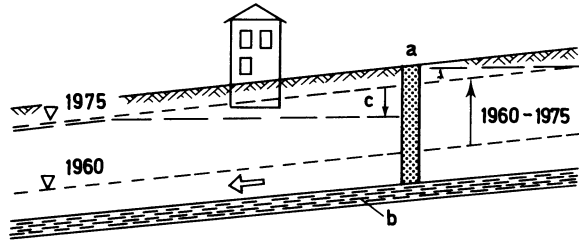


Fig. 376. The transmission of frictional force from concrete strips to foundation slab via a slip layer of bitumen

Fig. 377. Protecting a house on a hillside against rising ground water. (After Winkler, Heidel, and Lahr, 1974)



and

$$F_{t\max} = \sum F_{ti} \quad (403)$$

As these investigations show, frictional force with slip layers, in contrast to sand/concrete friction (Sect. 11.6), depends not only on the rate of ground displacement but also on ground temperature and the size of the area of contact. On this basis, a structure secured with viscous slip layers has to withstand a greater degree of friction in winter than in summer for the same amount of ground movement. One consequence of faster extraction is increased friction. When the ground is at rest again after movement, the elastic forces of deformation mobilized in the structure will gradually relax, against the resistance of the viscous slip layer, and the length change in the structure will return to zero. Later working will therefore again induce initial force and deformation in the elastic range. If there is simultaneous working in several seams, frictional forces having the same direction at the structure will become superimposed only up to a limit value determined essentially by the area of loaded slip layer and the degree of slip resistance (which will, it is true, be greater than with a single working because of the accelerated ground movement).

Finally, two further examples of precautionary measures may be mentioned. A house standing on a slope can be protected against a rise in the level of *ground water* in the forefield of an abandoned opencast working (Fig. 377) by sinking a wall "a" to form a dyke. This has the effect of causing the ground water to flow under it via the clay bed b, and thus of lowering the water level by the amount c below the cellar. Secondly, *jacks* built into a wall crack (Fig. 378). On further subsiding of the lower corner of the wall "a", the triangular portions "b" are forced outwards by spring pressure and in their turn force the bearing plate "c" upwards. When ground move-

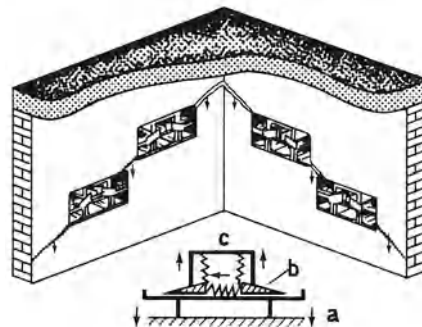


Fig. 378. Incorporating jacks in a subsiding wall corner. (After Mehrhoff, 1974)

ment has ended, a binder is injected into the open crack. Recently, spring mechanisms have even been built into the cellar walls of dwelling-houses as an after-measure, to facilitate the readjustment of stresses between subsoil and structure on flexure of the ground, and thereby avoid damage (Jung, 1978).

The most important *rules* to be observed in the construction of residential and office buildings in mining districts can be summed up as follows:

provide for small ground-plan areas, and divide up large building complexes (over 30 m long, or in steel-frame structures, over 50 m long) with wide and open expansion gaps;

avoid deep foundations and keying into the ground;

fill the foundation trench by the cellar wall lying at right angles to the foreseen direction of ground compression with loose material, such as peat;

include a flexible section in pipe mains where they pass through a wall;

preserve the distance between parts of the foundation with frames and plates secured against tension;

provide for foundations strong in tension and for low-friction slip layers along their base;

favour flexible construction methods, especially in walls, by using, for instance, framework bonding and lime mortar;

provide for later jacking of tilted buildings with pockets at intervals of 2.5 m and stiff beams or floor slabs above them;

stiffen floor slabs horizontally, and anchor or overlap reinforcing at sides;

preserve a clear space between neighbouring buildings.

Many countries prescribe guidelines for structural precautions in mining districts, which distinguish between several stages of making structures secure and enter into details of choice of materials, type of foundations, and countering deformations (Appendix I). The degrees of tilt, ground flexure, and linear deformation accepted in some countries in relation to specified linear changes in structures, and to be provided for in structural precautions, are set out in Table 30.

14.3.3 The Protection of Communications Installations and Public Utilities

The protection of communications installations and public-utility mains against mining damage is a matter of public interest. For *railway tracks* in mining districts, the butt-gap joint and short rails only 30 m long predominate. At a temperature of $+8^{\circ}\text{C}$, the butt gap permits of a linear movement in rails of up to ± 8 mm, equivalent to ± 0.26 mm/m (Table 21). In Germany it has been possible to reduce maintenance work on account of distortion in rails, and the costs associated with reduced-speed stretches of track, by the introduction of dilatation rails (Fig. 379). These tongue-shaped rail ends, initially introduced at an interval of 45 m and later of 180 m, have a maximum opening-and-closing width of 200 mm (on the avoidance principle). Trials with continuous welded rails (entail-

Table 30. Classification of structure and damage in certain mining districts (after the NCB 1963, Rimant 1968, and the VNIMI 1958)

Class	United Kingdom	Poland	Soviet Union	
			Donetz	Karaganda
I	Small cracks in plaster Linear change in structure: $\Delta s = 3$ cm; for 60 m length: 0.5 mm/m	Only hair cracks in plaster accepted Acceptable tilt: 2.5 mm/m Acceptable linear change: 1.5 mm/m $\rho_z > 20$ km	Accepted: Tilt 4 mm/m Radius of curvature 20 km Linear change 2 mm/m	Accepted: Tilt 6 mm/m Radius of curvature 3 km linear change 4 mm/m
II	Several small cracks in internal walls $\Delta s = 3 - 6$ cm $\epsilon = 0.5 - 1$ mm/m	Damage must be repairable $v'_z = 5$ mm/m $\epsilon = 3$ mm/m $\rho_z > 12$ km	Over 5 storeys $v'_z = 4.5$ mm/m $\rho_z = 18$ km $\epsilon = 2.5$ mm/m	$v'_z = 11$ mm/m $\rho_z = 1.5$ km $\epsilon = 7$ mm/m
III	Small cracks in external walls; doors jam $\Delta s = 6 - 12$ cm $\epsilon = 1 - 2$ mm/m	Damage should not affect functioning of structure $v'_z = 10$ mm/m $\epsilon = 6$ mm/m $\rho_z > 6$ km	3 & 4 storeys $v'_z = 5$ mm/m $\rho_z = 12$ km $\epsilon = 3.5$ mm/m	$v'_z = 16$ mm/m $\rho_z = 1$ km $\epsilon = 10$ mm/m
IV	Severe damage Open cracks $\Delta s = 12 - 20$ cm $\epsilon = 2 - 3$ mm/m	Structures must resist adequately $v'_z = 15$ mm/m $\epsilon = 9$ mm/m $\rho_z > 4$ km	2 storeys $v'_z = 8$ mm/m $\rho_z = 5.5$ km $\epsilon = 6$ mm/m	—
V	Very severe damage Partial reconstruction necessary Δs over 20 cm $\epsilon > 3$ mm/m	$v'_z > 15$ mm/m $\epsilon > 9$ mm/m $\rho_z < 4$ km	1 storey $v'_z = 10$ mm/m $\rho_z = 3$ km $\epsilon = 7.5$ mm/m	—
VI	—	—	1 storey $v'_z = 25$ mm/m $\rho_z = 1$ km $\epsilon = 14$ mm/m	—

ing constant monitoring of rail strain by means of attached strain gauges) have not yet been attempted in Germany because of the lateral distortion of track that could occur in summer under the combined influence of ground compression and thermal stress (see Sect. 12.4.1). As an experiment, however, mats of plastics material have been laid under track ballast to reduce friction from ground movement.

The question whether a route involving rails, or “permanent way”, is easier to protect against mining damage as an elevated track on pillar supports or as a tunnel underground is a matter of active discussion wherever a surface trace as an embankment or a cutting is out of the question in urban areas. A ban on mining within an elongated protective zone under the rail trace would not only interfere

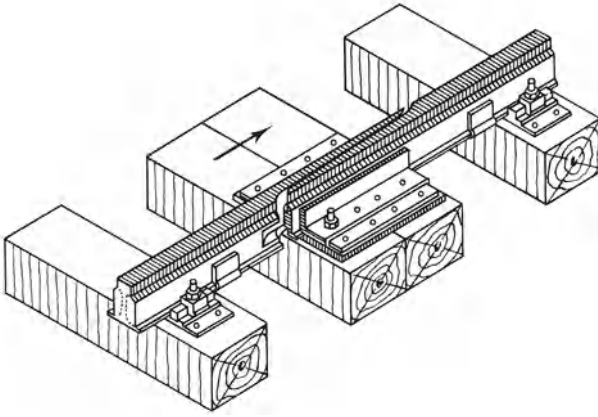


Fig. 379. Telescoping in the right-hand rail (after Rellensmann and Bauer). Dilatation movement ± 17 cm

with large-scale mine layouts but also lead to the formation of raised horsts between the adjoining subsidence troughs, with all the disadvantages that that would bring for the water system. The consequences of mining under a kilometres-long communications tunnel or elevated track must therefore be taken into account in design studies. For an *underground communications system*⁵, the following problems require to be solved:

- a) sealing the joints in sections of tunnel lying below the ground-water table;
- b) the difficulty in regulating track gradients in a tunnel, given the minimal room for jacking;
- c) deformation of the tunnel lining, especially in the longitudinal axis, as a result of linear deformation of the ground of up to ± 10 mm/m and flexure of maximally $\pm 1/10$ km;
- d) securing the stability of the tunnel against shearing within or over geological dislocation zones in the event of irregular strata movements (subsidence steps and cracks measuring tens of cm, and shear paths up to 2 cm in width in a zone of sand-to-rock transition);
- e) the effects of changes in tunnel profile on nearby surface structures; and
- f) the varying strength of surface strata passed through (the tunnel-and-bedrock system).

One proposal from the Ruhr (Duddek et al., 1973) envisages a continuous-welded, sheet-piling-like construction of corrugated steel sheeting, which would follow mining-induced deformations like a flexible tube but would be “low in deformation” with respect to surface structures (avoidance and resistance principles simultaneously). Arguments in favour of steel sheeting with cathodic rust protection are its high strength, its plasticity, and its weldability, which together offer good sealing properties against ground water and, in jointed rock, also against methane. Another consideration is that it permits easy repair by welding on extra sheets. The study in question does not look with favour on the notion of a slip layer between tunnel wall and surrounding soil, because with the large

⁵ Protection of a long bridge construction against mining damage, which is entirely possible, is dealt with in connection with another example (Fig. 381).

amount of surface area and great length of a tunnel system, a material with a very low coefficient of friction would have to be found, which at the same time would not flow away into joints in the strata. On the other hand, a form fit of the corrugated steel sheeting can be avoided by filling the hollows along the exterior of its profile with a deformable bituminous mixture. Subsidence steps expected in fault zones are taken care of with a larger excavation cross-section and external padding. In the transverse direction, however, some initial deformation and narrowing of the tunnel profile cannot be excluded (point e). Halting stations built as separate structures should be connected to the track tunnel on the stuffing-box principle of pipeline construction.

Where several seams are worked under a *street*, subsidence of several metres must be expected, as well as local tilt and ground extension and compression of up to 10 mm/m – in extreme cases up to 15 mm/m. The subsidence and alteration in gradient expected should be allowed for in the planning of new streets by providing for embanking and a “reserve” of gradient. If there is a risk that the street will subside to the average ground-water level, then only frost-resistant construction materials should be used and drain-pipes laid in the road foundation (Fig. 380, left). If subsidence and slope in existing streets become excessive, those streets should be banked up again. Water collecting at a trough centre should be conducted to a lower watercourse or pumped up to a higher one. Warning notices should draw attention to surfaces made slippery by water and to alterations in gradient. Changes in the lie of a street should be monitored by periodical measurement. Underground drainage pipes in an area of ground extension and compression should be laid with expandable and flexible adaptors at the places where they are held in the road foundation or the underlying sub-grade. Extension and compression in the road deck must be absorbed by transverse joints – compression by clearance joints and extension by butt joints (see Sect. 12.4.2). In an area of mining subsidence therefore, expansion joints should not be omitted, even in modern roads. Joint sealings should be soft during the period of mining influence. Bituminous road surfacing is the type most usually adopted in mining districts, being very flexible to linear change and easier to repair than a concrete deck. The roadbase load-bearing course should have a

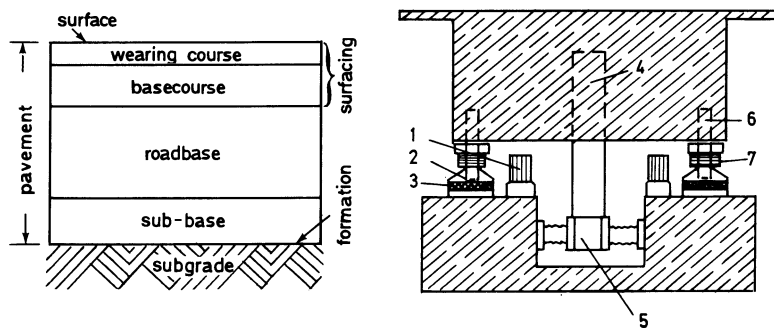


Fig. 380. *Left:* structural layers in a flexible road. *Right:* Section through a bridge bearing. (1) Jacking cylinder; (2) perforated block; (3) roller bearing; (4) spigot; (5) hydraulic cylinder; (6) pin in guide cylinder; (7) washers

similar deformation behaviour to that of the surfacing and should therefore also be bituminous-bound (Kutscher, 1980). In cement-bound basecourses, cm-wide cracks develop in a zone of extension, leading to break-up of the surfacing.

Although there is little or no scope for the incorporation of precautions in the construction of a *dam* to counter the effects of subsidence, the NCB suggest that, in order that a support pillar may be limited in size to protect the dam itself, other precautions should be taken. Take-off pipes and mains should be made flexible by the use of telescopic joints and the freeboard of reservoirs and surrounding drainage channels should leave room for some subsidence.

A *bridge* in a mining area should either be rigidly constructed to resist ground movement or so articulated that it can accommodate movement without damage. If the direction of workings can be so orientated as to move in line with the bridge, so that only two-dimensional movements have to be considered, precautions could be quite simple as in the following examples (NCB, 1975):

Short Span Bridges (up to 30 m) should be of a simply supported type. To allow for longitudinal and vertical movement the deck is fixed at one end and free at the other where an expansion joint is used. This allows the length of the bridge to vary and limits the failure of the road surface to one place. Jacking pockets may be provided to allow subsequent rellevelling.

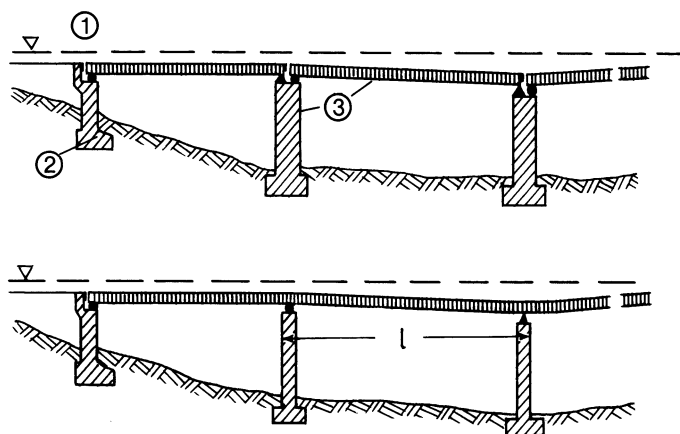
Multispan Bridges. If there is a series of short spans, as above, piers have to be hinged top and bottom to allow for tilting and change of length. Tilting or rocker bearings are required at each pier. Jacking pockets may be provided.

Long Span Bridges are, for economic reasons, generally not of the simply supported type. Examples of suitable types are cantilever and suspended-span construction, the Wichert Truss and the three pinned arch.

Usually, however, the direction of approach of workings to a bridge cannot always be anticipated and three-dimensional movement must generally be allowed for. This can be achieved by supporting the bridge on three points only, the movements being as in the two-dimensional movement mentioned above with the addition of the transverse tilting and strains which will produce torsion in the deck.

As an example of three-dimensional bridge precautions in a mining area, mention may be made of the Berlin Bridge in Duisburg on the Ruhr, which in fact consists of several steel and reinforced-concrete bridges (Schweden, 1963). In the piers of this 1824 m long series of bridges, the measures needed to compensate for mining-induced ground movements amounted to providing scope for displacement of ± 60 cm along the length of the bridge and ± 40 cm across it, angular movement longitudinally and transversely of $\tan 0.013$ and 0.005 respectively, and height adjustment of up to 80 cm. This requirement was fulfilled with a series of single-point tipping supports, movable in all directions and designed for a load of up to 1500 Mg (Fig. 380, right). Resting on two roller bearings disposed crosswise, each consisting of 11 rollers of 100 mm diameter, there is a perforated block, which receives a rubber pad of 100 cm diameter and 9 cm thickness and supports a tilting pin mounted on a spherical segment. Regulating

Fig. 381. Behaviour of a bridge in the subsidence zone: (*top*) as a chain of separately supported spans, and (*bottom*) as a single, continuous construction. (1) Reference height; (2) abutment pier; (3) extra-large dimensions



the height can be done either by inserting rings into the bearing as the bridge is raised with special jacks, or by turning nuts of 42 cm diameter on a spindle to achieve the adjustment. The bridge can shift laterally on spigots, and if the piers become displaced, these spigots can be brought back into their correct position laterally with hydraulic presses.

Ground subsidence, if it is accompanied by differential settlement of bridge piers, causes considerable twisting and stress redistribution in the deck structure because of the altered support conditions. The increased cross-sections required in the superstructure to counter this situation are economically justifiable in bridges of continuous construction, provided the differences in settlement remain less than $1/1000$ of the bridge span l – i.e. 7 cm on a span of 70 m, and 15 cm on one of 150 m (Domke, 1963). Where differences in settlement amount to $2/1000 l$, continuous constructions would have to be so greatly strengthened that they would be too stiff. Statically determinate constructions with three-point or four-point support must then be used, and these have the effect of dividing up an unified bridge structure into a chain of separate bridges (Fig. 381). In this way it is possible to isolate the deck from the forces set up by ground movement, but the bridge of separate spans has to be of thicker dimensions than a continuous bridge, and this means a heavier dead weight and therefore thicker piers. A further disadvantage is the doubled number of bearings on the tops of piers, the connecting constructions at the ends of sections, and the abrupt changes of gradient in the carriageway. If therefore major differences in settlement (over $1/1000 l$) can be prevented by arrangements enabling subsequent straightening of the bridge, the statically indeterminate continuous construction is more economic than a chain of separate decks.

Continuous constructions can be executed equally satisfactorily in steel or in pre-stressed concrete. In the case of pre-stressed concrete bridges in the form of box beams, which are somewhat stiffer than steel bridges, the moment of inertia, and with it the bend resistance, drops abruptly if the concrete's tensile strength is exceeded as a result of mining-induced bending stress. Acute additional tensile stresses can then develop in the peripheral zones of the cross-sections in cases where there are no bracing members. The additional tensile forces arising in the

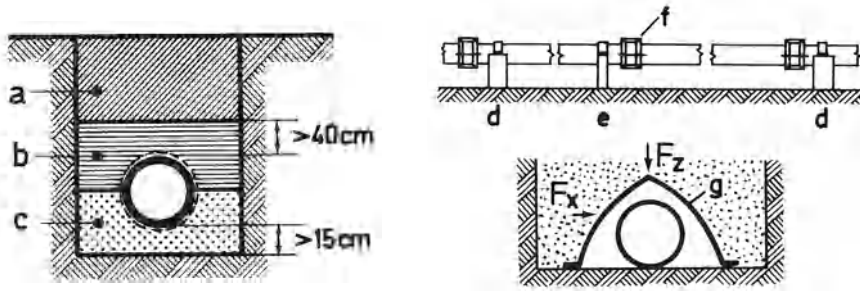


Fig. 382. *Left:* Pipeline in a trench. **a** Soil covering; **b** soil free of stones and lumps; **c** granular material. *Top right:* pipeline above ground. **d** Pipe anchored to ground; **e** pipe loose in guide support; **f** coupling with limited scope for movement in and out. *Bottom right:* pipeline on the ground with a covering hood **g**

fractured zone of tension in the concrete can be absorbed by non-pre-stressed steel reinforcing experiencing a stress of up to 20 kN/cm^2 (200 MN/m^2) or a strain of 1 mm/m . This guarantees a good distribution of the bending cracks in a large number of fine hair cracks, which close up again once the bridge has been straightened. In a steel construction, stresses and strains in the cross-sections increase uniformly with increasing differences in subsidence (no sudden drop in tensile strength as in concrete). The whole stress reserve for the absorption of additional bending moments is roughly equal for both constructional materials, up to the limit of proportionality.

Long-distance gas mains and oil *pipelines* are subject to legal provisions. The operator of a pipeline not provided with telescopic joints is obliged to ascertain the magnitude and direction of all forces acting upon it. Piping laid underground in the Ruhr is secured in the following ways according to choice (Lindner, 1972). The pipes are bedded on at least 15 cm of granular material and packed round with it up to half their height, then covered to at least 40 cm with soil free from stones (Fig. 382). In piping of 200 to 1500 mm diameter, sleeve joints – provided with extensometer wires and allowing for up to $\pm 80 \text{ cm}$ movement – are incorporated (Fig. 383). To avoid having the heavy sleeve-joints acting as fixed points owing to their enlarged outline, wrapping in polystyrene is recommended; and to avoid contamination of ground-water, care must be taken to ensure that the joints are tightly sealed. Continuous-welded crude-oil pipelines (e. g., the 700 mm dia. line from Wilhelmshaven to Cologne) are laid in the open in places where linear changes in the ground of over 1 mm/m have been observed, to enable any lengthening or shortening of the pipe to be measured to within $\pm 0.01 \text{ mm}$ by means of extensometers distributed at four positions around its circumference and at intervals of 24 m (Fig. 313). Piping exposed to over 0.85 mm/m compression, which corresponds to half the limit extension value of $360 \div 2 = 180 \text{ N/mm}^2$, are also laid in the open, to enable extension and compression of the pipe to be evened out over a distance of 300 to 600 m . In areas of intense ground compression it is often necessary to cut out up to 3-m length of piping, where-upon the cut-open ends converge by 10 to 30 cm . Once the stress is relaxed, a new piece of pipe is fitted and welded in. Horizontal elbow-bends, siphons, and vertically stacked loops are particularly prone to damage (see Fig. 310a). Tensile

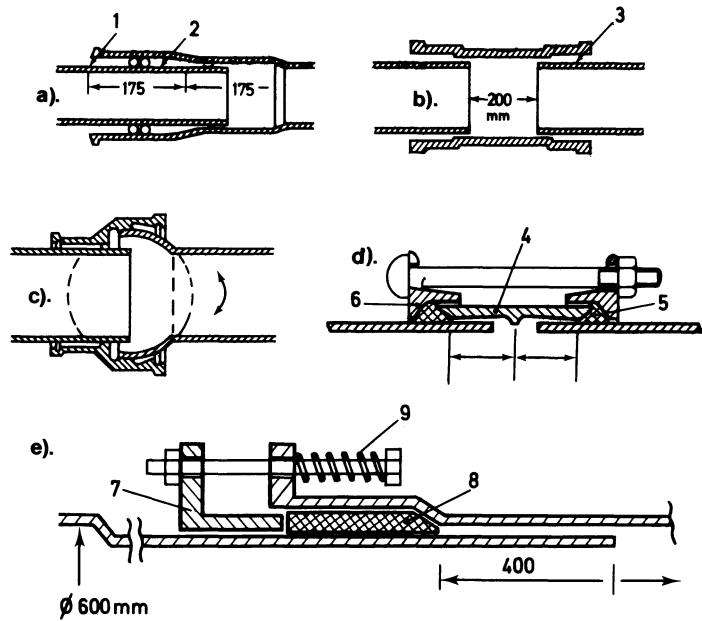


Fig. 383. Telescopic pipe connections as a precaution against ground in mining areas shifting. Small-diameter pipes: **a** Socket with cast lead (1) and tarred cord (2) as sealing; **b** sleeve connection with threaded ring (3) as sealing; **c** swivelling ball-joint socket. Large-diameter pipes: **d** typical sleeve joint for iron pipes, with gasket (4), sealing element (5), and reciprocally screwed pressure rings (6); **e** Ruhrgas-AG sleeve joint with pressure piece (7), sealing rings (8), and tensioning spring (9)

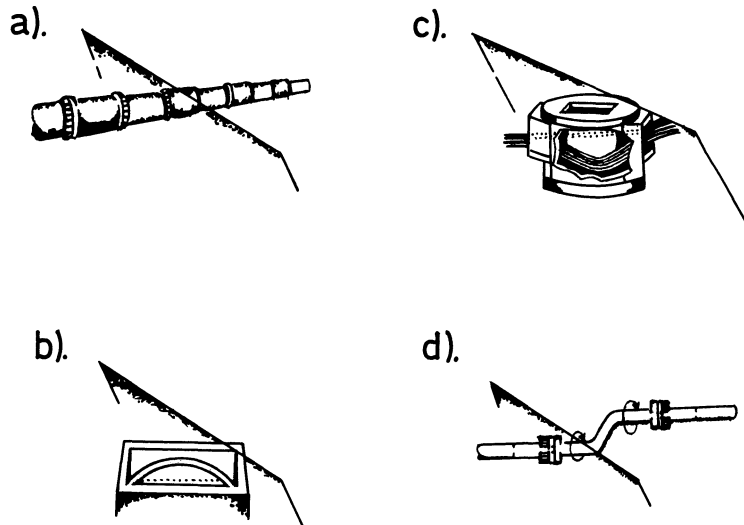


Fig. 384. Securing pipelines and cables at subsidence steps in ground (after Mehrhoff, 1974). **a** Close sequence of sleeve joints; **b** cable laid in an arc; **c** cable inspection point; **d** swivelling pipe joints on either side of step

strains have up to now made no relieving measures necessary. With flexible pipes under 200 mm in diameter, U-shaped loops are used which, for example, with a bow height of 4 m can absorb a linear change of approximately ± 9 cm. Protection against intense ground flexure and subsidence steps can be provided by ball-joint sockets (Fig. 383) or by short lengths of pipe held between two clamp-joint connections and capable of swivelling up to 10° . Other precautionary measures for pipelines and cable mains at geological faults or subsidence steps are shown in Fig. 384.

If deformation of the pipeline of up to 0.61 mm/m is accepted (see Sect. 12.4.3), a sleeve joint permitting movement usually of ± 40 cm would have to be incorporated at Station 68 in the pipeline represented in Fig. 311. If the linear changes to be expected in the ground are unknown, the sleeve joints should be at an interval, in silty soil, of at most

$$\begin{aligned} L_{\max} &= 2 \frac{\sigma_{\text{permissible}}}{\sigma} = 2 \frac{\epsilon_{\text{permissible}}}{\tan \alpha} \\ &= 2 \frac{0.61}{0.65} 100 = 188 \text{ m} \end{aligned} \quad (404)$$

(with $\tan \alpha$ related to 100 m), and in fine sandy soil of at most

$$L_{\max} = 2 \frac{0.61}{0.2} 100 = 610 \text{ m}$$

where axial stress in the pipe amounts to

$$\sigma = \frac{\tau A}{S} \quad (405)$$

(with A = external area of the pipe, and S = pipe cross-section). Here it is assumed that the joint's scope for movement is not completely exhausted, because a stiff joint would increase the effective length of pipeline to which frictional forces are transmitted, as in the case of a continuous-welded pipe string. The sleeve joint's required scope for movement can be derived from the curve of ground displacement (see Fig. 310c). Factor 2 takes into account the circumstance that deformation of a section of pipeline rises in a triangular outline from both sides and up to the centre (see Fig. 307). Accordingly, taking the example in Fig. 311, not knowing ground deformation would mean that, in addition to the sleeve joint at Station 68, two further sleeve joints would have to be incorporated – one at Station 8 ($\tan \alpha = 0.3$ mm/m for each 100 m) and the other at Station 56 ($\tan \alpha = 0.4$ mm/m for each 100 m). This makes clear the economic importance of pre-calculating ground movement.

In accordance with current practice, the intervals between sleeve joints are fixed on the basis of estimated ground deformation. Given expected ground extension or compression of 4 mm/m (2 mm/m), the intervals between sleeve joints capable of being shifted by $d = \pm 400$ mm are put at 100 m (200 m) by the Eq.

$$L_{\max} = \frac{d}{\epsilon}. \quad (406)$$

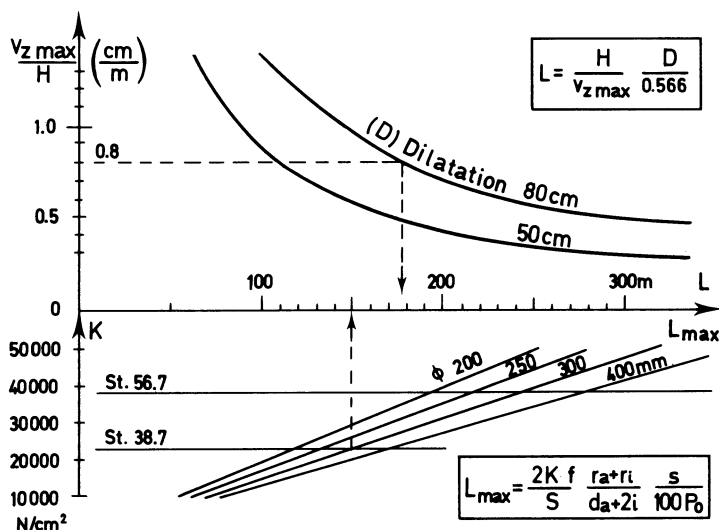


Fig. 385. Determining the distance between sleeve joints in pipelines. (After Ruhrgas-AG)

The knowledge that linear change in a section of pipe between two sleeve joints rises in a triangular outline to the mid-point (factor 2) can be used to save on the number of sleeve joints. In the Ruhrgas-AG group, the distance between sleeve joints is determined from the diagram in Fig. 385, either from the given values of maximum subsidence $v_{z\max}$, the depth of working H , and the dilatation D of the joint (top of Figure), or from the empirically ascertained stress [after Eq. (404)] on the pipe (bottom of Figure), given knowledge of the following values: the stretch limit K of the pipe in N/cm^2 (e. g., yield point S for $\epsilon = 0.2\%$ in Fig. 301), the weld factor f (1.0), the coefficient of safety S (1.7), the internal and external radii r_i and r_a of the pipe, the external diameter d_a of the insulated pipe, the thickness s of the pipe wall, the thickness i of the external insulation (all in cm), and the slip resistance P_0 of the pipeline (e. g., $1.15 \text{ N}/\text{cm}^2$ with bitumen insulation, or $0.6 \text{ N}/\text{cm}^2$ with plastics insulation).

In cases of slight ground movement, narrow trough width, bedding in ground causing little friction, and good quality steel, it is possible to do without sleeve joints altogether, as can be inferred from the similar example of a welded rail ($\tan \alpha = 0.3 \text{ mm}/\text{m}$ over 100 m) in Fig. 309. Maximum deformation in that case remains below the $0.9 \text{ mm}/\text{m}$ yield point of a steel of type St. 37 (safety factor 1.87). Sleeve joints in fact have certain disadvantages, quite apart from developing leaks and requiring maintenance. Because of their enlarged circumference, they prevent the pipeline slipping easily in the ground and thereby achieving stress relief. They act as a plate anchored in the ground and, having a surface area of 0.5 m^2 , offer a displacement resistance of at least 70 kN in the comparatively loose soil of the pipe trench. The stuffing-box type of sleeve joint does not react immediately to axial force by being pulled out or pushed in ($\pm 400 \text{ mm}$) but transmits the force, at the beginning of its action in full but also later to a large extent, to the adjoining section of the pipeline. This resistance of the sleeve joint to being telescoped out or in is difficult to estimate, because the packing

materials behave plastically (see Fig. 375) and their contact pressure on the inner pipe varies from sleeve to sleeve. In a sleeve joint for a gas pipeline of nominal diameter 600 mm as in Fig. 383e, measurements on a test bench showed an axial force absorption of 30–820 kN (average around 300 kN) on being closed up (compression) and of 60–355 kN (average around 200 kN) on being drawn out (extension) over a displacement distance of 800 mm (rate of displacement 30 mm/min), with the maximum forces appearing at the beginning of the telescoping action (Hollmann, 1982). As compared with mining-induced ground displacement and the rise in axial stress in the pipeline, closing-up or drawing-out of the sleeve joint occurs with a delay of 5–10 weeks; and the cancelling-out of stresses and strains in longer stretches of jointed pipeline may take up to half a year. This is something that needs to be looked out for in pipeline maintenance (Kelvin model; Fig. 61).

A large influence on the amount of expansion and contraction in the pipeline is exercised by the deflection points (elbow bends) at which it changes horizontal direction. It is at these places, which act as fixed points in the ground ($\Delta v = 0$), that frequently the maximum extension or compression in the pipe occurs, with the result that damage from cracking is first to be expected there. If, however, the deflection point lies over the inner edge zone of the working, the extension and compression effects in the pipe may cancel out in part and extreme values remain small. In tracing the route of new pipelines therefore, or in planning the layout of new workings, such a favourable position for deflection points in the trace should be borne in mind.

The extension, compression, and stress effects to be expected in a welded pipeline, with or without sleeve joints, can now be obtained with an electronic data-processing programme in accordance with Eq. (379), if ground displacement along the pipeline is precalculated (Leucht, 1982). By this means the choice of new traces through mining areas is greatly facilitated, and later maintenance of the pipeline can be concentrated on places determined in advance as being at risk. In extension of the graphical method (Sect. 12.4), it is possible with this programme to allow for a local variation in the slope of pipe strain ($\tan \alpha$) in the compression zone [F_n larger in Eq. (337)] and for traces containing frequent sharp bends. In the latter case the curves of ground displacement given for the individual sections of pipeline are no longer symmetrical as in Fig. 309 but to a greater or lesser degree unsymmetrical, depending on their route through the subsidence zone.

Laying pipes in an open trench or under a covering hood (Fig. 382) has been carried out in only isolated cases so far because of limitations on the use of land surface, although in both methods the area for frictional forces to act on the underside of pipelines is reduced and maintenance is facilitated. With pipelines laid above ground, ground displacement acts only through the points of anchorage d, and axial stresses may cancel out through the sleeve joints f and through temporary loosening of the pipes in the anchorage points.

14.4 Regional Planning

Associating the mining industry with the siting of major structures, communications installations, and housing estates should contribute to avoiding excessive mining-damage costs, repairs, and limitations on mining. Since mining is tied to deposits, future ground movements must be taken into consideration in the drawing up of land-use plans specifying areas for residential purposes, industry, and open spaces. For instance, no structures should be erected where tectonic faults outcrop at the surface: apart from recent geological movement at major fault planes, mining influences accumulate at these movement joints which pass through the caprock to the surface. Zones of important geological dislocation and cracks in the ground in the extension zone of subsidence (Fig. 143) are therefore the obvious place for siting open spaces.

The same applies to seams outcropping at the surface: being compliant strata they represent, like the slip planes of geological faults, areas of risk for building development. Development without structural precautions is also mostly out of the question over near-surface workings. The shallower the depth at which mining takes place, the more severe are its effects at the surface. The subsidence basin may not be very extensive, but the troughs are very acute and their flanks steep (Fig. 149). The strains which appear in the trough – compressive in the middle and tensile at the edge – are therefore considerable. At the joint planes the separate blocks of strata move against and away from each other, and any houses found in such an area develop cracks in the superstructure and shear fractures in the foundations tens of cm wide.

In general, areas of steep stratification are favourable for building development, because the steeper the dip the greater the area of influence (Fig. 157): thus, a working of a given size in steep-lying measures produces less deformation at the surface than if it were in flat-lying measures.

When new industries are settled in areas of earlier mining, the local authorities frequently take over the costs of securing foundations, or they may pay a premium on account of mining-damage hazards.

Communications structures of more than local significance should if possible be sited in areas having few seams or which are already worked out. In deciding the line of a canal, the possible later need for uniform lowering of a whole stretch of it must be borne in mind. Erecting locks over unworkable fault zones, with the object of sparing them from mining influences, rules out the possibility of later desirable lowering. In any case, massive lock structures are capable of withstanding the effects of mining without much damage.

The considerable costs for altering the level of a railway installation, a stretch of canal, or a motorway intersection can be reduced if the trace concerned takes account of mining. Tall buildings should not be sited near a mine boundary, because a mining layout requiring to be executed by several mines acting together often miscarries for operational reasons – e.g., inadequate development work in the boundary seams. The collaboration of the mining industry in zonal planning is often essential for the reason that a large part of the land surface in mining areas is owned by the mines.

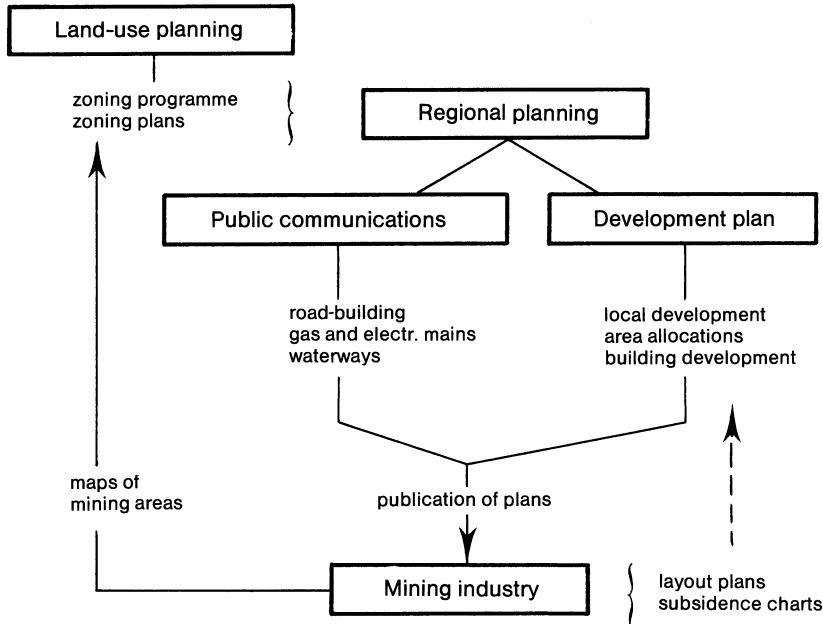


Fig. 386. Land-use planning and regional planning

There is no lack in most countries of *legal provisions* for regulating the inter-relationship between mining and land ownership (Fig. 386). The laws on land-use planning prescribe programmes and schemes for the zoning of the whole mining district. Areas overlying deposits may be used only in ways that can be justified from an economic, social, and cultural standpoint. There are special regulations covering protection zones, land procurement, highways, water supply, and building development. At the regional level, strategic plans covering development and zonal allocations in local districts, as well as public communications installations, are drawn up and discussed with the mining industry, which makes layout plans and subsidence charts available for this purpose. With their official announcement and publication, which are preceded by a public hearing of all interested parties, these plans acquire the force of law.

By the new mining law (Section 107) of the Federal Republic of Germany, the government can by regulation restrict building development in areas likely to be claimed for the exploration and exploitation of minerals, especially as extensive openast projects, within the ensuing 15 years. The loss in value of the land affected must be made good by the mining undertaker so favoured. For the purposes of planning and maintenance, the mining industry gives details of present and future subsidence zones, trough centres, maximum subsidence, tilt directions, and extension and compression zones to the building authorities, railway administration, water authorities, highway authorities, and pipeline operators. To building undertakers it suggests suitable measures for accommodating or securing structures planned in a mining area. For an area particularly at risk, it may issue an embargo on all building until working has passed underneath or been abandoned, or in the case of near-surface caving, without time-limit.

In the Ruhr there are associations of interests which regulate gradients in waste-water disposal and operate pumping installations in severely subsided areas. Surface subsidence can also bring benefits, however, of which the collaboration between the harbour authorities in Duisburg and the mining industry provides a good example. On the recommendation of the waterway authorities there, certain uninteresting seams under the harbour, which lies at the confluence of the Ruhr and the Rhine, have been extracted so as to lower the floor of the harbour basin, to the point where, despite the natural erosion of the river bed in the Rhine and the consequent drop in the water level of around 4 cm a year, the harbour can continue to fulfil its function (Schweden, 1963).

Under the heading of mining damage in the widest sense, and likewise regulated in the framework of regional planning, are waste dumps and abandoned opencast workings (which will only be touched upon here). *Waste dumps*, which are mostly established near mine shafts, disfigure the landscape with their pyramidal or conical outlines. Their content of acid and sulphur inhibits plant growth on steep slopes and affects the quality of ground water. Coal fragments left in them lead to smouldering combustion, and the wind carries their lighter particles into neighbouring localities. In many countries therefore, new waste dumps are now allowed only on condition that they fit into the landscape as terraced hills and that their slopes are planted with vegetation, so that later they may serve as recreational areas.

Abandoned opencast workings must be brought to some new use at the expense of the mining company. The deep surface excavation of a residual working is usually turned into a lake, the embanked sides and surrounding sand tips are levelled, planted up, and in the course of time afforested (with birch, poplar, alder). The surfaces of filled-in opencast workings, being free of waste tips, are recultivated as farmland (restoration, revegetation) and resettled with homesteads. The regulations promulgated to this end in all countries on surface mining control and land restoration have the object of minimizing damage to the environment during the post-mining stage. They include such matters as maintaining the stability of high walls and spoil heaps, the hydrological balance, the quality of air and water, rock blasting, the preservation of wildlife, and safeguarding cultural and historical resources.

Finally, one example concerning the choice of a road trace in a mining area, and another concerning the siting of a motorway intersection, may be mentioned. Under the selected motorway trace, and distributed over several seams, there are thicknesses of coal varying from a few metres up to 35 m (Fig. 387, left). Shifting the motorway to the north would have meant that significantly less coal, in a more uniform stratification, and a greater thickness of surface rock would have been encountered than in the section between Castrop-Rauxel (C. R. in the sketch map) and Kamen. In the planning of the branch-off towards Leverkusen, on the other hand, geological and tectonic circumstances in the underlying strata have been adequately taken into account. This motorway intersection was constructed near Kamen between two large faults, the planes of which reach the surface away from the structure (Fig. 387, right). The area of the site shown hatched in the sketch-plan is severely dislocated and is not capable of exploitation. Worthwhile quantities of coal are found again only outside the area of

influence. The factors considered here apply equally to canal installations, railways, pipelines, power stations, industrial installations, and large housing estates.

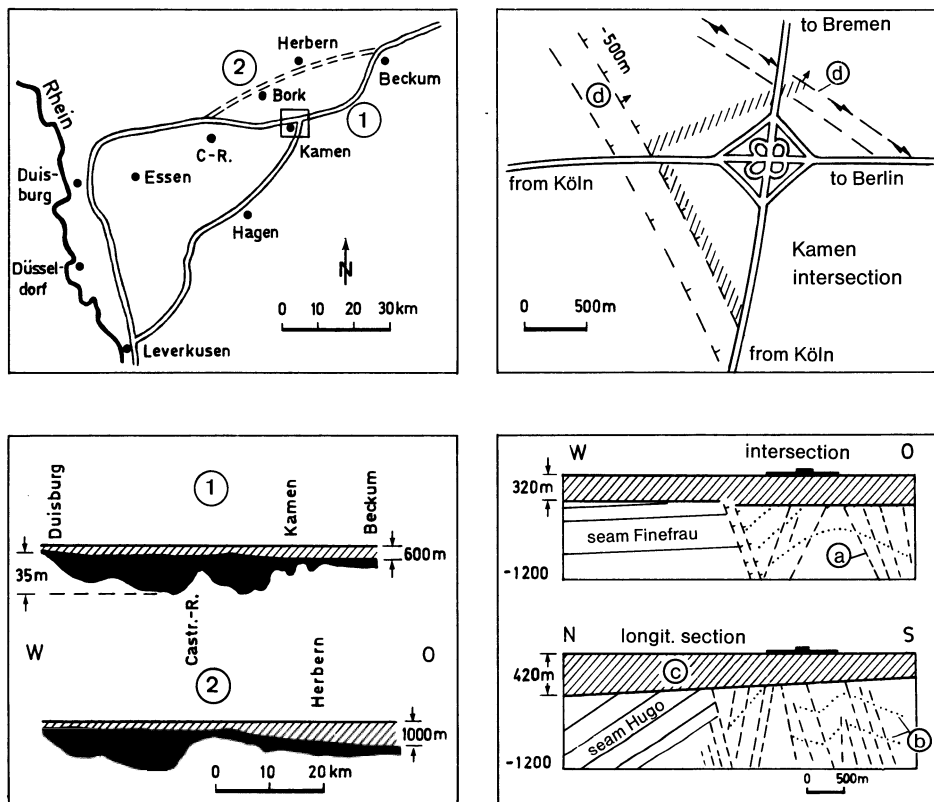


Fig. 387. The choice of trace for motorways in a mining area (after Treptow, 1963). *Left:* (1) the present trace, and (2) a more favourable line from the standpoint of mining damage. *Right:* the siting of a motorway intersection between geological faults (a) and over steep, unworkable seams b; c caprock; d line where fault planes reach the surface

Outlook

The 14 chapters of this book bear witness to the great advances that have been made in mining subsidence engineering since the turn of the century and even since the Second World War. Leaving aside extremes of working and strata conditions such as are to be found in steep-lying strata, in the area of geological faults, and where there is a high concentration of working in one place, it is possible today, with the known calculation methods, to predict ground movements with an operationally adequate precision of $\pm 10\%$ in flat-lying, and $\pm 20\%$ in steep-lying strata. These fruits of research are of value not only to mining but also to those areas of activity which stand in close technical association with it. Mention may be made here of construction engineering, communications, town and country planning, land ownership, and the administration of justice. In the common endeavour to uphold the safety of communications and the possibilities for working and living in localities exposed to the risk of mining damage, neither the mining industry nor associated agencies can afford not to pursue a constant improvement of techniques developed in the past. The work of research will never reach a definitive conclusion, because new factors are constantly being introduced into the calculations and considerations by developments in mining and construction techniques, such as concentration today on the best working areas economically, and the use of pre-stressed concrete or pre-fabricated construction elements in civil engineering – to mention only one or two innovations – but also by changes in legal and economic views.

As mentioned earlier, there are three routes principally which lead to the desired reduction in mining damage costs:

- a) correct pre-calculation of the ground movements to be expected;
- b) securing structures against mining damage and installing instruments to monitor movement;
- c) mine-layout planning, including regional planning, designed to reduce damaging ground movement.

In all three areas – ground movement, structural precautions, mine layouts – future research depends on what can be learnt in many directions. This applies, for example, to calculation processes based on mechanical models designed to take account of the stratification and the mechanical properties of the rock mass. What is required to be found is the rock-mechanical characteristics of the zones of strata, which alter with the degree of reworking and as a result of bed separation in strata series. According to the present state of knowledge, it looks as though the problems remaining in the pre-calculation of ground movement – such as the influence that old workings, dip, strata structure, mining method, and chronological sequence of working have on the stress/strain field in

the strata – can be solved only with a mechanical and rheological strata-model capable of incorporating time-related strata stresses into the calculation process. Analytical methods in which a rotationally symmetrical distribution of elementary mining influences is assumed for a homogeneous rock mass and is adhered to even after repeated reworking, or in which a formula for the trough profile is constructed solely as a function of the geometrical dimensions of the extraction panel, will have to be improved, as in the Ruhr, by supplementary adjustment functions or by asymmetrical influence functions.

Mine surveyors must assemble the new data by means of systematic measurements below and above ground. The lightening of the task of predicting strata and ground movements presented today by computers should be put to use by making frequent comparisons between measurement and prediction. With the aid of aerial photography, whole trough areas should periodically be surveyed. As is known, the difference between measured and calculated point movements enables valuable conclusions to be drawn on whether and how far the individual factors and strata characteristics have been correctly assessed and introduced. At the same time, for purposes of a rapid estimation of the movement and deformation values to be expected, the purely empirical methods employing, as in the U.K., conveniently laid out nomograms and simple maximum-value formulae should be further developed, as they are frequently quite sufficient for the planning stage. Evaluated observations provide the basis for regionally applicable diagrams and estimation formulae of this kind.

In protecting structures against mining damage the most common practice is to effect only partial precautions. Besides keeping ground plans small, the most effective means for preventing damage is the incorporation of separation gaps and slip joints. New investigations, which would have to be carried out with displacement pick-ups and pressure gauges applied to structural foundations themselves and also in the ground, could contribute to acquiring better knowledge of the strength both of the forces acting on the structure and of the deformation resistance evoked in it. This information should enable structural elements incorporated for protection, such as reinforced tension plates and joints, to be kept as small in size, and therefore as low in cost, as possible. Faulty conclusions on whether structures are damaged more by horizontal ground deformation (extension, compression) or by vertical deformation (flexure) lead to the adoption of the wrong constructions and protective measures, which can cause heavy additional costs without benefiting the structure. Comparative studies are required to improve the capacity to distinguish between pseudo and genuine mining damage.

For the shaft lining, the search is for an inexpensive slip liner to keep the forces of strata movement away from the external wall of the shaft cylinder, but also for a simple means of protecting shaft stations against axial forces. There are as yet no suitable methods for measuring the shear forces transmitted to the shaft lining, nor the relative vertical movements of the rock face behind the lining. Drawing the structure into the grid calculation with finite elements, and the graphical method of calculating shaft deformation, which is based on the transmission of frictional forces, are two promising directions for investigation. Methods at present solved theoretically must be perfected by the evaluation of further series of observations.

In designing mine layouts, account should be taken of the finding that approximately 70% of all mining damage can be traced to the vertical component of ground movement – namely, subsidence. To reduce the amplitude of subsidence, consideration should be given to changing from caving to stowage or chamber working in areas where disruption of the drainage system or significant changes of level in railway tracks or canals might be expected. There is a need above all for layout planning to be augmented by pre-calculation of ground movement in cases where working has to proceed under a densely populated land surface. Attention needs to be directed in particular to unstable areas in the trough profile where – as at geological faults and cracks in the ground – experience has shown that each reworking of the strata produces further severe damage at the places thus overstrained. In a zone of high-rise buildings or at a main highway, subsidence steps in the ground constitute a safety hazard. It also appears necessary, when designing mine layouts, to check what influence the degree of reworking and the mining technique (caving, rate of advance, etc.) will have on dynamic strata and ground movements, if stepped-face methods and a particular sequence of working are adopted locally with the object of equalizing tensile and compressive, or concave and convex, ground deformations at the structure, or vertically elongating and shortening strata deformations at the shaft, not only in the long term but also in the interim.

Mining regulations should contribute to ensuring that avoidable mining damage simply does not occur: regional development plans should have regard to zones of faulting and earth cracking and provide that sensitive structures shall not be erected at these places of risk. Areas rich in high-grade minerals should not be densely built up. Building undertakers in mining districts should be required to adapt their buildings, both in siting and in construction, to mining stresses to be expected later. The mining industry, for its part, should preferably pay for structural precautions, if damage can thereby be reduced, rather than leave everything to the later settlement of damages and compensation. In areas of flat terrain with a high ground-water table, low-subsidence mining methods should be the rule. The land-owner should everywhere have the right to inspect the records of ground-movement measurements undertaken by mine surveyors, as this facility, taken together with other legally required or jointly agreed arrangements, serves the aim of improving neighbourly relations between mining and land-ownership through greater trust and an equitable sharing of loss, advantage, and risk.

Appendix I Building Standards*

Preliminary Note

The present position in law is that the mining operator is free to decide whether or not to observe these standards and, if he does, to exceed or fall short of the individual recommendations in them. If the builder desires the mining operator to reimburse the extra costs arising from observing these standards, he must secure, before beginning construction, the written agreement of the mining operator to the measures proposed and to the costs arising out of them.

1. Mining and the Building Ground

If large, continuous cavities are created underground, as for example in coal-mining, the layers of rock over the deposit generally break and subside. The subsidence can be reduced in amount and severity by filling-in, or “stowing”, the mine cavity with waste material.

In the course of subsiding, the land surface deforms into a generally flat, plate-shaped trough, the edge of which travels ahead of the mine working.

The movement in the subsidence trough can be analysed into a vertical and a horizontal component. In the vertical direction there are subsidence movements alone, which increase in magnitude from the edge of the trough to the middle. In the horizontal direction there occurs in the margin area of the subsidence trough, when the centre of curvature of the subsidence profile lies below, a stretching, or “extension”, of the ground. In the interior of the trough, where the centre of curvature lies above, there occurs a contraction, or “compression”, of the ground. As mining advances, the zones of extension and compression travel with the subsidence trough.

The extent and nature of the deformation at the surface depend on the size of the cavities created by mining, their depth below ground, the nature of the overlying strata, and the thickness of the waste material stowed.

* Extract from West German Building Standards, Issued 10 September 1963

2. Load Assumptions

2.1 General

The effects of mining may simultaneously cause a structure to subside, tilt, bend, and be extended or compressed, with extension and compression sometimes alternating.

2.2 Forces Arising from Tilting of the Structure

An insufficiently reinforced structure must be of such dimensions as to absorb any directional horizontal force, besides possible other horizontal forces, of a magnitude equivalent to 1% of all loads acting vertically on the horizontal section in question.

2.3 Bending Forces

The smallest radius of curvature ρ_z of a subsidence trough is around 500 m. In estimating the uneven subsidence movements and vertical bending caused to a structure by mining, the radius of curvature ρ_z of the subsidence trough can in general be taken as 2000 m for convex curves and 5000 m for concave curves. The bending stress to be expected when foundations are left partially unsupported through steps developing in the ground is something to be investigated for borderline cases.

2.4 Horizontal Tensional and Compressive Forces

The frictional forces F_t transmitted to the structure can be ascertained by applying the coefficient of friction $\tan \rho = \frac{2}{3}$. In doing this, account must be taken, on the underside, of all vertical support forces F_n , and at the sides, of the earth pressure

$$E = \gamma \frac{h^2}{2} \tan^2 \left(45^\circ - \frac{\rho}{2} \right)$$

and any horizontal bearing forces.

Any interlocking with the foundation soil through pump sumps, bucket elevator or lift wells, etc. is especially to be allowed for.

The earth resistance E_p acting on whichever is the front side of the structure is to be taken – making allowance for cohesion – as being

$$E_p = \gamma h^2 \tan^2 \left(45^\circ + \frac{\rho}{2} \right).$$

With very light structures, adhesion stress must be allowed for at a scale of 1 Mg/m^2 of the surface in contact.

2.5 Direction and Limit Values of Forces

In proportioning structures it should be assumed that all forces set off by mining can act in any given direction, and that extension and compression, convex and concave flexure can occur anywhere in the structure.

3. Degree of Mining-damage Precautions

3.1 General

The damaging effects of mining on structures can be limited or prevented by constructional measures (precautions against mining damage).

In the choice of preventive constructional measures, three stages of precautions may be distinguished, of which Stages 1 and 2 are partial precautions, as they are not enough to exclude mining damage completely in all circumstances.

3.2 Stage 1 Precautions

In areas which in the view of the mining operator are threatened by the effects of mining, preventive measures as laid down in Sects. 4 and 5 below are generally sufficient, unless he considers more far-reaching precautions essential (cf. Sect. 3.1 above).

3.3 Stage 2 Precautions

Within this category belong all cases where the general measures of Stage 1 are insufficient – for example, on account of some special sensitivity of the structure concerned (cf. Sect. 6 below).

3.4 Stage 3 Precautions

This stage – total precautions – requires the structure to be protected against every type of damage through deformation resulting from the effects of mining.

4. General Considerations for the Positioning and Design of Structures

4.1 Planning and Positioning of Structures

4.1.1 Choice of Site. In the choice of building land, attention should be paid, within the town-planning framework, to the geological circumstances and the mining-induced deformations of the ground. This information should be obtained in good time from the mining operator, on submission of the general planning documents, before precise architectural drawings are set in hand.

4.1.2 Positioning the Main Axes. The longer horizontal dimension of a structure, and also the crane and track layout, should so far as possible run parallel to the strike of the seam.

4.1.3 Shape and Size. The aim should be simple buildings with as little division as possible. Attached and linking structures like garages, porter's lodges, and communicating wings must be avoided wherever possible.

In an area of mining subsidence, high buildings with a small ground area are to be preferred to lower and wider structures. All structures should therefore generally be no longer or wider than 30 m, or at the most 35 m; otherwise they should be divided by gaps into separate units (Sect. 4.4.3 below). In structures with steel frames, however, greater lengths of up to 50 m may be permitted.

4.2 Foundations

In general, a shallow foundation is to be preferred to a deep one, despite the greater natural settlement.

4.3 Type of Construction, and Building Materials

The type of construction and the building materials must be selected to suit the special demands imposed by the effects of mining.

4.4 Compensating for Ground Movement at the Structure

4.4.1 Compensating for Vertical Movements of the Ground Under a Structure. Structures which fracture at their weakest spot under major distortion must be given a statically determinate support or be so stiffened that they can bridge any unsupported stretches of ground that may develop (Sect. 2.3 above). Structures with little intrinsic rigidity should preferably be made as flexible as possible – i.e., yielding elastically or plastically – to enable them to accommodate vertical distortions of the ground built on.

4.4.2 Allowing for Extension and Compression in the Building Ground. Extension and compression in the ground under a structure should be rendered harmless so far as possible by providing the structure, or parts of it, with longitudinally displaceable supports and possibly also with articulated joints. Between a deep foundation and the structure there should be a slip plane.

4.4.3 Distance Between Buildings, and Width of Joints Dividing Them. Neighbouring buildings are to be sited at a distance from each other such that, if the ground under them experiences compression, or if they become tilted, they will not damage each other.

4.5 Structural Design

4.5.1 All separation joints should run in as straight a line as possible. They should pass through the whole structure, including roof, facing, and foundations.

4.5.2 Door and window lintels, and all structural members the supports of which can shift on movement of the ground, must be given an extra-large bearing length.

4.5.3 Girderless concrete ceilings and masonry vaults or arches are unacceptable in structures secured under Stage 1 or 2 precautions.

4.5.4 The seatings of machinery, boilers, furnaces, single chimneys, etc. standing directly on the ground should be separated from the remainder of the structure and its foundations by gaps of adequate width (Sects. 4.4.1 and 4.4.3 above).

4.5.5 If there are any facilities in a structure which cannot tolerate tilting of that structure, such as lifts, crane rails, machinery and the like, their anchoring must be made adjustable.

4.5.6 Sealing a structure against ground water always creates difficulties in an area of mining subsidence. If at all possible therefore, the cellar floor should be laid above the level of the higher water-table expected by the mine surveyors or, if connection with a drainage system is possible, be provided with pumping facilities.

5. Special Construction for Stage 1 Precautions

5.1 Masonry Structures

5.1.1 In masonry structures the lowest floor should always be in jointless reinforced concrete.

5.1.2 Absorption of tensional and compressive forces in the plane of the foundation or of the lowest floor (e.g. the cellar floor): precautions in the plane of the

foundation are effected by placing a reinforced-concrete slab under the cellar bottom, so stiffened that it can absorb the expected longitudinal forces arising from extension and compression but not the vertical bending moments. Strip foundations under the slab are separated from it by slip joint (a card-board layer).

5.1.3 With masonry structures of three or more full storeys it is recommended that, at every floor, tie-beams be placed in all enclosing walls, all floor-bearing internal walls, and all min 25 cm thick, bracing transverse walls. These tie-beams are to be joined together at their intersections in a tension- and compression-proof manner and anchored to the floors. They are usually manufactured of reinforced concrete and can usefully be combined with window lintels.

5.2 Skeleton Structures and Continuous Girders

In skeleton structures the girders should be joined to the stays and bearers with metal straps, and in the case of reinforced-concrete frames it should be ensured that there is an adequate number of reinforcing rods over the stays. All large areas of glass should be protected against damage by means of suitable joints or an articulated suspension.

5.3 Stiff Slabs

Stiff slabs of a structure, of great length and with considerable rigidity in a vertical direction, should be divided up by additional separation gaps, unless they have statically determinate supports.

5.4 Frames

For the design of frames, cf. Sect. 4.4.2 above.

Columns directly endangered by horizontal movements of the ground should usually be attached top and bottom with hinged joints.

5.5 Warehouses and Similar Engineering Structures

To avoid costly spacing at the truss level, use is made of bearer units with statically determinate supports.

5.6 Gutters and Pipes

With all pipes and gutters serving to carry away fluids, except those under pressure, falls must be 1% greater than strictly necessary. All mains, with the exception of lead piping, must incorporate adequate expansion arrangements – for

example, at joints between buildings, at bends, and at places where cracking may be expected, likewise where mains in the ground meet buildings. Pipes for gas, water, etc. should if possible be welded at joins or be provided with yielding sleeve joints.

5.7 Interior Facilities

Machinery facilities, including lifting gear, rails, conveyor mechanisms, leaf or sliding gates, and pipes for heating, gas, or fluids, must be able to adjust to deformation of the ground and to consequent distortion of the structure.

5.8 Structures Standing in Ground Water

If a cellar cannot be sited above ground-water level, it must be provided at its underside with arrangements to secure it against extension (cf. Sects. 2.4 and 4.4.2 above).

Separation gaps between two reinforced-concrete buildings should be sealed off with two folded sheets of metal, preferably copper.

5.9 Bridges

Bridges should incorporate facilities to enable jacking up later (cf. also Sect. 6.7 below) and be statically determinate in their supports. Three-hinged arches and others reckoned as such are unacceptable, however. To compensate for horizontal ground movement, gaps at horizontally shifting bearings must allow for play of $\pm 1\%$ of the distance to the nearest bearing or junction, with a minimum of ± 20 cm.

In designing the bridgeway deck, it should be borne in mind that differential tilting of piers and abutments will produce distortion.

6. Special Standards for Structures in Stage 2 Precautions

6.1 General

For Stage 2 precautions the prescriptions of Sect. 5 apply unless otherwise specified below.

6.2 Masonry and Skeleton Structures

Masonry structures and other structures with bearing masonry walls and free-laying ceiling joists are unacceptable. They should be replaced by skeleton structures.

6.3 Frame Supports

With frame supports, in contrast to Stage 1 precautions, utilizing spring deformation to avoid the use of hinge joints (point or line rocker-bearings, etc.) is unacceptable.

6.4 Joints

The joints between two structures or parts of a structure may not show any throw, vertically or laterally, but must be in a single plane.

6.5 Flexure of the Building Ground

The effect of flexure of the building ground (cf. Sect. 2.3 above) must be statically determined for the bearing members. For this purpose it is sufficient if flexure of the building ground in accordance with Sect. 2.3 is assumed in the direction of both main axes and not diagonally as well. With skeleton structures the bracing effect of bricking-in is not included in the formulation. For bearing masonry walls such a determination is not required for the present (Sect. 8.1 below).

6.6 Allowing for Extension and Compression of the Building Ground

Forces resulting from extension and compression of the building ground must be absorbed throughout the whole extent of the ground plan in a single horizontal plane (a reinforced-concrete slab above the foundations).

6.7 Adjustability of Structures

Facilities for relevering the whole structure must include in the jacking possibilities all essential parts of the structure, the protection of which makes classification in the higher stage of precautions necessary.

The foundations of these structures are to be made wide enough to permit points of support to be shifted laterally without exceeding the permissible edge compression, and also to permit jacking devices to be set up.

7. Special Standards for Structures in Stage 3 Precautions

7.1 Varieties of Total Precautions

For areas of application, cf. Sect. 3 above. In total precautions, four varieties can be distinguished.

7.1.1 Single-area support is suitable for structures having a plan area of relatively modest extent – e.g., bins and silos, machinery beds of compact shape, calcining kilns, etc.

7.1.2 Two-area support can be employed in the main only for structures of elongated ground plan.

7.1.3 Three-point support is particularly suitable for structures having, above the foundations, a continuous slice covering the whole ground area, the bearers of which require to be in a triangular arrangement so far as possible.

The three separate foundations of this category require either the maintenance of proper spacing or special sliding bearings, in order that on horizontal deformation of the ground only the frictional forces of the bearings are transmitted to the structure. The sliding bearings can also be replaced by rocking pillars or walls.

7.1.4 A structure should be divided up by means of gaps or hinge joints into separate, fully secured units, each of which can be supported in accordance with Sects. 7.1.1 to 7.1.3 above.

7.2 Loading the Building Ground, and Statical Determination

If the building ground is very hard and brittle, it is advisable, in the case of heavy structures exerting a high ground compression, to introduce a compensating layer of plastic clay or the like, to allow the foundation to cut into the ground in the event of uneven subsidence. This measure is successful, however, only if the compensating layer remains permanently moist enough.

If the development of subsidence steps is expected (cf. Sects. 1 and 2.3 above), a ground compression value as high as possible within the permissible range should be used for the calculation, in order that a part of the difference in subsidence may be rendered harmless by virtue of the foundation's pressing into the back of the subsidence steps.

Limit cases of the losing of support from foundations (free standing areas) as a result of ground curvature, must be investigated in the direction of the structure's principal axes and also diagonally. Bending moments should be followed up in each case to the point of change-over in transverse load.

8. Acceptable Stresses, Allowing for the Effects of Mining

8.1 Structural Steel:

Type 33	20 kN/cm ²
Type 37	22 kN/cm ²
Type 52 – 53	34 kN/cm ²

8.2 Reinforcing Steel:

Reinforcing steel, category I	22 kN/cm ²
Reinforcing steel, category II	34 kN/cm ²
Reinforcing steel, category III	40 kN/cm ²
Reinforcing steel, category IV	50 kN/cm ²

8.3 Reinforced Concrete:

Double the values of acceptable stresses stated in standard specifications. In the case of pre-stressed concrete: the acceptable stresses given in the standard specifications (no increases).

8.4 Unreinforced concrete structures:

The values given in the standard specifications (no increases).

Appendix II Calculation of Ground Movement by the Computer System of Ruhrkohle AG

INPUT

co-ordinates in m

panel

point	y	x	z
100	11400	24000	-700
101	12000	24230	-700
102	12220	24000	-740
103	11500	23580	-740

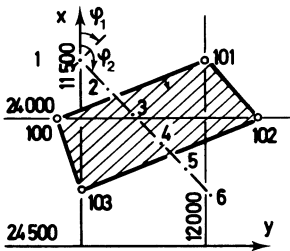
surface points

point	y	x	z
1	11500	24230	20
2	11600	24130	20
3	11710	24015	20
4	11810	23915	20
5	11915	23810	20
6	12020	23700	20

data for calculation

thickness of seam	2.0	m
subsidence factor	0.90	
time factor	1.00	
limit angle	60	gr
angle of dip	10	gr

mean depth	740	m
radius R of critical area	530	m
rotatable cross:		
distance P - P ₁	20	m



OUTPUT

point	subsidence cm	tilt mm/m		curvature radius km		displacement cm		extension (+) direction compres. (-) gr		mm/m	
		phi 1	phi 2	phi 1	phi 2	phi 1	phi 2	phi 1	phi 2	phi 1	phi 2
1	35	-0.2	-0.6	-650	70	-3.5	-10.2	-0.3	2.7	50	150
2	75	-0.5	-1.6	-500	450	-8.7	-28.0	-0.4	0.4	50	150
3	120	-0.1	-0.5	-50	-38	-1.8	-8.7	-3.8	-5.1	50	150
4	105	0.3	1.8	-200	-60	5.3	31.1	-1.0	-3.2	50	150
5	55	0.2	1.1	-800	90	3.5	20.3	-0.2	2.1	50	150
6	30	0.1	0.6	-1000	65	1.8	11.2	-0.2	2.9	50	150

seam inclination 0 - 20 gr

subsidence $k_z(r)$ for point P

↓
tilt $k'_z(r)$ → prop. displacement

↓
curvature $k''_z(r)$ → prop. strain

$$v_x = 0.033 R v'_z$$

cm m mm/m

$$\epsilon = 362 R 1/\rho_z$$

mm/m m m

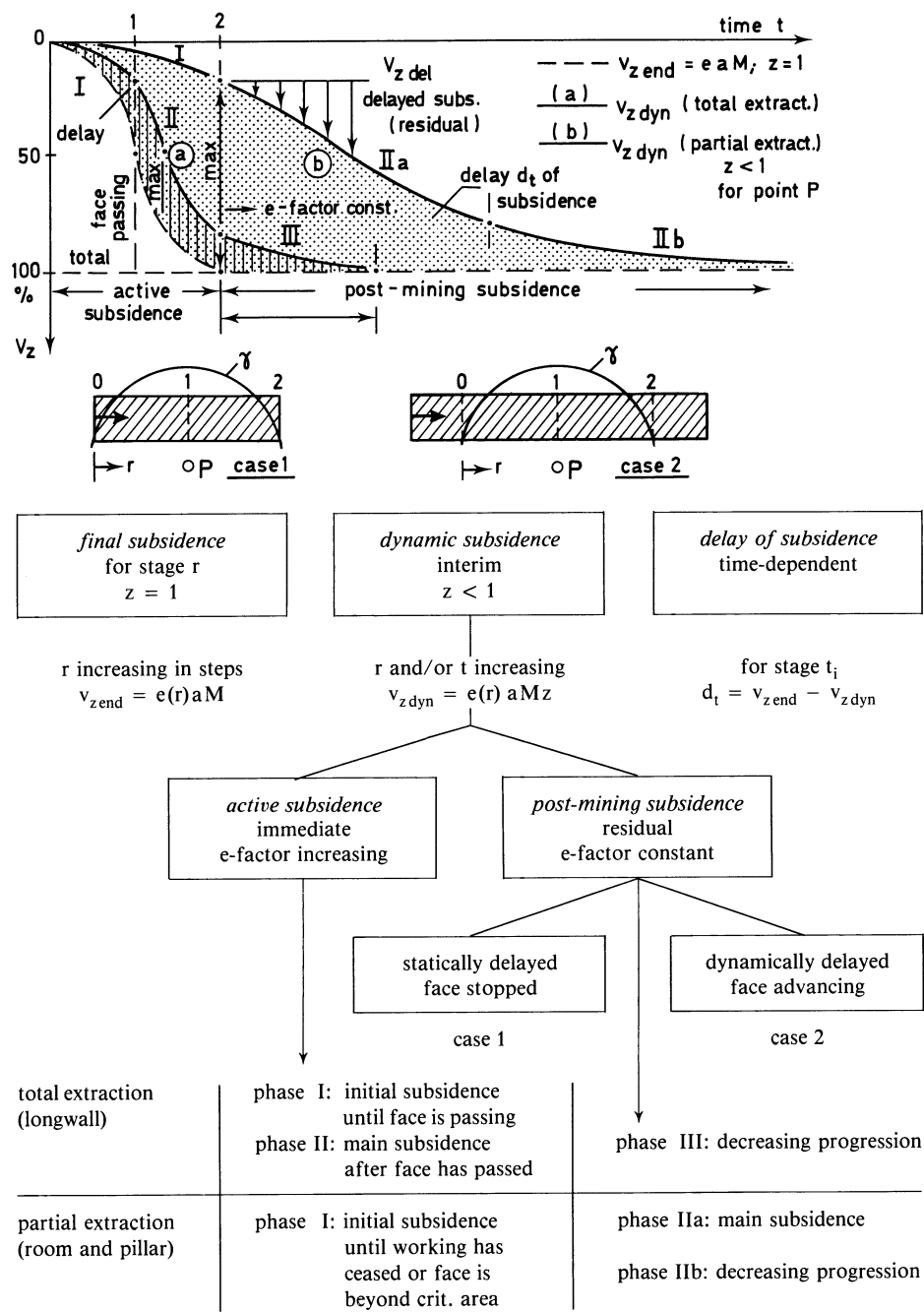
seam inclination 20 - 80 gr

subsidence $k_z(r)$ for P and P₁ - P₄

↓
tan μ tilt and curvature

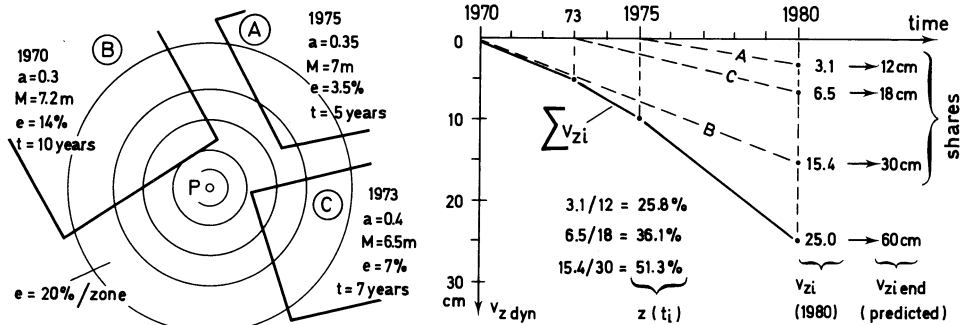
↓
displacement → strain

Appendix III Classification of Interim Ground Movement in Terms of Time t, Influence Factor e and Extraction Stage r

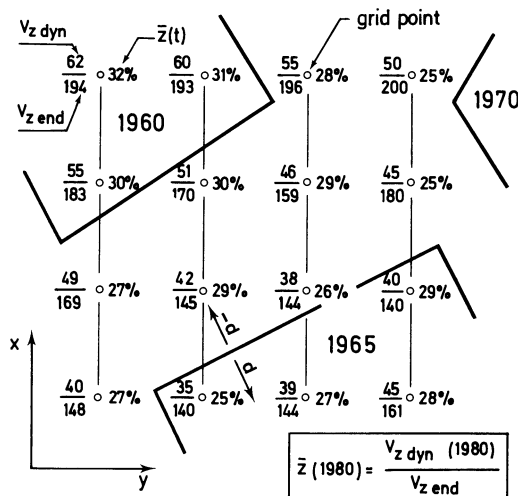


Appendix IV Determination of the Time Factor in Mining with Yielding, Permanent Pillars

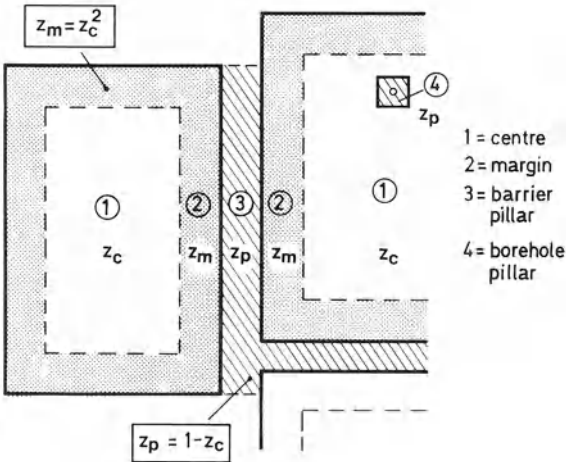
1. In accordance with Eqs. (307), and (308), Section 10.2.1.3. Subsidence at the surface point P measured 1980: 25 cm.



2. By placing a grid with 100 m lines, and representing the amounts of subsidence measured $v_{z \text{ dyn}}$ as a proportion of final subsidence pre-calculated $v_{z \text{ end}}$, Eq. (308). Solution of the function $z = f(t, d)$ by means of regression analysis.

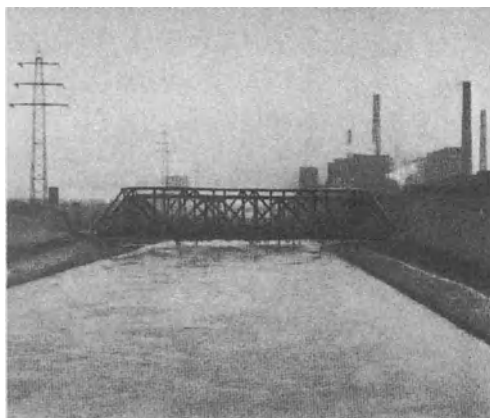


3. By using differing time factors for the inner zone and the margin of the extraction area (see example in Section 10.3.5).



4. By extrapolation into the future by means of Eq. (323), see Fig. 251, right.

Appendix V Examples of Mining Damage



Figs. 1 and 2. Before and after raising a bridge and embankments at a canal where the ground had subsided while the water level remained the same



Figs. 3 and 4. Before and after rebuilding the end house of a terrace damaged by concave curvature in a subsidence trough. (Note manner of correcting for tilt at join with next house)



Fig. 5

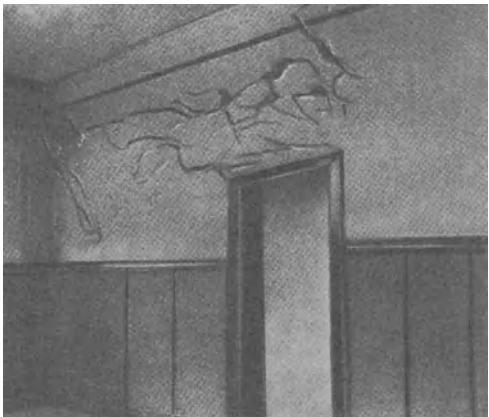


Fig. 6

Fig. 5. Gap created between gables (left) in a terrace of houses by convex ground curvature
Fig. 6. Shear cracks from mining subsidence which is affecting a partition wall



Fig. 7

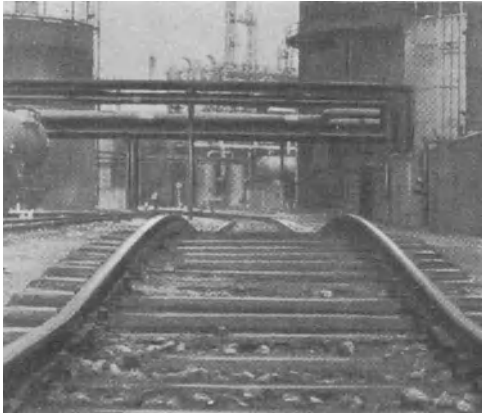


Fig. 8 ▲



Fig. 9 ►

Figs. 7, 8 and 9. Mining damage to external masonry walls. Causes: ground flexure (*left*), compression (*top right*), and extension (*bottom right*)



Figs. 10 and 11. Damage to railway tracks caused by ground compression: vertical distortion (*left*) and lateral distortion (*right*)



Fig. 12

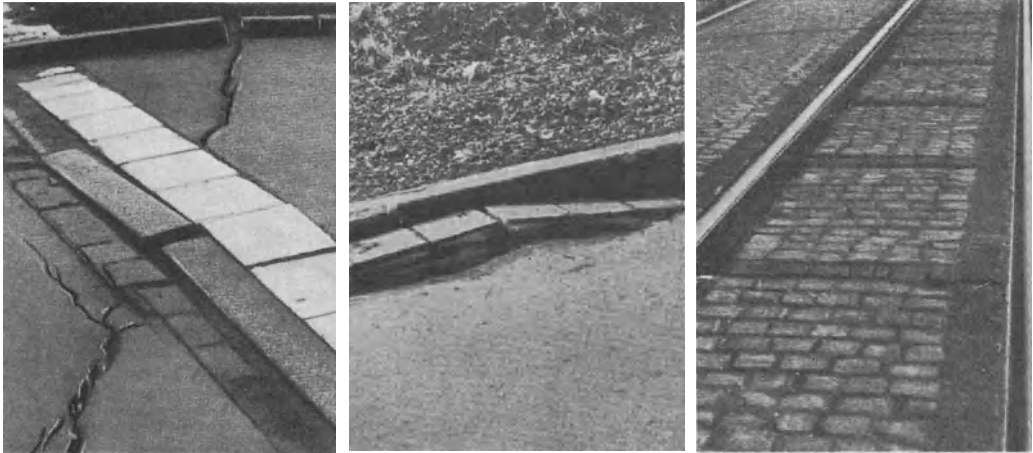
Figs. 12, 13 and 14. Compression damage (*left, and bottom left*) and extension damage (*bottom right*) to pipes laid in the ground



Fig. 13



Fig. 14



Figs. 15, 16 and 17. Damage to streets from ground extension (*left*) and ground compression (*centre*); and displacement of tramway rails (see gauge rods) in street cobbles (*right*)



Fig. 18



Fig. 19

Fig. 20

Fig. 18. House damage over a subsidence step. *Left:* a house corner dropping. *Right:* a semi-basement dragged sideways by shear movement

Fig. 19. An enclosure wall damaged by a step in the ground (road level already repaired)

Fig. 20. Wide tension crack in ground over the face edge of a shallow working





Fig. 21. A stream overflows its banks and floods the subsidence zone



Fig. 22. Subsidence funnel over a potash working collapsed by water penetration

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Note on the SI Units Used

Use is made not only of the basic SI units (e.g., m for metre, kg for kilogram) and the derived units (e.g., N for newton, Pa for pascal) but also of some extended international units like kN, MPa, N/cm², cm, mm, and gr, which make the magnitudes concerned clearer.

Force

$$1 \text{ N (newton)} = 1 \text{ kg m/s}^2$$

$$1 \text{ kN} = 1000 \text{ N}$$

Pressure, tension, stress (perpendicular to a surface)

$$1 \text{ Pa (pascal)} = 1 \text{ N/m}^2$$

$$1 \text{ MPa} = 1,000,000 \text{ Pa} = 1 \text{ N/mm}^2 = 100 \text{ N/cm}^2 = 10 \text{ bar}$$

$$1 \text{ N/cm}^2 = 0.01 \text{ MPa} = 0.1 \text{ bar}$$

Shear stress (tangential to a surface)

$$1 \text{ N/m}^2$$

$$1 \text{ N/cm}^2 = 10,000 \text{ N/m}^2 = 10 \text{ kN/m}^2$$

Weight

$$1 \text{ kg (kilogram)} = 1000 \text{ g}$$

$$1 \text{ Mg} = 1000 \text{ kg (1 tonne)}$$

Linear measurement

$$1 \text{ m (metre)} = 10 \text{ dm} = 100 \text{ cm} = 1000 \text{ mm}$$

$$1 \text{ cm} = 0.01 \text{ m}$$

Linear strain (extension, compression)

$$1 \text{ m/m}$$

$$1 \text{ mm/m} = 0.001 \text{ m/m} = 1\text{‰}$$

Area

$$1 \text{ m}^2 = 10,000 \text{ cm}^2$$

Volume

$$1 \text{ m}^3 = 1,000,000 \text{ cm}^3$$

Angular measurement

1 rad (radian) = 1 m of arc/1 m of radius

1 gr (grade) = $\pi/200$ rad = $90^\circ/100$

Prefixes to SI units

M (mega-) = 10^6

k (kilo-) = 10^3

d (deci-) = 10^{-1}

c (centi-) = 10^{-2}

m (milli-) = 10^{-3}

List of Symbols

a	subsidence factor; year	g	acceleration of free fall (9.81 m/s^2); weights; in- fluence function
A	area (subscripts: critical, subcritical)	gr	grade (angular)
dA	elementary area, surface element	G	weight force (in N); shear modulus; modulus of rigidity; impulse
b	width	h	depth; height
c	convergence; cohesion	H	depth of working
c_d	daily increment of convergence	I	polar moment of inertia, impulsive moment
C	total convergence	k_z	infinitesimal subsidence influence
C_b	bedding index	k_{xy}	radial displacement influence
D	deformation resistance	K_z, K_{xy}	components of finite influence
d	diameter; thickness; day; distance; differential	l, L	length; spacing
d_t	delay, delay factor (short: d), retardation	l_s	length of panel along strike
e	influence factor or development factor; Napier constant 2.718...	l_d	length of panel downdip
de	effective influence of an elementary area	m	index of transverse strain $1/\nu$; metre; mass in kg
E	modulus of elasticity (Young's modulus); earth pressure $< E_p$	mo	month
E_0	earth pressure at rest	M	thickness of seam mined; moment
E_p	passive earth pressure, earth resistance, maximal	n	voids volume (porosity); safety coefficient
E_s	stiffness index	N	rigidity, bend-resistance; newtons
f	height of arch; nodal- point force	p	compressive force; normal pressure; cover load; uniformly distributed load (Pa)
F	force in N; point load, concentrated load	P	a point
F_t	frictional force (tangential)	q	force of reaction
F_n	normal force (perpendicular)	Q	transverse load, shearing force; quantity
F'_n	load per running metre of length (N/m)		

r	radius; position of face	$v''_{z\max}$	maximum flexure/curvature
R	radius of critical area	$v''_{z\text{Max}}$	greatest possible flexure/curvature
s	horizontal distance between points; degree of flexibility	dv_z	subsidence differential
S	cross-section	dv_{xy}	displacement differential
t	time; duration	$v_{z\text{dyn}}$	short-term subsidence, interim
u	infinitesimal displacement, horizontal	$v_{z\text{end}}$	final, long-term subsidence
v_t, v_d	face advance in metres per time, day; rate of deformation (short: v)	V	volume
$\bar{V}$	three-dimensional displacement vector	V_L	sacrifice (loss) of mineral, extraction forgone
v	(short for v_{xy}): horizontal displacement	V_w	extraction ratio (worked out volume in %)
v_x, v_y	components of displacement, horizontal movement	w	sag; infinitesimal subsidence
$v_{\max}$	maximum displacement	W	work; moment of resistance
$v_{\max 1}$	max. displacement over lower face edge	z	time factor; depth axis
$v_{\max 2}$	max. displacement over upper face edge	α	angle of dip
v_{Max}	greatest possible displacement (full displacement)	$\tan \alpha$	gradient of deformation-line (mm/m per 100 m)
v_z	subsidence (negative value if distinguished from heave), vertical movement	β	angle of break
$v_{z\max}$	maximum subsidence	γ	limit angle (γ_L, γ_H); angle of slide; weight force (in N/m^3)
$v_{z\text{Max}}$	greatest possible subsidence (full subsidence)	γH	cover load (in N/m^2), vertical
$\Delta v_z(t)$	rate of subsidence (short: v_i)	δ	movement (displacement) of nodal points; angle
v'_z	tilt, subsidence slope, from horizontal (in mm/m)	$+\epsilon, -\epsilon$	extension, compression horizontal (in mm/m)
v'_{xz}, v'_{yz}	tilt, deviation, from vertical	$\pm \epsilon_1, \pm \epsilon_2$	principal linear strain, principal extension or compression (in mm/m)
$v'_{z\max}$	maximum/principal tilt, slope	$\pm \epsilon_x, \pm \epsilon_y$	extension or compression in the x (or y) direction
$v'_{z\text{Max}}$	greatest possible tilt, slope	$\pm \epsilon_{\max}$	maximum extension or compression
v''_{z1}, v''_{z2}	principal flexures of the ground surface	$\pm \epsilon_{\text{Max}}$	greatest possible (full) extension or compression
$v''_{z(x)}, v''_{z(y)}$	flexure/curvature in the x (or y) direction	ϵ_{dyn}	short-term, interim strain
v''_{xz}, v''_{yz}	curvature of the shaft axis	ϵ_{end}	long-term, final strain
		$\pm \epsilon_z$	vertical extension (expansion) or vertical compression (contraction, compaction)

ε_{Br}	rupture-deformation, breaking limit	σ, σ_n	normal stress (in N/m^2)
Δs	horizontal linear deformation/change	$\sigma_{z,max}$	abutment pressure, additional pressure
ζ	zonal angle from the vertical	σ_{Ap}	spezific pillar load
η	viscosity; angle of inclination; zonal angle from the seam normal	σ_1, σ_2	principal (main) stress, normal
λ	compression index, ratio; slenderness index	σ_{Br}	breaking stress; breaking strength (also: σ_B)
μ	nadir angle; coefficient of friction	τ	shear stress
ν	Poisson's ratio (0–0.5); zenith angle; safety factor	τ_{Br}	shear strength
π	pi (circle ratio 3.14159...)	τ_0	flow point
ρ, ρ_z	density (in kg/m^3), angle of inner friction; radius of vertical curvature	φ	angle of direction horizontally
		ω	trough angle
		Δ	increment (in magnitude), difference
		$\triangleq$	to be equivalent to

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T. J. Blachut, A. Chrzanowski, J. H. Saastamoinen

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**Rock Mechanics
and
Rock Engineering**

Continuing Rock Mechanics
Felsmechanik
Mécanique des Roches

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Mécanique des Roches"

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Beginning with 1983 "Rock Mechanics and Rock Engineering" appears under a new Editorial Board, which has set itself the task of promoting the discipline of rock mechanics by means of this international journal. The editors are fortunate to do this by building upon a proud tradition starting with the founding of this technical journal in 1929.

The express wish of the new editors is to see a close connection between theory and practice. They will endeavour to preserve a balance between articles dealing with fundamental of rock mechanics, with engineering geology and with problems arising from constructions practice. The new title of the journal has been chosen to reflect this goal.

Fields of interest: Rock mechanics in all its varied aspects including laboratory testing, field investigations, computational methods and design principles. Applications in tunnelling, rock slopes, large dam foundations, mining engineering and engineering geology.

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